KASIDIH HIGH SCHOOL

POST: SAKCHI, JAMSHEDPUR

(Affiliated up to +2 level to C.B.S.E., New Delhi)

Affiliation No. - 3430143

XII-MATHEMATICS

Chapterwise Topicwise Worksheets with Solution

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Website: www.jemfoundation.com; E-mail: principal.khs@jemfoundtion.in

Ph. 0657-2428165

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CBSE TEST PAPER-01

CLASS - XII MATHEMATICS (Relations and Functions)

Topic: <u>Relation and function</u>

1. A Relation is said to be Reflexive if ------------ every $a \in A$ where A is non empty set. [1]

2. A Relation is said to be Symmetric if ------------ $\forall$ a,b,$\in$ A [1]

3. A Relation is said to be Transitive if ------------ $(a,c) \in R$, $\forall$ a,b,c $\in A$ [1]

4. Define universal relation? Give example. [1]

5. What is trivial relation? [1]

6. Let T be the set of all triangles in a plane with R a relation in T given by [4]
   $R = \{(T_1, T_2): T_1$ is congruent to $T_2\}$.
   Show that R is an equivalence relation.

7. Show that the relation R in the set Z of integers given by [4]
   R = {(a, b) : 2 divides a-b}.

8. Let L be the set of all lines in plane and R be the relation in L define if [4]
   $R = \{(l_1, L_2): L_1$ is $\perp$ to $L_2\}$.
   Show that R is symmetric but neither reflexive nor transitive.

9. Check whether the relation R defined in the set {1, 2, 3, 4, 5, 6} as [4]
   R = {(a, b): b = a+1} is reflexive, symmetric or transitive.

10. Let L be the set of all lines in Xy plane and R be the relation in L define as [4]

    $R = \{(L_1, L2): L_1 \| L_2\}$
    Show then R is on equivalence relation.
    Find the set of all lines related to the line Y=2x+4.

CBSE TEST PAPER-01

CLASS - XII MATHEMATICS (Relations and Functions)

[ANSWERS]

Ans 1. $(a, a) \in R$

Ans 2. $(a, b) \in R$, $(b, a) \in R$

Ans 3. $(a, b) \in R$, and $(b, c) \in R \Rightarrow (a, c) \in R$.

Ans 4. A Relation R in a set A called universal relation if each element of A is related to every element of A. Ex. Let = {2,3,4}

$R = (A \times A) = \{(2,2),(2,3)\ (2,4)\ (3,2)\ (3,3)\ (3,4)\ (4,2)\ (4,3)\ (4,4)\ \}$

Ans 5. Both the empty relation and the universal relation are some time called trivial relation.

Ans 6. R is reflexive, since every Δ is congruent to itself.

$(T_1T_2) \in R$ similarly $(T_2T_1) \in R$

$\Rightarrow$ since $T_1 \cong T_2$

$(T_1T_2) \in R$, and $(T_2,T_3) \in R$

$\Rightarrow$ $(T_1T_3) \in R$ Since three triangles are congruent to each other.

Ans 7. R is reflexive , as 2 divide a-a = 0

$((a,b) \in R$,(a-b) is divide by 2

$\Rightarrow$ (b-a) is divide by 2 Hence $(b,a) \in R$ hence symmetric.

Let $a,b,c \in Z$

If $(a,b) \in R$

And $(b,c) \in R$

Then a-b and b-c is divided by 2

$\Rightarrow$ a-b +b-c is even

$\Rightarrow$ (a-c is even

$\Rightarrow$ (a,c) $\in$ R

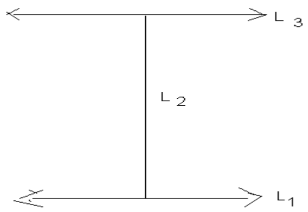
Hence it is transitive.

Ans 8. R is not reflexive , as a line L_1 cannot be $\perp$ to itself i.e $(L_1,L_1) \notin R$

$\Rightarrow L_1 \perp L_2$

$\Rightarrow L_2 \perp L_1$

$\Rightarrow (L_2,L_1) \in R$



$L_1 \perp L_2$ and $L_2 \perp L_3$

Then L_1 can never be $\perp$ to L_3 in fact $L_1 \parallel L_3$

i.e $(L_1,L_2) \in R$, $(L_2,L_3) \in R$.

But $(L_1, L_3) \notin R$

Ans 9. R = {(a,b): b= a+1}

Symmetric or transitive

R = {(1,2) (2,3) (3,4) (4,5) (5,6) }

R is not reflective , because $(1,1) \notin R$

R is not symmetric because $(1,2) \in R$ but $(2,1) \notin R$

$(1,2) \in R$ and $(2,3) \in R$

But $(1,3) \notin R$ Hence it is not transitive

Ans 10. $L_1 || L_1$ i.e $(L_1, L_1) \in R$ Hence reflexive

$L_1 || L_2$ then $L_2 || L_1$ i.e $(L_1L_2) \in R$

$\Rightarrow (L_2,L) \in R$ Hence symmetric

We know the

L1||L2 and L2||L3

Then L1|| L3

Hence Transitive . y = 2x+K

When K is real number.

CBSE TEST PAPER-02

CLASS - XII MATHEMATICS (Relations and Functions)

<u>Topic:- Relation and function</u>

1. Prove that the function f: R → R, given by f(x) = 2x, is one – one. [1]
2. State whether the function is one – one, onto or bijective f: R → R defined by f(x) = 1+ x^2 [1]
3. Let S = {1, 2, 3} [1]

   Determine whether the function f: S → S defined as below have inverse.

   f = {(1, 2), (2, 1), (3, 1)}
4. Find gof f(x) = |x|, g(x) = |5x + 1| [1]
5. Let f, g and h be function from R to R show that (f + g) oh = foh = goh [1]
6. If a * b = a + 3b^2, then find 2 * 4 [1]
7. Show that the relation in the set R of real no. defined R = {(a, b) : a$\leq$ b^3 }, is neither reflexive nor symmetric nor transitive. [4]
8. Let A = N$\times$N and * be the binary operation on A define by (a, b) * (c, d) = (a + c, b + d) [4]

   Show that * is commutative and associative.
9. Show that if f: $R-\left\{\frac{7}{5}\right\}\rightarrow R-\left\{\frac{3}{5}\right\}$ is defining by f(x) = $\frac{3x+4}{5x-7}$ and g: $R-\left\{\frac{3}{5}\right\}\rightarrow R-\left\{\frac{7}{5}\right\}$ is define by g(x) = $\frac{7x+4}{5x-3}$, then fog = I_A and gof = I_B when $A=R-\left\{\frac{3}{5}\right\}$, $B=R-\left\{\frac{7}{5}\right\}$; I_A (x) = x, for all x$\in$ A, I_B(x) = x, for all x$\in$ B are called identify function on set A and B respectively. [4]
10. Let f: N → N be defined by f(x) = $\begin{cases} \frac{n+1}{2}, if\ \ n\ is\ odd\ \ for\ \text{all n} \in N \\ \frac{n}{2} if\ \ n\ is\ even \end{cases}$ Examine wether the function f is onto, one – one or bijective [4]

CBSE TEST PAPER-02

CLASS - XII MATHEMATICS (Relations and Functions)

[ANSWERS]

Topic:- Relations and functions

1. f is one – one as $f(x_1) = f(x_1)$
   $2x_1 = 2x_2$
   $x_1 = x_2$
   Prove.

2. Let $x_1, x_2 \in x$
   If $f(x_1) = f(x_2)$
   $1+x_1^2 = 1+x_1^2$
   $x_1^2 = x_1^2$
   $x_1 = \pm x_2$
   Hence not one – one
   $y = 1+x^2$
   $x = \pm\left(\sqrt{1-y}\right)$
   $f\left(\sqrt{1-y}\right) = 1+(1-y) = 2-y \neq y$

3. f(2) = 1 f(3) = 1,
   f is not one – one, So that that f is not invertible.

4. gof (x) = g [f(x)]
   = g [(x)]
   = $\left|5|x|-2\right|$

5. L.H.S = (f + g) oh
   = {(f + g) oh} (x)
   = (f + g) h (x)
   = f [h (x)] + g [h (x)]
   = foh + goh

6. $2 * 4 = 2 + 3\,(4)^2$
   $= 2 + 3 \times 16$
   = 2 + 48
   = 50

7. (i) (a, a) $\notin R$ as $a \le a^2$ Which is false R is not reflexive.
(ii) $a \le b^2$ and $b \le a^2$ Which is false R is not symmetric.
(iii) $a \le b^2$, $b \le c^2$ then $a \le c4$ Which is false

8. (i) (a, b) * (c, d) = (a + c, b + d)
= (c + a, d + b)
= (c, d) * (a, b)
Hence commutative

(ii) (a, b) * (c, d) * (e, f)
= (a + c, b + d) * (e, f)
= (a + c + e, b + d + f)
= (a, b) * (c + e, d + f)
= (a, b) * (c, d) * (e, f)
Hence associative.

9. gof (x) = $g\left(\frac{3x+4}{5x+7}\right) = \frac{7\left(\frac{3x+4}{5x+7}\right)+4}{5\left(\frac{3x+4}{5x+7}\right)-3} = x$

$$fog(x) = f\left(\frac{3x+4}{5x-3}\right) = \frac{3\left(\frac{7x+4}{5x-3}\right)+4}{5\left(\frac{7x+4}{5x-3}\right)-7} = x$$

Thus gof(x) = x, for all x $\in$ B
fog (x) = x, for all x $\in$ A
Which implies that gof = I_B
And Fog = I_A

10. $f(1) = \frac{n+1}{2} = \frac{1+1}{2} = 1$

$f(2) = \frac{n}{2} = \frac{2}{2} = 1$

f is not one – one
1 has two pre images 1 and 2
Hence f is onto
f is not one – one but onto.

CBSE TEST PAPER-03

CLASS - XII MATHEMATICS (Relations and Functions)

<u>Topic:-</u> Relations and Functions

1. Show that function f: N → N, given by f(x) = 2x, is one – one. [1]

2. State whether the function is one – one, onto or bijective f: R → R defined by f(x) = 3 – 4x [1]

3. Let S = {1, 2, 3} [1]

   Determine whether the function f: S → S defined as below have inverse.

   f = {(1, 1), (2, 2), (3, 3)}

4. Find got f(x) = |x|, g(x) = |5x -2| [1]

5. Consider f: {1, 2, 3} → {a, b, c} given by f(1) = a, f(2) = b and f(3) = c find f^{-1} and show that

   $(f^{-1})^{-1}$ = f [1]

6. If f(x) = x + 7 and g(x) = x – 7, x∈ R find (fog) (7) [1]

7. Show that the relation R in the set all books in a library of a collage given by R {(x, y) : x and y have same no of pages}, is an equivalence relation. [4]

8. Let * be a binary operation. Find the binary operation a * b = a – b + ab is [4]

   (a) Commutative

   (B) Associative

9. Let f: R → R: f (x) = 2x + 1 and g: R → R: g(x) = x^2 – 2 find (I) gof (II) fog [4]

10. Let A = R – {3} and B = R- {1}. Consider the function of f: A → B defined by f(x) = $\frac{x-2}{x-3}$.

    is f one – one and onto. [4]

CBSE TEST PAPER-03

CLASS - XII MATHEMATICS (Relations and Functions)

[ANSWERS]

Topic:- Relations and Functions

1. the function f is one – one, for
$f(x_1) = f(x_2)$
$2x_1 = 2x_2$
$x_1 = x_2$

2. is $x_1, x_2 \in R$
$f(x_1) = f(x_2)$
$3 - 4x_1 = 3 - 4x_2$
$x_1 = x_2$
Hence one – one
$Y = 3 - 4x$
$$x = \left(\frac{3-4}{4}\right)$$
$$f\left(\frac{3-4}{4}\right) = 3 - 4\left(\frac{3-4}{4}\right)$$
= y
Hence onto also.

3. f is one – one and onto, so that f is invertible with inverse $f^{-1} = \{(1, 1)\ (2, 2)\ (3, 3)\}$

4. fog (x) = f(g x)
$= f\{|5x - 2|)$
$= |5x - 2|$

5. $f = \{(1, a)\ (2, b)\ (3, c)\}$
$f^{-1} = \{\ (a, 1)\ (b, 2)\ (c, 3)\}$
$(f^{-1})^{-1} = \{(1, a)\ (2, b)\ (3, c)\}$
Hence $(f^{-1})^{-1} = f$.

6. (fog) (x) = f[g(x)]
$= f(x - 7)$
$= x - 7 + 7$
$= x$
(fog) (7) = (7)

7. (i) $(x, x) \in$ R, as x and x have the same no of pages for all $x \in$ R $\therefore$ R is reflexive.
(ii) (x, y) R
x and y have the same no. of pages
y and x have the same no. of pages
$\Rightarrow (y, x) \in R$
$\Rightarrow (x, y) = (y, x)$ R is symmetric.
(iii) if $(x, y) \in R$, $(y, y) \in R$
$(x, z) \in R$
$\therefore$ R is transitive.

8. (i) a * b = a – b + ab
b * a = b – a + ab
a * b $\neq$ b * a
(ii) a * (b * c) = a * (b – c + bc)
= a – (b – c + bc) + a. (b – c + bc)
= a – b + c – bc + ab – ac + abc
(a * b) * c = (a – b + ab) * c
= [(a – b + ab) – c] + (a – b + ab)
= a- b + ab – c + ac – bc + abc
a * (b * c) $\neq$ (a * b) * c.

9. (i) gof (x) = g[f(x)]
= g (2x + 1)
= $(2x + 1)^2 - 2$
(ii) fog (x) = f (fx)
= f (2x + 1)
= 2(2x + 1) + 1
= 4x + 2 + 1 = 4x + 3

10. Let x_1 $x_2 \in$ A
Such that $f(x_1) = f(x_2)$

$$\frac{x_1 - 2}{x_1 - 3} = \frac{x_2 - 2}{x_2 - 3}$$

$$x_1 = x_2$$

f is one – one

$$\frac{y}{1} = x\frac{x-2}{x-3}$$

$$x = \frac{2y-2}{y-1}$$

$$f\left(\frac{3y-2}{y-1}\right) = y$$

Hence onto

CBSE TEST PAPER-04

CLASS - XII MATHEMATICS (Relations and Functions)

<u>Topic:-</u> Relations and Functions

1. What is bijective function? [1]
2. f: R → R be define as f(x) = x^4 whether the above function is one – one onto, or other. [1]
3. Let S = {1, 2, 3} [1]

   Determine whether the function f: S → S defined as below have inverse.

   f = {(1, 3) (3, 2) (2, 1)}
4. Find gof f(x) = $8x^3$, g(x) = $x^{1/3}$ [1]
5. Let f, g and h be function from R + R. Show that (f.g) oh = (foh). (goh) [1]
6. Let * be a binary operation defined by a * b = 2a + b – 3. find 3 * 4 [1]
7. Show that the relation R defined in the set A of all triangles as R = {$(T_1, T_2) : T_1$ is similar to T_2}, is an equivalence relation. Consider three right angle triangles T_1 with sides 3, 4, 5. T_2 with sides 5, 12, 13 and T_3 with sides 6, 8, 10. Which triangles among T_1, T_2 and T_3 are relations?

   [4]
8. Determine which of the following operation on the set N are associative and which are commutative. [4]

   (a) a * b = 1 for all a, b ∈ N

   (B) a * b = $\frac{a+b}{2}$ for all a, b, ∈ N
9. Let A and B be two sets. Show that f: A ×B → B ×A such that f(a, b) = (b, a) is a bijective function. [4]

CBSE TEST PAPER-04

CLASS - XII MATHEMATICS (Relations and Functions)

[ANSWERS]

<u>Topic:-</u> Relations and Functions

1. A function f: X → Y is said to be one – one and onto (bijective), if f is both one – one and onto.

2. Let $x_1, x_2 \in R$
If $f(x_1) = f(x_2)$
$x_1^4 = x_2^4$
$x_1^2 = x_1^2$
$\pm x_1 = \pm x_2$
Not one – one
$y = x^4$
$x = \pm y^{1/4}$
$f(y^{1/4}) = y$
Not onto.
$f(-y^{1/4}) = y$

3. f is one – one and onto, Ao that f is invertible with $f^{-1} = \{(3,1)\ (2, 3)\ (1, 2)\}$

4. gof (x) = g[f(x)]
$= g\ (8x^3)$
$= \left(8x^3\right)^{\frac{1}{3}}$
$= 2x$

5. (f. g) oh
(f. g) h (x)
f[h(x)]. g[h(x)]
foh. goh

6. 3 * 4 = 2 (3) + 4-3 = 7

7. (i) Each triangle is similar to at well and thus $(T_1, T_1) \in R$
$\therefore$ R is reflexive.

(ii) $(T_1, T_2) \in R$

$\Rightarrow$ T_1 is similar to T_2

$\Rightarrow$ T_2 is similar to T_1

$(T_2, T_1) \in R$

R is symmetric

(iii) T_1 is similar to T_2 and T_2 is similar to T_3

$\Rightarrow$ T1 is similar to T_3

$\Rightarrow (T_1, T_3) \in R$

$\therefore$ R is transitive.

Hence R is equivalence

(II) part $T_1 = 3, 4, 5$

$T_2 = 5, 12, 13$

$T_3 = 6, 8, 10$

$\frac{3}{6} = \frac{4}{8} = \frac{5}{10} = \frac{1}{2}$ T_1 is relative to T_3.

8. (a) a * b = 1

b * a = 1

for all a, b $\in$ N also

(a * b) * c = 1 * c = 1

a * (b * c) = a * (1) = 1 for all, a, b, c R N

Hence R is both associative and commutative

(b) a * b = $\frac{a+b}{2}$, b * a = $\frac{b+a}{2}$

Hence commutative.

(a * b) * c = $\left(\frac{a+b}{2}\right) * c$

$= \left(\frac{a+b}{2}\right) + c = \frac{a+b+2c}{4}$

$= a*(b*c) = a*\left(\frac{a+b}{2}\right) = \frac{a+\left(\frac{a+b}{2}\right)}{2}$

$= \frac{2a+b+c}{4}$

* is not associative.

9. Let $(a_1 b_1)$ and $(a_2, b_2) \in A \times B$

(i) $f(a_1 b_1) = f(a_2, b_2)$
$b_1 = b_2$ and $a_1 = a_2$
$(a_1 b_1) = (a_2, b_2)$
Then $f(a_1 b_1) = f(a_2, b_2)$
$(a_1 b_1) = (a_2, b_2)$ for all
$(a_1 b_1) = (a_2, b_2) \in A \times B$

(ii) f is injective,
Let (b, a) be an arbitrary
Element of $B \times A$. then $b \in B$ and $a \in A$
$\Rightarrow (a, b)\ ) \in (A \times B)$
Thus for all $(b, a) \in B \times A$ their exists $(a, b)\ ) \in (A \times B)$
Hence that
$f(a, b) = (b, a)$
So $f: A \times B \rightarrow B \times A$
Is an onto function.
Hence bijective

CBSE TEST PAPER-05
CLASS - XII MATHEMATICS (Relations and Functions)

<u>Topic:-</u> Relations and Functions

1. show that a one – one function f: {1, 2, 3} → {1, 2, 3} must be onto. [1]
2. f: R → R be defined as f(x) = 3x whether the function is one – one onto or other [1]
3. Let S = {1, 2, 3} [1]

   Determine whether the function f: S → S defined as below have inverse.

   f = { (1, 2) (2, 1) (3, 1) }
4. Find fog

   $f(x) = 8x^3$, $g(x) = x^{1/3}$ [1]
5. If f: R → R be given by f(x) = $\left(3-x^3\right)^{\frac{1}{3}}$, find fof (x) [1]
6. If f(x) is an invertible function, find the inverse of f(x) = $\frac{3x-2}{5}$ [1]
7. Show that the relation R defined by (a, b) R (c, d) $\Rightarrow$ a + b = b + c on the set N $\times$N is an equivalence relation. [4]
8. Let * be the binary operation on H given by a * b = L. C. M of a and b. find [4]

   (a) 20 * 16

   (b) Is * commutative

   (c) Is * associative

   (d) Find the identity of * in N.
9. If the function f: R → R is given by f(x) = $\frac{x+3}{2}$ and g: R → R is given by g(x) = 2x – 3, Find

   (i) fog (ii) gof. Is $f^{-1} = g$ [4]
10. Let L be the set of all lines in Xy plane and R be the relation in L define as [4]

    R = {(L_1, L2): $L_1 \parallel L_2$}

    Show then R is on equivalence relation.

    Find the set of all lines related to the line Y=2x+4.

CBSE TEST PAPER-05

CLASS - XII MATHEMATICS (Relations and Functions)

[ANSWERS]

Topic:- Relations and Functions

1. Since f is one – one three element of {1, 2, 3} must be taken to 3 different element of the co – domain {1, 2, 3} under f. hence f has to be onto.

2. Let $x_1, x_2 \in R$

$3x_1 = 3x_2 \quad [if\ f(x_1) = f(x_2)]$

$x_1 = x_2$

f is one - one

$y = 3x$

$x = \frac{y}{3}$

$f\left(\frac{y}{3}\right) = \frac{\cancel{3}y}{\cancel{3}} = y$

3. f(2) = 1, f(3) =1
f is not one – one so that f is not invertible
Hence no inverse

4. fog (x) = f(gx)

$= f\left(x^{\frac{1}{3}}\right)$

$= 8\left(x^{\frac{1}{3}}\right)^3$

= 8x

5. $f\left[f\left(x\right)\right] = \left[\left[3 - \left(3 - x^3\right)^{3\frac{1}{3}}\right]^{\frac{1}{3}}\right]^{\frac{1}{3}}$

$= \left(3 - 3 + x^3\right)^{\frac{1}{3}}$

= x

6. Let f(x) = y

$$\frac{3x-2}{5} = y, \Rightarrow x = \frac{5y+2}{3}$$

$$\Rightarrow f^{-1}(y) = \frac{5y+2}{3}$$

7. (a, b) R (c, d) $\Rightarrow$ a + b = b + c where a, b, c, d $\in$ N
(a, b) R (a, b) $\Rightarrow$ a + b = b + a (a, b) $\in$ N $\times$ N
R is reflexive
(a, b) R (c, d) $\Rightarrow$ a + b = b + c (a, b) (c, d) $\in$ N $\times$ N
$\Rightarrow$ d + a = c + b
$\Rightarrow$ c + b = d + a
$\Rightarrow$ (c, d) R (a, b) (a, b), (c, d) $\in$ N $\times$ N
Hence reflexive.
(a, b) R (c, d) $\Rightarrow$ a + d = b + c (1) (a, b), (c, d) $\in$ N $\times$ N
(c, d) R (e, f) $\Rightarrow$ c + f = d + e (2) (c, d), (e, f) $\in$ N $\times$ N
Adding (1) and (2)
(a + b) + [(+f)] = (b + c) + (d + e)
a + f = b + e
(a, b) R (e, f)
Hence transitive
So equivalence

8. (i) 20 * 16 = L. C.M of 20 and 16
= 80

$$\left[\begin{array}{l} HCF = 4 \\ LCM = \dfrac{p(x)\times q(x)}{HCF} \end{array}\right]$$

(ii) a * b = L.C.M of a and b
= L.C.M of b and a
= b * a

(iii) a * (b * c) = a * (L.C.M of b and c)
= L.C.M of (a and L.C.M of b and c)
= L.C.M of a, b and c
Similarity
(a * b) * c = L. C.M of a, b, and c

(iv) a * 1 = L.C.M of a and 1
= a

$\boxed{\text{Ans} = 1}$

9. (i) fog (x) = f [g(x)]

$= f(2x - 3)$

$= \dfrac{2x-3+3}{2}$

= x

(ii) gof (x) = g [f(x)]

$= g\left(\dfrac{x+3}{2}\right)$

$= 2\left(\dfrac{x+3}{2}\right) - 3$

= x

(iii) fog = gof = x

Yes,

10. $L_1 || L_1$ i.e $(L_1, L_1) \in R$ Hence reflexive

$L_1 || L_2$ then $L_2 || L_1$ i.e $(L_1 L_2) \in R$

$\Rightarrow (L_2, L) \in R$ Hence symmetric

We know the

L1||L2 and L2||L3

Then L1|| L3

Hence Transitive . y = 2x+K

When K is real no .

CBSE TEST PAPER-06

CLASS - XII MATHEMATICS (Relations & Functions)

Topic:-Inverse Trigonometric Functions

1. Find the principal value of $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$ [1]

2. Find the value of $\sin^{-1}\left(\sin\frac{3\pi}{5}\right)$ [1]

3. Find the value of $\tan^{-1}\sqrt{3}-\cot^{-1}(-\sqrt{3})$ [1]

4. Find the value of $\sin\left(\sin^{-1}a+\cos^{-1}a\right)$ [1]

5. $\tan^{-1}\left(\frac{x}{y}\right)-\tan^{-1}\left(\frac{x-y}{x+y}\right)$ [1]

6. Find the value of $\tan^{-1}(1)+\cos^{-1}\left(-\frac{1}{2}\right)+\sin^{-1}\left(-\frac{1}{2}\right)$ [4]

7. Show that $\sin^{-1}\frac{3}{5}-\sin^{-1}\frac{8}{17}=\cos^{-1}\frac{84}{85}$ [4]

8. Prove that $\tan^{-1}\frac{1}{5}+\tan^{-1}\frac{1}{7}+\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{8}=\frac{\pi}{4}$ [4]

9. Prove that $\tan^{-1}x+\tan^{-1}\frac{2x}{1-x^2}=\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$ [4]

10. Simplify $\sin^{-1}\left(\frac{\sin x+\cos x}{\sqrt{2}}\right)$ *or* $\cos^{-1}\left(\frac{3}{5}\cos x+\frac{4}{5}\sin x\right)$ [4]

CBSE TEST PAPER-06

CLASS - XII MATHEMATICS (Relations & Functions)

[ANSWERS]

Topic:-Inverse Trigonometric Functions

Ans: 1 Let $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)=\theta$

$\sin\theta=\frac{1}{\sqrt{2}}$

We know that $\theta\in\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$

$\sin\theta=\sin\frac{\pi}{4}$

$$\boxed{\theta=\frac{\pi}{4}}$$

There for P.V. of $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) is \frac{\pi}{4}$

Ans:2 $\sin^{-1}\left(\sin\frac{3\pi}{5}\right)=?$

$\sin^{-1}\left(\sin\frac{3\pi}{5}\right)=\sin^{-1}\left[\sin\left(\pi-\frac{3\pi}{5}\right)\right]\left[\because\sin^{-1}\left(\sin\theta\right)\right]=\theta$

When $\theta\in\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$

$=\boxed{\frac{2\pi}{5}}$

Ans: 3 $\tan^{-1}\sqrt{3}-\cot^{-1}\left(-\sqrt{3}\right)=?$

$\tan^{-1}\sqrt{3}-\cot^{-1}\left(-\sqrt{3}\right)$

$= \tan^{-1}\sqrt{3}-\left(\pi-\cot^{-1}\sqrt{3}\right)\left[\because \cot^{-1}(-x)=\pi-\cot^{-1}x\right]$

$= \tan^{-1}\sqrt{3}-\pi+\cot^{-1}\sqrt{3}$

$= \left(\tan^{-1}\sqrt{3}+\cot^{-1}\sqrt{3}\right)-\pi\left[\because \tan^{-1}x+\cot^{-1}x=\frac{\pi}{2}\right]$

$= \frac{\pi}{2}-\frac{\pi}{1}= \quad \boxed{\frac{-\pi}{2}}$

Ans: 4 $\quad \sin(\sin^{-1}a+\cos^{-1}a)$

$\sin\frac{\pi}{2}\left[\because \sin^{-1}a+\cos^{-1}a=\frac{\pi}{2}\right]$

=1

Ans: 5 $\quad \tan^{-1}\left(\frac{x}{y}\right)-\tan^{-1}\left(\frac{\frac{x}{y}-1}{\frac{x}{y}+1}\right)$

$\tan^{-1}\left(\frac{x}{y}\right)-\tan^{-1}\left(\frac{\frac{x}{y}-1}{1+\frac{x}{y}}\right)\left[\because \tan^{-1}x-\tan^{-1}y=\tan^{-1}\left(\frac{x-y}{1+xy}\right)\right]$

$\tan^{-1}\left(\frac{x}{y}\right)-\left[\tan^{-1}\left(\frac{x}{y}\right)-\tan^{-1}(1)\right]$

$\tan^{-1}\left(\frac{x}{y}\right)-\tan^{-1}\left(\frac{x}{y}\right)+\tan^{-1}(1)$

$\tan^{-1}\left(\tan\frac{\pi}{4}\right)$

$= \quad \boxed{\frac{\pi}{4}}$

Ans: 6 $\tan^{-1}(1) = \tan^{-1}\tan\left(\frac{\pi}{4}\right) = \frac{\pi}{4}$

As $\theta \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

$$\cos^{-1}\left(\frac{-1}{2}\right) = \theta$$

$$\cos\theta = \frac{-1}{2}$$

$$\theta \in [0, \pi]$$

$$\therefore \cos\theta = \cos\left(\pi - \frac{\pi}{3}\right)$$

$$\theta = \frac{2\pi}{3}$$

$$\sin{-1}\left(\frac{-1}{2}\right) = \theta$$

$$\sin\theta = \frac{-1}{2}$$

$$\theta \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$$

$$\sin\theta = \sin\left(\frac{-\pi}{6}\right)$$

$$\theta = \frac{-\pi}{6}$$

$$= \frac{\pi}{4} + \frac{2\pi}{3} - \frac{\pi}{6}$$

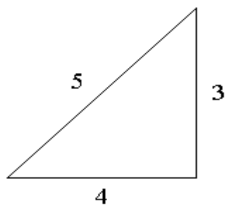
$$= \frac{3\pi + 8\pi - 2\pi}{12}$$

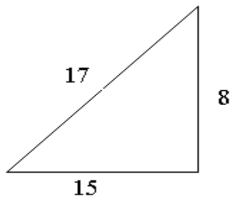
$$= \frac{9\pi}{12}$$

$$= \boxed{\frac{3\pi}{4}}$$

Ans: 7 Let $\sin^{-1}\frac{3}{5}=x$ $\qquad\qquad$ $\sin_{-1}\frac{8}{17}=y$

$\sin x=\frac{3}{5}$ $\qquad\qquad$ $\sin y=\frac{8}{17}$





$\cos(x-y)=\cos x.\cos y+\sin x.\sin y$

$\cos(x-y)=\frac{4}{5}\times\frac{15}{17}+\frac{3}{5}\times\frac{8}{17}$

$=\frac{60}{85}+\frac{24}{85}$

$=\frac{60+24}{85}$

$=\frac{84}{85}$

$x-y=\cos^{-1}\left(\frac{84}{85}\right)$

$\sin^{-1}\frac{3}{5}-\sin^{-1}\frac{8}{17}=\cos^{-1}\left(\frac{84}{85}\right)$

Ans: 8 L.H.S = $\tan^{-1}\left(\frac{1}{5}\right)+\tan^{-1}\left(\frac{1}{7}\right)+\tan^{-1}\left(\frac{1}{3}\right)+\tan^{-1}\left(\frac{1}{8}\right)$

$=\tan^{-1}\left(\frac{\frac{1}{5}+\frac{1}{7}}{1-\frac{1}{5}\times\frac{1}{7}}\right)+\tan^{-1}\left(\frac{\frac{1}{3}+\frac{1}{8}}{1-\frac{1}{3}\times\frac{1}{8}}\right)$

$=\tan^{-1}\left(\frac{\frac{7+5}{35}}{\frac{35-1}{35}}\right)+\tan^{-1}\left(\frac{\frac{8+3}{24}}{\frac{24-1}{24}}\right)$

$=\tan^{-1}\left(\frac{12}{35}\times\frac{35}{34}\right)+\tan^{-1}\left(\frac{11}{24}\times\frac{24}{23}\right)$

$$= \tan{-}^{1}\left(\frac{6}{17}\right)+\tan{-}1\left(\frac{11}{23}\right)$$

$$= \tan^{-1}\left(\frac{\frac{6}{17}+\frac{11}{23}}{1-\frac{6}{17}\times\frac{11}{23}}\right)$$

$$= \tan^{-1}\left(\frac{\frac{6\times23+11\times17}{17\times23}}{\frac{17\times23-6\times11}{17\times23}}\right)$$

$$= \tan^{-1}\left(\frac{325}{325}\right)$$

$$= \tan^{-1}(1)$$

$$= \tan^{-1}\left(\tan\frac{\pi}{4}\right)$$

$$= \frac{\pi}{4}$$

<u>Ans: 9</u> L.H.S = $\tan^{-1}x+\tan^{-1}\frac{2x}{1-x^2}$

$$= \tan^{-1}\left[\frac{x+\frac{2x}{1-x^2}}{1-x\left(\frac{2x}{1-x^2}\right)}\right]$$

$$= \tan^{-1}\left[\frac{\frac{x-x^3+2x}{1-x^2}}{\frac{(1-x^2)-2x^2}{1-x^2}}\right]$$

$$= \tan^{-1}\left[\frac{3x-x^3}{1-3x^2}\right]$$

L.H.S = R.H.S

Ans: 10 (i) $\sin^{-1}\left[\frac{1}{\sqrt{2}}\sin x+\frac{1}{\sqrt{2}}\cos x\right]$

$$\sin^{-1}\left[\cos\frac{\pi}{4}.\sin x+\sin\frac{\pi}{4}.\cos x\right]$$

$$\sin^{-1}\left[\sin\left(x+\frac{\pi}{4}\right)\right]=x+\frac{\pi}{4}$$

Or

$\frac{3}{5}=r\cos\theta,\frac{4}{5}=r\sin\theta$ squaring and adding

$$r^2\cos^2\theta+r^2\sin^2\theta=\frac{3}{25}+\frac{16}{25}$$

$$r^2(1)=\frac{25}{25}$$

$$r=1$$

$\frac{3}{5}=\cos\theta$ | $\frac{4}{5}=\sin\theta$

$$\tan\theta=\frac{4}{3}$$

$$\cos^{-1}\left[\frac{3}{5}\cos x+\frac{4}{5}\sin x\right]$$

$$=\cos^{-1}\left[\cos\theta.\cos x+\sin\theta.\sin x\right]$$

$$=\cos^{-1}\left[\cos(x-\theta)\right]$$

$$=x-\theta$$

$$=x-\tan^{-1}\frac{4}{3}$$

CBSE TEST PAPER-07

CLASS - XII MATHEMATICS (Relations & Functions)

Topic:-Inverse Trigonometric Functions

1. Find the principal value of $\cot^{-1}(-\frac{1}{\sqrt{3}})$. [1]

2. Find the value of $\cos^{-1} \cos(\frac{13\pi}{6})$. [1]

3. Find the value of $\sin(\frac{\pi}{3}-\sin^{-1}(\frac{-1}{2}))$ [1]

4. Prove that [1]

$$\cot^{-1}(\frac{ab+1}{a-b})+\cot^{-1}(\frac{bc+1}{b-c})+\cot^{-1}(\frac{ca+1}{c-a}) = 0.$$

5. $\sin(\tan^{-1} x) = ?$ [1]

6. Explore $\tan^{-1}(\frac{\cos x}{1-\sin x})$ in the simplest form. [4]

7. Show that $\sin^{-1}\frac{12}{13}+\cos^{-1}\frac{4}{5}+\tan^{-1}\frac{63}{16}=\pi$. [4]

8. Prove that $\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right) = \frac{x}{2}$ [4]

9. Write in simplest form that $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ [4]

10. Prove [4]

$$2\tan^{-1}\frac{1}{5}+\sec^{-1}\frac{5\sqrt{2}}{7}+2\tan^{-1}\frac{1}{8} = \frac{\pi}{4}$$

Or

Prove that $\cos^{-1} x = 2\sin^{-1}\sqrt{\frac{1-x}{2}} = 2\cos^{-1}\sqrt{\frac{1+x}{2}}$

CBSE TEST PAPER-07

CLASS - XII MATHEMATICS (Relations & Functions)

[ANSWERS]

Topic:-Inverse Trigonometric Functions

1. Let $\cot^{-1}\left(\frac{-1}{\sqrt{3}}\right)=\theta$

$\cot\theta=\frac{-1}{\sqrt{3}}$

We know that $\theta\ \in(0,\pi)$

$\cot\theta=\cot\left(\pi-\frac{\pi}{3}\right)$

$\theta=\frac{2\pi}{3}$

$There$ four p.v of $\cot^{-1}\left(\frac{-1}{\sqrt{3}}\right)=\frac{2\pi}{3}$

2. $\cos^{-1}\cos\left(\frac{13\pi}{6}\right)=\frac{13\pi}{6}$

$but\ \frac{13\pi}{6}\ \notin[0,\pi]$

Which is principal branch of $\cos^{-1}x$

$\cos\left(\frac{13\pi}{6}\right)=\cos\left(2\pi+\frac{\pi}{6}\right)$

$=\frac{\pi}{6}$

3. $\sin\left[\frac{\pi}{3}-\sin^{-1}\left(\frac{-1}{2}\right)\right]$

$\sin\left[\frac{\pi}{3}+\sin^{-1}\left(\frac{1}{2}\right)\right]\left[\because\sin^{-1}(-x)=-\sin x\right]$

$\sin\left[\frac{\pi}{3}+\sin^{-1}\left(\sin\frac{\pi}{6}\right)\right]$

$\sin\left[\frac{\pi}{3}+\frac{\pi}{6}\right]$

$$\sin\left[\frac{2\pi+\pi}{6}\right]=\sin\left[\frac{3\pi}{6}\right]$$

$$=\sin\frac{\pi}{2}=1$$

4. $$\tan^{-1}\left(\frac{a-b}{1+ab}\right)+\tan^{-1}\left(\frac{b-c}{1+bc}\right)+\tan^{-1}\left(\frac{c-a}{1+ca}\right)$$

$$\left[\cot^{-1}x=\tan^{-1}\frac{1}{x}, x>0\right]$$

$$=\tan^{-1}(a)-\tan^{-1}b+\tan^{-1}b-\tan^{-1}c+\tan^{-1}c-\tan^{-1}a$$

$$=0$$

5. $$\sin(\tan^{-1}(x)]=?$$

$$Let\ \ \tan^{-1}x=\theta$$

$$\frac{x}{1}=\tan\theta$$

$$\sin\theta=\frac{x}{\sqrt{1+x^2}}$$

$$\theta=\sin^{-1}\frac{x}{\sqrt{1+x^2}}$$

$$\Rightarrow \tan^{-1}x=\sin^{-1}\frac{x}{\sqrt{1+x^2}}$$

$$\sin(\tan^{-1}x)=\sin\left(\sin^{-1}\frac{x}{\sqrt{1+x^2}}\right)$$

$$=\frac{x}{\sqrt{1+x^2}}$$

6. $$\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right)$$

$$\tan^{-1}\left(\frac{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}{\cos^2\frac{x}{2}+\sin^2\frac{x}{2}-2\sin\frac{x}{2}\cos\frac{x}{2}}\right)$$

$$\left[\begin{array}{l}\because \cos 2\theta = \cos^2\theta - \sin^2\theta \\ \sin 2\theta = 2\sin\theta\cos\theta\end{array}\right]$$

$$\tan^{-1}\left(\frac{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)}{\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)^2}\right)$$

Dividing N and b by $\cos\frac{x}{2}$

$$\tan^{-1}\left[\frac{1+\tan\frac{x}{2}}{1-\tan\frac{x}{2}}\right]$$

$$\tan^{-1}\left[\frac{\tan\frac{\pi}{4}+\tan\frac{x}{2}}{1-\tan\frac{\pi}{4}.\tan\frac{x}{2}}\right]$$

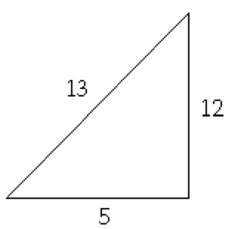
$$\tan^{-1}\left[\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right]$$

$$=\frac{\pi}{4}+\frac{x}{2}$$

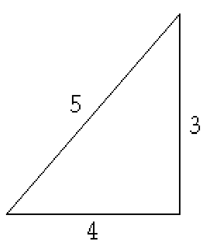
7. $Let\ \sin^{-1}\frac{12}{13}=x,$ | $\cos^{-1}\frac{4}{5}=y,$ | $\tan^{-1}\frac{63}{16}=z,$

$\sin x=\frac{12}{13}$ | $\cos y=\frac{4}{5}$ | $\tan z=\frac{63}{16}$

$\tan x=\frac{12}{5}$



$\tan y=\frac{3}{4}$



$$\tan(x+y).=\frac{\tan x+\tan y}{1-\tan x.\tan y}$$

$$=\frac{\frac{12}{5}+\frac{3}{4}}{1-\frac{12}{5}\times\frac{3}{4}}$$

$$=\frac{\frac{45+15}{20}}{\frac{20-36}{20}}$$

$$=\frac{63}{16}\left[\because \tan z=\frac{63}{16}\right]$$

$$\tan(x+y)=-\tan z$$
$$\tan(x+y)=\tan(-z)$$
$$x+y=-z$$
$$x+y\neq -z$$

$$\tan(x+y)=\tan(\pi-z)$$
$$x+y=\pi-z$$
$$x+y+z=\pi$$

$$\sin^{-1}\frac{12}{13}+\cos^{-1}\frac{4}{5}+\tan^{-1}\frac{63}{16}=\pi$$

8. $$1\pm\sin x=\cos^2\frac{x}{2}+\sin^2\frac{x}{2}\pm 2\sin\frac{x}{2}\cos\frac{x}{2}$$

$$=\left(\cos\frac{x}{2}\pm\sin\frac{x}{2}\right)^2$$

$$\cot^{-1}\left[\frac{\sqrt{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2}+\sqrt{\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)^2}}{\sqrt{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2}-\sqrt{\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)^2}}\right]$$

$$\cot^{-1}\left(\frac{\cos\frac{x}{2}+\sin\frac{x}{2}+\cos\frac{x}{2}-\sin\frac{x}{2}}{\cos\frac{x}{2}+\sin\frac{x}{2}-\cos\frac{x}{2}+\sin\frac{x}{2}}\right)$$

$$\cot^{-1}\left(\frac{2\cos\frac{x}{2}}{2\sin\frac{x}{2}}\right)$$

$$\cot^{-1}\left(\cot\frac{x}{2}\right)$$

$$=\frac{x}{2}$$

9. $$\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$$

$but\ x=\tan\theta$

$$\tan^{-1}\left(\frac{\sqrt{1+\tan\theta^2}-1}{\tan\theta}\right)$$

$$\tan^{-1}\left(\frac{\sqrt{\sec^2\theta}-1}{\tan\theta}\right)$$

$$\tan^{-1}\left(\frac{\sec\theta-1}{\tan\theta}\right)$$

$$\tan^{-1}\left(\frac{\frac{1}{\cos\theta}-1}{\frac{\sin\theta}{\cos\theta}}\right)$$

$$\tan^{-1}\left(\frac{\frac{1-\cos\theta}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}}\right)$$

$$\tan^{-1}\left(\frac{1-\cos\theta}{\sin\theta}\right)$$

$$\tan^{-1}\left(\frac{2\sin^2\frac{\theta}{2}}{2\sin^2\frac{\theta}{2}\cos\frac{\theta}{2}}\right)$$

$$\tan^{-1}\left(\tan\frac{\theta}{2}\right)=\frac{\theta}{2}$$

$$=\frac{1}{2}\tan^{-1}x$$

10. $L.H.S = 2\tan^{-1}\frac{1}{5} + 2\tan^{-1}\frac{1}{8} + \sec^{-1}\frac{5\sqrt{2}}{7}$

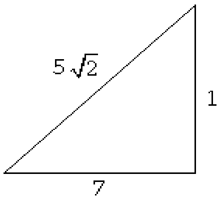
$= 2\left(\tan^{-1}\frac{1}{5} + 2\tan^{-1}\frac{1}{8}\right) + \sec^{-1}\frac{5\sqrt{2}}{7}$

$= 2\tan^{-1}\left(\frac{\frac{8+5}{40}}{\frac{40-1}{40}}\right) + \sec^{-1}\frac{5\sqrt{2}}{7}$

$= 2\tan^{-1}\left(\frac{13}{39}\right) + \sec^{-1}\frac{5\sqrt{2}}{7}$

$\left[\begin{array}{l} \because \sec^{-1}\frac{5\sqrt{2}}{7} = \theta \\ \sec\theta = \frac{5\sqrt{2}}{7} \end{array}\right]$

$\tan\theta = \frac{1}{7}$



$= 2\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{7}$

$\left[2\tan^{-1}x = \tan^{-1}\frac{2x}{1-x^2}\right]$

$= \tan^{-1}\frac{2\times\frac{1}{3}}{1-\left(\frac{1}{3}\right)^2} + \tan^{-1}\frac{1}{7}$

$= \tan^{-1}\left(\frac{\frac{2}{3}}{\frac{9-1}{9}}\right) + \tan^{-1}\frac{1}{7}$

$= \tan^{-1}\left(\frac{2}{3}\times\frac{9}{8}\right) + \tan^{-1}\frac{1}{7}$

$$\tan^{-1}\left(\frac{3}{4}\right)+\tan^{-1}\left(\frac{1}{7}\right)$$

$$\tan^{-1}\left(\frac{\frac{3}{4}+\frac{1}{7}}{1-\frac{3}{4}\times\frac{1}{7}}\right)$$

$$\tan^{-1}\left(\frac{\frac{21+4}{28}}{\frac{28-3}{28}}\right)=\tan^{-1}\left(\frac{25}{25}\right)$$

$$=\tan^{-1}\left(\tan\frac{\pi}{4}\right)$$

$$=\frac{\pi}{4}$$

OR

Put $\cos^{-1}x=\theta$

$x=\cos\theta$

$$\cos^{-1}(\cos\theta)=2\sin^{-1}\sqrt{\frac{1-\cos\theta}{2}}=2\cos^{-1}\sqrt{\frac{1+\cos\theta}{2}}$$

$$\theta=2\sin^{-1}\sqrt{\frac{2\sin^2\frac{\theta}{2}}{2}}=2\cos^{-1}\sqrt{\frac{2\cos^2\frac{\theta}{2}}{2}}$$

$$\theta=2\sin^{-1}\left(\sin\frac{\theta}{2}\right)=2\cos^{-1}\left(\cos\frac{\theta}{2}\right)$$

$$\theta=2.\frac{\theta}{2}=2.\frac{\theta}{2}$$

$\theta=\theta=\theta$ Prove.

CBSE TEST PAPER-08

CLASS - XII MATHEMATICS (Relations & Functions)

<u>Topic:-Inverse Trigonometric Functions</u>

1. Find the principal value of $\cos^{-1}(-\frac{1}{2})$. [1]

2. Find the value of $\tan^{-1} \tan(\frac{7\pi}{6})$. [1]

3. Find the value of $\cot(\tan^{-1} a + \cot^{-1} a)$ [1]

4. Prove that $\tan^{-1}\sqrt{x} = \frac{1}{2}\cos^{-1}(\frac{1-x}{1+x})$ [1]

5. $\tan^{-1}(\tan\frac{3\pi}{4}) = ?$ [1]

6. Prove $2\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{7} = \tan^{-1}\frac{31}{17}$ [4]

7. Simplify $\tan^{-1}\left(\frac{a\cos x - b\sin x}{b\cos x + a\sin x}\right)$. [4]

8. Prove that $\tan^{-1}\left(\frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt{1+x} + \sqrt{1-x}}\right) = \frac{\pi}{4} - \frac{1}{2}\cos^{-1} x$ [4]

9. If $\sin(\sin^{-1}\frac{1}{5} + \cos^{-1} x) = 1$ find x. [4]

10. If a>b>c>0 prove that [4]
$\cot^{-1}(\frac{ab+1}{a-b}) + \cot^{-1}(\frac{bc+1}{b-c}) + \cot^{-1}(\frac{ca+1}{c-a}) = \pi$

CBSE TEST PAPER-08

CLASS - XII MATHEMATICS (Relations & Functions)

[ANSWERS]

Topic:-Inverse Trigonometric Functions

1. Let $\cos^{-1}\left(\frac{-1}{2}\right)=\theta$

$\cos\theta=\frac{-1}{2}$

$\theta\in[0,\pi]$

$\cos\theta=\cos\left(\pi-\frac{\pi}{3}\right)$

$\theta=\frac{2\pi}{3}$

$P.V$ is $\frac{2\pi}{3}$

2. $\tan^{-1}\left(\tan\frac{7\pi}{6}\right)=\frac{7\pi}{6}$

but $\frac{7\pi}{6}\notin\left(\frac{-\pi}{2},\frac{\pi}{2}\right)$

$\because \tan^{-1}\left(\tan\frac{7\pi}{6}\right)=\tan^{-1}\left[\tan\left(\pi+\frac{\pi}{6}\right)\right]$

$=\tan^{-1}\left(\tan\frac{\pi}{6}\right)$

$=\frac{\pi}{6}$

3. $\cot\frac{\pi}{2}=0$

4. Put $\tan^{-1}\sqrt{x}=\theta$

$$\tan\theta = \sqrt{x}$$

$$\tan^2\theta = x$$

$$R.H.S = \frac{1}{2}\cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)$$

$$= \frac{1}{2}\cos^{-1}(\cos 2\theta) = \frac{1}{2}\times 2\theta = \theta$$

$$= \tan^{-1}\sqrt{x}$$

5. $$\tan^{-1}\left(\tan\frac{3\pi}{\pi}\right) = ?$$

$$\tan^{-1}\left(\tan\frac{3\pi}{\pi}\right) \neq \frac{3\pi}{4}$$

$$as\,\frac{3\pi}{4} \notin \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$$

$$\because \tan^{-1}\left(\tan\frac{3\pi}{4}\right) = \tan^{-1}\left[\tan\left(\pi - \frac{\pi}{4}\right)\right]$$

$$= \tan^{-1}\left[\tan\left(-\frac{\pi}{4}\right)\right]$$

$$= -\frac{\pi}{4}$$

6. $$L.H.S = 2\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{7}$$

$$= \tan^{-1}\frac{2\times\frac{1}{2}}{1-\left(\frac{1}{2}\right)^2} + \tan^{-1}\frac{1}{7}\left[2\tan^{-1}x = \tan^{-1}\frac{2x}{1-x^2}\right]$$

$$= \tan^{-1}\frac{1}{\frac{4-1}{4}} + \tan^{-1}\left(\frac{1}{7}\right)$$

$$\tan^{-1}\left(\frac{4}{3}\right) + \tan^{-1}\left(\frac{1}{7}\right)$$

$$= \tan^{-1}\left(\frac{\frac{4}{3}+\frac{1}{7}}{1-\frac{4}{3}\times\frac{1}{7}}\right) = \tan^{-1}\left(\frac{\frac{28+3}{21}}{\frac{21-4}{21}}\right)$$

$$= \tan^{-1}\left(\frac{31}{17}\right)$$

7. $$\tan-1\left(\frac{\frac{a\cos x}{b\cos x}-\frac{b\sin x}{b\cos x}}{\frac{b\cos x}{b\cos x}+\frac{a\sin x}{b\cos x}}\right)$$

$$\tan-1\left(\frac{\frac{a}{b}-\tan x}{1+\frac{a}{b}\tan x}\right)$$

$$= -\tan^{-1}\left(\frac{a}{b}\right)-\tan^{-1}\left(\tan x\right)$$

$$= \tan^{-1}\left(\frac{a}{b}\right)-x$$

8. Put $x = \cos 2\theta$

$$L.H.S = \tan^{-1}\left(\frac{\sqrt{1+\cos 2\theta}-\sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta}+\sqrt{1-\cos 2\theta}}\right)$$

$$= \tan^{-1}\left(\frac{\sqrt{2\cos^2\theta}-\sqrt{2\sin^2\theta}}{\sqrt{2\cos^2\theta}+\sqrt{2\sin^2\theta}}\right)$$

$$= \tan^{-1}\left(\frac{\sqrt{2}\cos\theta-\sqrt{2}\sin\theta}{\sqrt{2}\cos\theta+\sqrt{2}\sin\theta}\right)$$

$$= \tan^{-1}\left(\frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}\right)$$

$$= \tan^{-1}\left(\frac{1-\tan\theta}{1+\tan\theta}\right)$$

$$= \tan^{-1}\left[\tan\left(\frac{\pi}{4}-\theta\right)\right]$$

$$= \frac{\pi}{4}-\theta$$

$$=\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x$$

9. $$\sin\left(\sin^{-1}\frac{1}{5}+\cos^{-1}x\right)=1$$

$$\sin\left(\sin^{-1}\frac{1}{5}+\cos^{-1}x\right)=\sin\frac{\pi}{2}$$

$$\sin^{-1}\frac{1}{5}+\cos^{-1}x=\frac{\pi}{2}$$

$$\sin^{-1}\frac{1}{5}+\cos^{-1}\frac{1}{5}-\cos^{-1}\frac{1}{5}+\cos^{-1}x=\frac{\pi}{2}$$

$$\frac{\pi}{2}-\cos^{-1}\frac{1}{5}+\cos^{-1}x=\frac{\pi}{2}$$

$$\cos^{-1}x=\cos^{-1}\frac{1}{5}$$

$$x=\frac{1}{5}$$

10. $$\tan^{-1}\left(\frac{a-b}{1+ab}\right)+\tan^{-1}\left(\frac{b-c}{1+bc}\right)+\pi+\tan^{-1}\left(\frac{c-a}{1+ca}\right)$$

$$\tan^{-1}a-\tan^{-1}b+\tan^{-1}b-\tan^{-1}c+\pi+(\tan^{-1}c+\tan^{-1}a)$$

$$=\pi$$

$$\left[\because \cot^{-1}x=\pi+\tan^{-1}\left(\frac{1}{x}\right)\ \mathit{for}\ \mathrm{x}\in 0\right]$$

CBSE TEST PAPER-09

CLASS - XII MATHEMATICS (Relations and Functions)

Topic:-Inverse Trigonometric Functions

1. Find the principal value of $\cos ec^{-1}(-\sqrt{2})$. [1]

2. Find the value of $\sin^{-1} \sin(\frac{2\pi}{3})$. [1]

3. Find the value of $\cos(\sec^{-1} x + \cos ec^{-1} x)$ [1]

4. Prove that $\tan^{-1}\sqrt{\frac{1-\cos x}{1+\cos x}} = \frac{x}{2}$ [1]

5. $\tan^{-1}\left(\frac{3a^2x - x^3}{a^3 - 3ax^2}\right) = ?$ [1]

6. Find the value $\tan\frac{1}{2}\left[\sin^{-1}\frac{2x}{1+x^2} + \cos^{-1}\frac{1-y^2}{1+y^2}\right]$ [4]

7. Solve $\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$. [4]

8. Prove that [4]

$$\frac{9\pi}{8} - \frac{9}{4}\sin^{-1}\frac{1}{3} = \frac{9}{4}\sin^{-1}\frac{2\sqrt{2}}{3}$$

9. Find the value of $\tan\left(\sin^{-1}\frac{3}{5} + \cot^{-1}\frac{3}{2}\right)$ [4]

10. If $\cos^{-1}\frac{x}{a} + \cos^{-1}\frac{y}{b} = \alpha$ [4]

Prove that $\frac{x^2}{a^2} - \frac{2xy}{ab}\cos x + \frac{y}{b} = \sin^2\alpha$

CBSE TEST PAPER-09

CLASS - XII MATHEMATICS (Relations and Functions)

[ANSWERS]

Topic:-Inverse Trigonometric Functions

1. Let $\cos ec^{-1}\left(-\sqrt{2}\right)=\theta$

$\cos ec=-\sqrt{2}$

$\theta\in\left[\frac{-\pi}{2},\frac{\pi}{2}\right]-\{0\}$

$\cos ec=\cos ec\left(\frac{-\pi}{4}\right)$

$\theta=\frac{-\pi}{4}$

$P.V$ is $\frac{-\pi}{4}$

2. $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)\neq\frac{2\pi}{3}$

$as\ \frac{2\pi}{3}\in\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$

$\sin^{-1}\left(\sin\frac{2\pi}{3}\right)=\sin^{-1}\left[\sin\left(\pi-\frac{\pi}{3}\right)\right]$

$=\sin^{-1}\left[\sin\left(\frac{\pi}{3}\right)\right]$

$=\frac{\pi}{3}$

3. $\cos(\sec^{-1}x+\cos ec^{-1}x)$

$\cos\frac{\pi}{2}=0$

4. $\tan^{-1}\left(\sqrt{\dfrac{2\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}}}\right)=\tan^{-1}\left(\sqrt{\tan^2\frac{x}{2}}\right)$

$=\tan^{-1}\left(\sqrt{\tan\frac{x}{2}}\right)=\frac{x}{2}$

5. Put $x=a\tan\theta$

$\tan^{-1}\left(\dfrac{3a^3\tan\theta-\tan^3\theta}{a^3-3a^3\tan^2\theta}\right)$

$\tan^{-1}\left[\dfrac{a^3\left(3\tan\theta-\tan^3\theta\right)}{a^3\left(1-3\tan^2\theta\right)}\right]$

$\tan^{-1}\left(\tan 3\theta\right)$

$=3\theta$

$=3\tan^{-1}\frac{x}{a}$

6. $\tan\frac{1}{2}\left[\sin^{-1}\frac{2x}{1+x^2}+\cos^{-1}\frac{1-y^2}{1+y^2}\right]$

Put $x=\tan\theta, y=\tan\phi$

$\tan\frac{1}{2}\left[\sin^{-1}\frac{2\tan\theta}{1+\tan^2\theta}+\cos^{-1}\frac{1-\tan^2\phi}{1+\tan^2\phi}\right]$

$\tan\frac{1}{2}\left[\sin^{-1}\left(\sin 2\theta\right)+\cos^{-1}\left(\cos 2\phi\right)\right]$

$\tan\frac{1}{2}\left[2\theta+2\phi\right]$

$\tan\frac{1}{2}2\left(\theta+\phi\right)$

$\dfrac{\tan\theta+\tan\phi}{1-\tan\theta.\tan\phi}=\dfrac{x+y}{1-xy}$

7. $\tan^{-1}2x+\tan^{-1}3x=\dfrac{\pi}{4}$

$\tan^{-1}\left(\dfrac{2x+3x}{1-2x.3x}\right)=\dfrac{\pi}{4}$

$\dfrac{5x}{1-6x^2}=\tan\dfrac{\pi}{4}$

$\dfrac{5x}{1-6x^2}=\times\dfrac{1}{1}$

$1-6x^2=5x$

$6x^2+5x-1=0$

$(6x-1)(x+1)=0$

$x=\dfrac{1}{6}, x=-1$

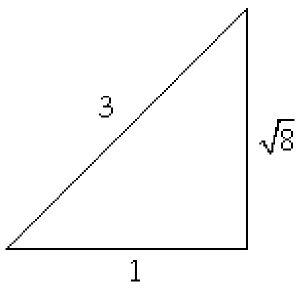
$\boxed{x=\dfrac{1}{6}}$

8. $L.H.S=\dfrac{9}{4}\left[\dfrac{\pi}{2}-\sin^{-1}\dfrac{1}{3}\right]$

$=\dfrac{9}{4}\cos^{-1}\dfrac{1}{3}\left[\because \sin^{-1}\dfrac{1}{3}+\cos^{-1}\dfrac{1}{3}=\dfrac{\pi}{2}\right]$

$\cos^{-1}\dfrac{1}{3}=\theta$

$\cos\theta=\dfrac{1}{3}$



$\sin\theta=\dfrac{2\sqrt{2}}{3}$

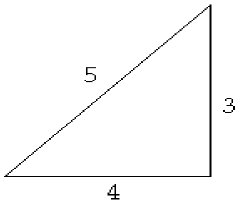
$\theta=\sin^{-1}\dfrac{2\sqrt{2}}{3}$

$\Rightarrow \cos^{-1}\dfrac{1}{3}=\sin^{-1}\dfrac{2\sqrt{2}}{3}$

$=\dfrac{9}{4}.\sin^{-1}\dfrac{2\sqrt{2}}{3}$

9. Let $\sin^{-1}\frac{3}{5}=x$

$\sin x=\frac{3}{5}$



$\tan x=\frac{3}{4}$

$\cot^{-1}\frac{3}{2}=y$

$\cot y=\frac{3}{2}$

$\tan y=\frac{2}{3}$

$$\tan\left(\sin^{-1}\frac{3}{4}+\cot^{-1}\frac{3}{2}\right)=\tan(x+y)$$

$$=\frac{\tan x+\tan y}{1-\tan x.\tan y}$$

$$=\frac{\frac{3}{4}+\frac{2}{3}}{1-\frac{3}{4}\times\frac{2}{3}}$$

$$=\frac{\frac{9+8}{12}}{\frac{12-6}{12}}$$

$$=\frac{17}{6}$$

10. $\cos^{-1}\frac{x}{a}+\cos^{-1}\frac{y}{b}=\alpha$

$$\left[\because \cos^{-1}x+\cos^{-1}y=\cos^{-1}\left(xy-\sqrt{1-x^2}\sqrt{1-y^2}\right)\right]$$

$$\cos^{-1}\left[\frac{x}{a}.\frac{y}{b}-\sqrt{1-\frac{x^2}{a^2}}.\sqrt{1-\frac{y^2}{b^2}}\right]=\alpha$$

$$\frac{xy}{ab}-\sqrt{1-\frac{x^2}{a^2}}.\sqrt{1-\frac{y^2}{b^2}}=\cos\alpha$$

$$\frac{xy}{ab}-\cos\alpha=\sqrt{1-\frac{x^2}{a^2}}\sqrt{1-\frac{y^2}{b^2}}$$

Squaring both side

$$\left(\frac{xy}{ab}-\cos\alpha\right)^2=\left(\sqrt{1-\frac{x^2}{a^2}}\sqrt{1-\frac{y^2}{b^2}}\right)^2$$

$$\frac{x^2y^2}{a^2b^2}+\cos^2\alpha-2.\frac{xy}{ab}.\cos\alpha=\left(1-\frac{x^2}{a^2}\right)\left(1-\frac{y^2}{b^2}\right)$$

$$\frac{x^2y^2}{a^2b^2}+\cos^2\alpha-2\frac{xy}{ab}\cos\alpha=1-\frac{y^2}{b^2}-\frac{x^2}{a^2}+\frac{x^2y^2}{a^2b^2}$$

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-2\frac{xy}{ab}\cos\alpha=1-\cos^2\alpha$$

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-2\frac{xy}{ab}\cos\alpha=\sin^2\alpha$$

CBSE TEST PAPER-10

CLASS - XII MATHEMATICS (Relations and Functions)

<u>Topic:-Inverse Trigonometric Functions</u>

1. Find the principal value of $\sec^{-1}\left(\frac{3}{\sqrt{3}}\right)$. [1]

2. Find the value of $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$ [1]

3. Find the value of $\cot^{-1}\frac{1}{\sqrt{x^2-1}}$. [1]

4. Find $\tan^{-1}\frac{x}{\sqrt{a^2-x^2}}$. [1]

5. Find the value of $\tan^{-1}\left[2\cos\left(2\sin^{-1}\left(\frac{1}{2}\right)\right)\right]$. [1]

6. If $\tan^{-1}\frac{x-1}{x-2}+\tan^{-1}\frac{x+1}{x+2}=\frac{\pi}{4}$ find x. [4]

7. Show that $\sin^{-1}\left(\frac{8}{17}\right)+\sin^{-1}\left(\frac{3}{5}\right)=\tan^{-1}\frac{77}{36}$ [4]

8. Solve $\tan^{-1}\frac{1-x}{1+x} = \frac{1}{2}\tan^{-1}x$ [4]

9. Find x if $\sin^{-1}(1-x) - 2\sin^{-1}x=\frac{\pi}{2}$. [4]

10. Prove that : $\tan\left(\frac{\pi}{4}+\frac{1}{2}\cos^{-1}\frac{a}{b}\right)+\tan\left(\frac{\pi}{4}-\frac{1}{2}\cos^{-1}\frac{a}{b}\right)=\frac{2b}{a}$ [4]

CBSE TEST PAPER-10

CLASS - XII MATHEMATICS (Relations and Functions)

[ANSWERS]

Topic:-Inverse Trigonometric Functions

1. Let $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)=\theta$

$\sec\theta=\frac{2}{\sqrt{3}}$

$\theta\in[0,\pi]-\left\{\frac{\pi}{2}\right\}$

$\sec\theta=\sec\frac{\pi}{6}$

$\theta=\frac{\pi}{6}$

$P.V$ of $\sin^{-1}\left(\frac{2}{\sqrt{3}}\right)=\frac{\pi}{6}$

2. $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)\neq\frac{7\pi}{6}$

$as\frac{7\pi}{6}\notin[0,\pi]$

$\therefore$

$\cos^{-1}\cos\left(\frac{7\pi}{6}\right)=\left(\pi+\frac{\pi}{6}\right)$

$\cos^{-1}\left[\cos\left(2\pi-\frac{5\pi}{6}\right)\right]$

$=\frac{5\pi}{6}$

3. $\cot^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right)$

$put\ x = \sec\theta$

$$\cot^{-1}\left(\frac{1}{\sqrt{\sec^2 - 1}}\right) = \cot^{-1}\left(\frac{1}{\tan\theta}\right) = \cot^{-1}(\cot\theta)$$

$$= \theta = \sec^{-1} x$$

4. $$\tan^{-1}\left(\frac{x}{\sqrt{a^2 - x^2}}\right)$$

$Put\ x = a\sin\theta$

$$\tan^{-1}\left(\frac{a\sin\theta}{\sqrt{a^2 - a^2\sin^2\theta}}\right)$$

$$\tan^{-1}\left(\frac{a\sin\theta}{\sqrt{a^2\left(1 - \sin^2\theta\right)}}\right)$$

$$\tan^{-1}\left(\frac{a\sin\theta}{a\cos\theta}\right)$$

$$\tan^{-1}\left(\tan\theta\right) = \theta$$

$$= \sin^{-1}\left(\frac{x}{a}\right)$$

5. $$\tan^{-1}\left[2\cos\left(2\sin^{-1}\left(\sin\frac{\pi}{6}\right)\right)\right]$$

$$\tan^{-1}\left[2\cos\left(2\frac{\pi}{6}\right)\right]$$

$$\tan^{-1}\left(2\cos\frac{\pi}{3}\right)$$

$$\tan^{-1}\left(2.\frac{1}{2}\right)$$

$$\tan^{-1}(1)$$

$$\tan^{-1}\left(\tan\frac{\pi}{4}\right)$$

$$= \frac{\pi}{4}$$

6. $\tan^{-1}\left[\dfrac{\dfrac{x-1}{x-2}+\dfrac{x+1}{x+2}}{1-\left(\dfrac{x-1}{x-2}\right)\left(\dfrac{x+1}{x+2}\right)}\right]=\dfrac{\pi}{4}$

$$\frac{\dfrac{(x-1)(x+2)+(x+1)(x-2)}{(x-2)(x+2)}}{\dfrac{(x-2)(x+2)-(x-1)(x+1)}{(x-2)(x+2)}}=\tan\frac{\pi}{4}$$

$$\frac{x^3+2x-x-2+x^2-2x+x-2}{(x^2+2x-2x-4)-)(x^2-1)}=1$$

$$\frac{2x^2-4}{x^2-4-x^2+1}=1$$

$$\frac{2x^2-4}{-3}=\frac{1}{1}$$

$$2x^2-4=-3$$

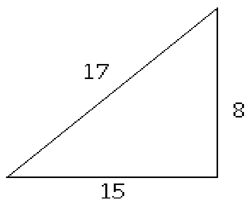
$$2x^2=-3+4$$

$$2x^2=1$$

$$x=\pm\frac{1}{\sqrt{2}}$$

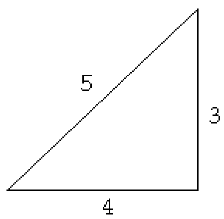
7. Let $\sin^{-1}\dfrac{8}{17}=x$

$\sin x=\dfrac{8}{17}$



$\sin^{-1}\dfrac{3}{5}=y$

$\sin y=\dfrac{3}{5}$



$$\tan(x+y)=\frac{\tan x+\tan y}{1-\tan x.\tan y}$$

$$=\frac{\dfrac{8}{15}+\dfrac{3}{4}}{1-\dfrac{8}{15}\times\dfrac{3}{4}}$$

$$= \frac{\frac{32+45}{60}}{\frac{60-24}{60}}$$

$$\tan(x+y) = \frac{77}{36}$$

$$\sin^{-1}\frac{8}{17} + \sin^{-1}\frac{3}{5} = \tan^{-1}\frac{77}{36}$$

8. $2\tan^{-1}\frac{1-x}{1+x} = \tan^{-1} x$

$$\tan^{-1}\frac{2\left(\frac{1-x}{1+x}\right)}{1-\left(\frac{1-x}{1+x}\right)^2} = \tan^{-1} x \left[2\tan^{-1} x = \tan^{-1}\frac{2x}{1-x^2}\right]$$

$$\frac{2\left(\frac{1-x}{1+x}\right)}{\frac{(1+x)^2-(1-x)^2}{(1+x)^2}} = x$$

$$\frac{2\left(\frac{1-x}{1+x}\right)}{\frac{(1+x+2x)^2-(1+x-2x)^2}{(1+x)^2}} = x$$

$$2\left(\frac{1-x}{1+x}\right)\times\frac{(1+x)^2}{4x} = x$$

$$\frac{2(1-x^2)}{4x} = \frac{x}{1}$$

$$1-x^2 = 2x^2$$

$$3x^2 = 1$$

$$x = \pm\frac{1}{\sqrt{3}}$$

9. $\sin^{-1}(1-x) = \frac{\pi}{2} + 2\sin^{-1} x$

$$(1-x) = \sin\left(\frac{\pi}{2} + 2\sin^{-1} x\right)$$

$$1-x=\cos(2\sin^{-1}x)$$
$$1-x=\cos 2(\sin^{-1}x)$$
$$\left[\cos^2\theta=1-2\sin^2\theta\right]$$
$$=1-2\sin^2(\sin^{-1}x)$$
$$=1-2\left[\sin\left(\sin^{-1}x\right)\right]^2$$
$$1-x=1-2x^2$$
$$2x^2-x=0$$
$$x(2x-1)=0$$
$$x=0, x=\frac{1}{2}$$

10. $\frac{1}{2}\cos^{-1}\frac{a}{b}=\theta$

$$\tan\left(\frac{\pi}{4}+\theta\right)+\tan\left(\frac{\pi}{4}-\theta\right)$$
$$=\frac{1+\tan\theta}{1-\tan\theta}+\frac{1-\tan\theta}{1+\tan\theta}$$
$$=\frac{\left(1+\tan\theta\right)^2+\left(1-\tan\theta\right)^2}{1-\tan^2\theta}$$
$$=\frac{2+2\tan^2\theta}{1-\tan^2\theta}$$
$$=2\left(\frac{1+\tan^2\theta}{1-\tan^2\theta}\right)$$
$$=\frac{2}{\cos 2\theta}\left[\begin{array}{l}\because \cos^{-1}\frac{a}{b}=2\theta \\ \cos 2\theta=\frac{a}{b}\end{array}\right]$$
$$=\frac{2}{\frac{a}{b}}$$
$$=\frac{2b}{a}$$

CBSE TEST PAPER-01

CLASS - XII MATHEMATICS (Algebra)

Topic:- Matrices

1. If a matrix has 8 elements, what are the possible orders it can have. [1]

2. Identity matrix of orders n denoted by. [1]

3. Define square matrix [1]

4. The no. of all possible metrics of order 3×3 with each entry 0 or 1 is [1]

5. $A = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$ Write (1) a_{33}, a_{12} (ii) what is order [1]

6. Find x and y if x + y = $\begin{bmatrix} 5 & 2 \\ 0 & 9 \end{bmatrix}$ and x – y = $\begin{bmatrix} 3 & 6 \\ 0 & -1 \end{bmatrix}$ [4]

7. $f(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$ Show that f(x). f(y) = f(x+y) [4]

8. If $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$ $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ Find K. So that $A^2 = KA - 2I$ [4]

9. $A = \begin{bmatrix} -2 \\ 4 \\ 5 \end{bmatrix}$ B = $\begin{bmatrix} 1 & 3 & -6 \end{bmatrix}$ Prove $(AB)' = B'A'$ [4]

10. $A = \begin{bmatrix} 0 & -\tan\dfrac{\alpha}{2} \\ \tan\dfrac{\alpha}{2} & 0 \end{bmatrix}$, Prove $I + A = (I - A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$ [4]

CBSE TEST PAPER-01

CLASS - XII MATHEMATICS (algebra)

[ANSWERS]

Topic:- Matrices

1. $1\times8,\quad 8\times1,\quad 4\times2,\quad 2\times4,$

2. I_n

3. A matrix in which the no. of rows are equal to no. of columns i.e. m = n

4. 512

5. (i) $a_{33} = 9,\quad a_{12} = 4$

   (ii) 4×3

6. $$x+y+x-y=\begin{bmatrix}5 & 2\\ 0 & 9\end{bmatrix}+\begin{bmatrix}3 & 6\\ 0 & -1\end{bmatrix}$$

$$2x=\begin{bmatrix}8 & 8\\ 0 & 8\end{bmatrix}$$

$$x=\begin{bmatrix}4 & 4\\ 0 & 4\end{bmatrix}$$

$$(x+y)-(x-y)=\begin{bmatrix}5 & 2\\ 0 & 9\end{bmatrix}-\begin{bmatrix}3 & 6\\ 0 & -1\end{bmatrix}$$

$$x+y-x+y=\begin{bmatrix}2 & -4\\ 0 & 10\end{bmatrix}$$

$$y=\begin{bmatrix}1 & -2\\ 0 & 5\end{bmatrix}$$

7. L.H.S = f(x). f(y)

$$=\begin{bmatrix}\cos x & -\sin x & 0\\ \sin x & \cos x & 0\\ 0 & 0 & 1\end{bmatrix}-\begin{bmatrix}\cos y & -\sin y & 0\\ \sin y & \cos y & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}\cos x\cos y-\sin x.\sin y+0 & -\sin y\cos x-\sin x\cos y+0 & 0+0+0\\ \sin x\cos y+\cos x.\sin y+0 & -\sin x.\sin y+\cos x.\cos y+0 & 0+0+0\\ 0+0+0 & 0+0+0 & 0+0+1\end{bmatrix}$$

$$\begin{bmatrix} \cos(x+y) & -\sin(x+y) & 0 \\ \sin(x+y) & \cos(x+y) & 0 \\ 0 & 0 & 1 \end{bmatrix} = F(x+y)$$

8. $A^2 = A.A$

$$= \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 9-8 & -6+4 \\ 12-8 & -8+4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix}$$

$$A^2 = KA - 2I$$

$$\begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = K \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} - 2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = \begin{bmatrix} 3K & -2K \\ 4K & -2K \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 3 & -2 \\ 4 & -4 \end{bmatrix} = \begin{bmatrix} 3K & -2K \\ 4K & -2K \end{bmatrix}$$

$$K = 1$$

9. $$AB = \begin{bmatrix} -2 & -6 & 12 \\ 4 & 12 & -24 \\ 5 & 15 & -30 \end{bmatrix}$$

$$A' = \begin{bmatrix} -2 & 4 & 5 \end{bmatrix}$$

$$B' = \begin{bmatrix} 1 \\ 3 \\ -6 \end{bmatrix}$$

$$B'A' = \begin{bmatrix} 1 \\ 3 \\ -6 \end{bmatrix} \begin{bmatrix} -2 & 4 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 4 & 5 \\ -6 & 12 & 15 \\ 12 & -24 & -30 \end{bmatrix}$$

$$AB' = \begin{bmatrix} -2 & 4 & 5 \\ -6 & 12 & 15 \\ 12 & -24 & -30 \end{bmatrix}$$

$$AB' = B'A'$$

10. Put $\tan\frac{\alpha}{2} = t$

$$A = \begin{bmatrix} 0 & -t \\ t & 0 \end{bmatrix}$$

$$I + A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & -t \\ t & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -t \\ t & 1 \end{bmatrix}$$

$$I - A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & -t \\ t & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & t \\ -t & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & t \\ -t & 1 \end{bmatrix}$$

$$L.H.S = (I - A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$$= (I - A)\begin{bmatrix} \dfrac{1-\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}} & \dfrac{-2\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}} \\ \dfrac{2\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}} & \dfrac{1-\tan^2\frac{\alpha}{2}}{1+\tan^2\frac{\alpha}{2}} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & t \\ -t & 1 \end{bmatrix}\begin{bmatrix} \dfrac{1-t^2}{1+t^2} & \dfrac{-2t}{1+t^2} \\ \dfrac{-2t}{1+t^2} & \dfrac{1-t^2}{1+t^2} \end{bmatrix}$$

$$=\begin{bmatrix} \frac{1-t^2}{1+t^2}+\frac{t.2t}{1+t^2} & \frac{-2t}{1+t^2}+\text{t}\left(\frac{1-t^2}{1+t^2}\right) \\ \text{-t}\left(\frac{1-t^2}{1+t^2}\right)+\frac{2t}{1+t^2} & \text{-t}\left(\frac{-2t}{1+t^2}\right)+\left(\frac{1-t^2}{1+t^2}\right) \end{bmatrix}$$

$$=\begin{bmatrix} \frac{1-t^2+2t^2}{1+t^2} & \frac{-2t+t-t^3}{1+t^2} \\ \frac{-t+t^3+2t}{1+t^2} & \frac{2t^2+1-t^2}{1+t^2} \end{bmatrix}$$

$$=\begin{bmatrix} \frac{1+t^2}{1+t^2} & \frac{-t^3-t}{1+t^2} \\ \frac{t^3+t}{1+t^2} & \frac{t^2+1}{1+t^2} \end{bmatrix}$$

$$=\begin{bmatrix} 1 & \frac{-t(1+t^2)}{1+t^2} \\ \frac{t(1+t^2)}{1+t^2} & \frac{t^2+1}{1+t^2} \end{bmatrix}$$

$$=\begin{bmatrix} 1 & \text{-t} \\ \text{t} & 1 \end{bmatrix}$$

L.H.S = R.H.S

Hence prove

CBSE TEST PAPER-02

CLASS - XII MATHEMATICS (algebra)

Topic:- Matrices

1. Two matrices A = [aij] and B = [bij] are said to be equal if [1]

2. Define scalar matrix [1]

3. Every diagonal element of a skew symmetric matrix is [1]

4. If $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}, them\ A + A' = I$ Find α [1]

5. $A = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix} Find\ A + A'$ [1]

6. Construct a 3 $\times$4 matrix, whose element are given by aij = $\frac{1}{2}\left|-3i + j\right|$ [4]

7. Obtain the inverse of the following matrix using elementary operations [4]

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$

8. Let $A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 5 & 2 \\ 7 & 4 \end{bmatrix}$, $C = \begin{bmatrix} 2 & 5 \\ 3 & 8 \end{bmatrix}$ Find a matrix D such that CD – AB = 0 [4]

9. If $A = \begin{bmatrix} 3 & -4 \\ 1 & 1 \end{bmatrix}$, then prove that $A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$ where n is any positive integer [4]

10. for what values of x $\begin{bmatrix} 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = 0$ [4]

CBSE TEST PAPER-02

CLASS - XII MATHEMATICS (algebra)

[ANSWERS]

Topic:- Matrices

1. They are of the same order.
2. A square matrix in which every non – diagonal element is zero is called diagonal matrix.
3. Zero.
4. $A+A'=\begin{bmatrix}\cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha\end{bmatrix}+\begin{bmatrix}\cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha\end{bmatrix}$

$=\begin{bmatrix}2\cos & 0 \\ 0 & 2\cos\end{bmatrix}$

$A+A'=I(Given)$

$\begin{bmatrix}2\cos & 0 \\ 0 & 2\cos\end{bmatrix}=\begin{bmatrix}1 & 0 \\ 0 & 1\end{bmatrix}$

$2\cos\alpha=1$

$\cos\alpha=\dfrac{1}{2}$

$\cos\alpha=\cos\dfrac{\pi}{3}$

$\alpha=\dfrac{\pi}{3}$

5. $A+A'=\begin{bmatrix}1 & 5 \\ 6 & 7\end{bmatrix}+\begin{bmatrix}1 & 6 \\ 5 & 7\end{bmatrix}$

$=\begin{bmatrix}2 & 11 \\ 11 & 14\end{bmatrix}$

6. Let $A=\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}_{3\times 4}$

$$a_{11}=1, a_{12}=\frac{1}{2}, a_{13}=0,\ a_{14}=\frac{1}{2}$$

$$a_{21}=\frac{5}{2}, a_{22}=2, a_{23}=\frac{3}{2}, a_{24}=1$$

$$a_{31}=4, a_{32}=\frac{7}{2}, a_{33}=3, a_{34}=\frac{5}{2}$$

$$A=\begin{bmatrix} 1 & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{5}{2} & 2 & \frac{3}{2} & 1 \\ 4 & \frac{7}{2} & 3 & \frac{5}{2} \end{bmatrix}_{3\times 4}$$

7. $A=IA$

$$\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.A$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 3 & 1 & 1 \end{bmatrix}=\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}.A \qquad R_1 \Leftrightarrow R_2$$

$$D=\begin{bmatrix} -191 & -110 \\ 77 & 44 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & -5 & -8 \end{bmatrix}=\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & -3 & 1 \end{bmatrix}.A \qquad R_3 \to R_3-3R_1$$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & -5 & -8 \end{bmatrix}=\begin{bmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & -3 & 1 \end{bmatrix}.A \qquad R_1 \to R_1-2R_2$$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix}=\begin{bmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ 5 & -3 & 1 \end{bmatrix}.A \qquad R_3 \to R_3+5R_2$$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ \frac{5}{2} & \frac{-3}{2} & \frac{1}{2} \end{bmatrix}.A \qquad R_3 \to \frac{1}{2}R_3$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{-1}{2} & \frac{1}{2} \\ 1 & 0 & 0 \\ \frac{5}{2} & \frac{-3}{2} & \frac{1}{2} \end{bmatrix}.A \qquad \mathrm{R}_1 \to \mathrm{R}_1 + \mathrm{R}_3$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{-1}{2} & \frac{1}{2} \\ -4 & 3 & -1 \\ \frac{5}{2} & \frac{-3}{2} & \frac{1}{2} \end{bmatrix}.A$$

$$A^{-1} = \begin{bmatrix} \frac{1}{2} & \frac{-1}{2} & \frac{1}{2} \\ -4 & 3 & -1 \\ \frac{5}{2} & \frac{-3}{2} & \frac{1}{2} \end{bmatrix}$$

8. Let $D = \begin{bmatrix} a & \mathrm{b} \\ \mathrm{c} & \mathrm{d} \end{bmatrix}$

$$\begin{bmatrix} 2 & 5 \\ 3 & 8 \end{bmatrix}\begin{bmatrix} a & \mathrm{b} \\ \mathrm{c} & \mathrm{d} \end{bmatrix} - \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 5 & 2 \\ 7 & 4 \end{bmatrix} = 0$$

$$\begin{bmatrix} 2a+5c & \mathrm{2b+5d} \\ \mathrm{3a+8c} & \mathrm{3b+8d} \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 43 & 22 \end{bmatrix} = 0$$

$$\begin{bmatrix} 2a+5c-3 & \mathrm{2b+5d} \\ \mathrm{3a+8c-43} & \mathrm{3b+8d-22} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

2a + 5c – 3 = 0

2b + 5d = 0

3a + 8c – 43 = 0

3b + 8d – 22 = 0

a = -191, b = -110, c = 77, d = 44

$$D = \begin{bmatrix} -191 & -110 \\ 77 & 44 \end{bmatrix}$$

9. For n = 1

$$\therefore A' = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$$

Hence result is true for n =1

Let result is true for n = k

$$A^K = \begin{bmatrix} 1+2K & -4K \\ K & 1-2K \end{bmatrix} \qquad (i)$$

now, we prove their result is true for n = k + 1

$$A^{t+1} = A.A^K$$

$$= \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} . \begin{bmatrix} 1+2K & -4K \\ K & 1-2K \end{bmatrix}$$

$$= \begin{bmatrix} 2K+3 & -4K-4 \\ K+1 & -2K-1 \end{bmatrix}$$

$\therefore$ P (K + 1) is true Hence P (n) is true.

10. $$\begin{bmatrix} 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 0 + 4 + 0 \\ 0 + 0 + x \\ 0 + 0 + 2x \end{bmatrix} = 0$$

$$\begin{bmatrix} 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ x \\ 2x \end{bmatrix} = 0$$

4 + 2x + 2x = 0

4x =-4

x = -1

CBSE TEST PAPER-03

CLASS - XII MATHEMATICS (algebra)

Topic:- Matrices

1. If $A=\begin{bmatrix}\alpha & \beta \\ \gamma & -\alpha\end{bmatrix}$ and A^2 = I. Find relation. [1]

2. If the matrix A is both symmetric and skews symmetric, then A will be. [1]

3. Matrices A and B will be inverse of each other only if [1]

4. If A, B are symmetric matrices of same order, them AB – BA is a [1]

5. Diagonal of skew symmetric matrix are [1]

6. Find the matrix X so that $X\begin{bmatrix}1 & 2 & 3 \\ 4 & 5 & 6\end{bmatrix}=\begin{bmatrix}-7 & -8 & -9 \\ 2 & 4 & 6\end{bmatrix}$ [4]

7. $A=\begin{bmatrix}0 & 1 \\ 0 & 0\end{bmatrix}$, Show that [4]

   $(aI+bA)^n = a^n I + na^{n-1}bA$, Where I is the identify matrix of order 2 and $n \in N$

8. Find the values of x, y, z if the matrix [4]

   $A=\begin{bmatrix}0 & 2y & z \\ x & y & -z \\ x & -y & z\end{bmatrix}$ Satisfy the equation $A'A = I_3$

9. If $A=\begin{bmatrix}3 & 1 \\ -1 & 2\end{bmatrix}$, Show that $A^2 - 5A = 7I = 0$ [4]

10. If A is a square matrix such that $A^2 = A$, then $(I + A)^3 - 7A$ is equal to [4]

CBSE TEST PAPER-01

CLASS - XII MATHEMATICS (algebra)

[ANSWERS]

Topic:- Matrices

1. $$A^2 = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}\begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$$

$$= \begin{bmatrix} \alpha^2 + \beta\gamma & 2\beta - \alpha\beta \\ \alpha\gamma - \alpha\gamma & \beta\gamma + \alpha^2 \end{bmatrix}$$

ATQ. $$\begin{bmatrix} \alpha^2 + \beta\gamma & \alpha \\ \alpha\gamma - \alpha\gamma & \beta\gamma + \alpha^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\alpha^2 + \beta\gamma = 1$$

$$\alpha^2 + \beta\gamma - 1 = 0$$

2. $A^1 = A$
$A^1 = -A$
$\Rightarrow A = -A$
$2A = 0$
$A = 0$

3. AB = BA = I

4. P = AB – BA
$P' = (AB - BA)'$

$P' = (AB)' - (BA)'$

$$= B'A' - A'B' = \begin{bmatrix} \because A' = A \\ B' = B \end{bmatrix}$$

$= BA - AB$
$= -(AB - BA)$
$= -P$

5. Zero

6. Let $X = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$\therefore \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$$

$$\begin{bmatrix} a+4b & 2a+5b & 3a+6b \\ c+4d & 2c+5d & 3c+6d \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$$

a = 1, b = -2, c = 2, d = 0

$$X = \begin{bmatrix} 1 & -2 \\ 2 & 0 \end{bmatrix}$$

7. When n = 1

$(aI + ba)^1 = a^1 I + 1.a^{1-1}.ba$

aI + bA = aI + bA

L.H.S = R.H.S

When n = k

$(aI + bA)^K = A^K I + Ka^{K-1}bA$……….. (i)

Result is true for n = k

When n = k + 1

$(aI + bA)^{k+1} = (aI + bA).\ (aI + bA)^k$

$= (aI + bA).\ (a^kI + ka^{k-1}ba)$ [From (i)]

$= aI\ (a^kI + ka^{k-1}ba) + bA\ (a^kI + ka^{k-1}\ bA)$

$= a^{k+1}I + ka^kba + a^kba + ka^{k-1}\ b^2A^2$

$$\begin{bmatrix} \because I\ I = I \\ IA = A = AI \end{bmatrix}$$

$= a^{k+1} + (k+1)\ a^kbA \ \left[\because A^2 = 0\right]$

Hence result is true for n = k+1

When eves it is true for n = k

8. $A'A = I_3 (Given)$

$$\begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix}\begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2y^2 & 0 & 0 \\ 0 & 6y^2 & 0 \\ 0 & 0 & 3z^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$x = \pm\frac{1}{\sqrt{2}}, y = \pm\frac{1}{\sqrt{6}}, z = \pm\frac{1}{\sqrt{3}}$$

9. $A^2 - 5A + 7I = 0$

$$L.H.S = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = R.H.S$$

$A^2 = 5A - 7I$

$A^2 = A^2.A$

$= (5A - 7I) .A$

$= 5A^2 - 7AI$

$= 5A^2 - 7A \; [\because IA = A]$

$= 5(5A - 7I) - 7A$

$= 25A - 35I - 7A$

$= 18A - 35I$

$A^4 = A^3.A$

$= (18A - 35I).A$

$=18A^2 - 35IA$

$= 18(5A - 7I) - 35A$

$= 90A - 126I - 35A$

$= 55A - 126I$

$$= 55\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} - 126\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 165 & 55 \\ -55 & 110 \end{bmatrix} + \begin{bmatrix} -126 & 0 \\ 0 & -126 \end{bmatrix}$$

$$= \begin{bmatrix} 39 & 55 \\ -55 & -16 \end{bmatrix}$$

10. $(I + A)^3 - 7A = I^3 + A^3 + 3IA\,(I + A) - 7A$

$= I + A^3 + 3I^2A + 3IA^2 - 7A$

$= I + A^3 + 3A + 3A^2 - 7A$

$= I + A^3 + 3A + 3A - 7A \; \{A^2 = A\}$

$= I + A^3 - A \; \begin{Bmatrix} A^2 = A \\ A^3 = A^2 \end{Bmatrix}$

$= I + A^2 - A$

$= I + A - A \; \{A^2 = A\}$

$= I$

CBSE TEST PAPER-04

CLASS - XII MATHEMATICS (algebra)

<u>Topic:- Matrices</u>

1. If A and B are symmetric matrices of the same order, prove that AB + BA is symmetric [1]

2. If $A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$, Prove that $A - A^t$ is a skew – symmetric matrix [1]

3. If A is any square matrix, prove that AA^1 is symmetric [1]

4. Solve for x and y given that $\begin{bmatrix} 2 & -3 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ 4 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$ [1]

5. Give example of matrices such that AB = 0, BA = 0, $A \neq 0$, $B \neq 0$ [1]

6. Construct 2×3 matrix whose element aij are given by [4]

$$aij = \begin{bmatrix} \text{2i+j} & \text{when} & \text{i<j} \\ \text{4i.j} & \text{when} & \text{i=j} \\ \text{i+2j} & \text{when} & \text{i>j} \end{bmatrix}$$

7. If $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$, then show that $A^3 - 23A - 40I = 0$ [4]

8. Express the matrix $B = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ as the sum of a symmetric and a skew symmetric matrix. [4]

9. If $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$, *prove* that $A^n = \begin{bmatrix} 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \end{bmatrix}$ [4]

10. If $A = \begin{bmatrix} 8 & 0 \\ 4 & -2 \\ 3 & 6 \end{bmatrix}$ $B = \begin{bmatrix} 2 & -2 \\ 4 & 2 \\ -5 & 1 \end{bmatrix}$ then find the matrix X such that 2A + 3X = 5B. [4]

CBSE TEST PAPER-04

CLASS - XII MATHEMATICS (algebra)

[ANSWERS]

Topic:- Matrices

1. Let $P = AB + BA$

$P' = (AB + BA)'$

$= (AB)' + (BA)'$

$= B'A' + A'B$

$= BA + AB\left[A' = A, B' = B\right]$

$= AB + BA$

$= P$

2. $P = A - A^t$

$= \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} + \begin{bmatrix} -2 & -4 \\ -3 & -5 \end{bmatrix}$

$= \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

$P' = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

$P' = -\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

$P' = -P$

Prove

3. Let $P = AA'$

$P' = (AA)'$

$= \left[(A')'A'\right]$

$= AA'$

$= P$ Pr*ove*

4. $\begin{bmatrix} 2x-3y \\ x+y \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

$2x - 3y = 1$

x + y = 3
x = 3 – y
2 (3 – y) – 3y = 1
-5y = -5
y = 1
x = 3 – 1
x = 2

5. $A=\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix}, B=\begin{bmatrix}0 & 0\\ 0 & 1\end{bmatrix}$

$AB=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}, BA=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$

6. $A=\begin{bmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\end{bmatrix}_{2\times 3}$

For i = j
aij = 4i.j
$a_{11} = 4 \times 1 = 4$
$a_{22} = 4\times 2 \times 2 = 16$
For i < j
aij = 2i + j
$a_{12} = 2 \times 1 + 2 = 4$
$a_{13} = 2 \times 1 + 3 = 5$
$a_{23} = 2 \times 2 + 3 = 7$
For i > j
aij = I + 2j
$a_{21} = 2 + 2 \times 1 = 4$

$A=\begin{bmatrix}4 & 4 & 5\\ 4 & 16 & 7\end{bmatrix}$

7. $A^2 = A.A\begin{bmatrix}19 & 4 & 8\\ 1 & 12 & 8\\ 14 & 6 & 15\end{bmatrix}$

$$A^3 = A.A^2\begin{bmatrix}1 & 2 & 3\\ 3 & -2 & 1\\ 4 & 2 & 1\end{bmatrix}\begin{bmatrix}19 & 4 & 8\\ 1 & 12 & 8\\ 14 & 6 & 15\end{bmatrix}$$

$$=\begin{bmatrix}63 & 46 & 69\\ 69 & -6 & 23\\ 92 & 46 & 63\end{bmatrix}$$

$$A^3-23A-40I=\begin{bmatrix}63 & 46 & 69\\ 69 & -6 & 23\\ 92 & 46 & 63\end{bmatrix}-23\begin{bmatrix}1 & 2 & 3\\ 3 & -2 & 1\\ 4 & 2 & 1\end{bmatrix}-40\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}=0$$

8. $$B'=\begin{bmatrix}2 & -1 & 1\\ -2 & 3 & -2\\ -4 & 4 & -3\end{bmatrix}$$

$$\text{Let } P=\frac{1}{2}(B+B')=\begin{bmatrix}2 & \frac{-3}{2} & \frac{-3}{2}\\ \frac{-3}{2} & 3 & 1\\ \frac{-3}{2} & 1 & -3\end{bmatrix}$$

$$P'=\begin{bmatrix}2 & \frac{-3}{2} & \frac{-3}{2}\\ \frac{-3}{2} & 3 & 1\\ \frac{-3}{2} & 1 & -3\end{bmatrix}=P$$

Thus P = $\frac{1}{2}$(B+B') is a symmetric matrix

$$\text{Let } Q=\frac{1}{3}(B-B')=\begin{bmatrix}0 & \frac{-1}{2} & \frac{-5}{2}\\ \frac{1}{2} & 0 & 3\\ \frac{5}{2} & -3 & 0\end{bmatrix}$$

$$Q' = \begin{bmatrix} 0 & \frac{-1}{2} & \frac{5}{2} \\ \frac{-1}{2} & 0 & -3 \\ \frac{-5}{2} & 3 & 0 \end{bmatrix}$$

$$Q' = \begin{bmatrix} 0 & \frac{-1}{2} & \frac{-5}{2} \\ \frac{1}{2} & 0 & 3 \\ \frac{5}{2} & -3 & 0 \end{bmatrix}$$

$Q' = -Q$

Thus $Q = \frac{1}{2}(B - B')$ is a skew symmetric matrix

$$P + Q = \begin{bmatrix} 2 & \frac{-3}{2} & \frac{-3}{2} \\ \frac{-3}{2} & 3 & 1 \\ \frac{-3}{2} & 1 & -3 \end{bmatrix} + \begin{bmatrix} 0 & \frac{-1}{2} & \frac{-5}{2} \\ \frac{1}{2} & 0 & 3 \\ \frac{5}{2} & -3 & 0 \end{bmatrix}$$

9. For n = 1

$$A' = \begin{bmatrix} 3^{1-1} & 3^{1-1} & 3^{1-1} \\ 3^{1-1} & 3^{1-1} & 3^{1-1} \\ 3^{1-1} & 3^{1-1} & 3^{1-1} \end{bmatrix} = \begin{bmatrix} 3^0 & 3^0 & 3^0 \\ 3^0 & 3^0 & 3^0 \\ 3^0 & 3^0 & 3^0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Result is true for n = 1

Let it be true for n = k

$$A^k = \begin{bmatrix} 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \end{bmatrix}$$

$$\therefore A^{k+1} = A.Ak$$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \end{bmatrix}$$

$$= \begin{bmatrix} 3.3^{k-1} & 3.3^{k-1} & 3.3^{k-1} \\ 3.3^{k-1} & 3.3^{k-1} & 3.3^{k-1} \\ 3.3^{k-1} & 3.3^{k-1} & 3.3^{k-1} \end{bmatrix}$$

$$\begin{bmatrix} 3^k & 3^k & 3^k \\ 3^k & 3^k & 3^k \\ 3^k & 3^k & 3^k \end{bmatrix}$$

Thus result is true for n = k+1

Whenever it is true for n = k

10. 3X = 5B – 2A

$$= 5\begin{bmatrix} 2 & -2 \\ 4 & 2 \\ -5 & 1 \end{bmatrix} - 2\begin{bmatrix} 8 & 0 \\ 4 & -2 \\ 3 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & -10 \\ 20 & 10 \\ -25 & 5 \end{bmatrix} + \begin{bmatrix} -16 & 0 \\ -8 & 4 \\ -6 & -12 \end{bmatrix}$$

$$\begin{bmatrix} -6 & -10 \\ 12 & 14 \\ -31 & -7 \end{bmatrix}$$

$$X = \frac{1}{3}\begin{bmatrix} -6 & -10 \\ 12 & 14 \\ -31 & -7 \end{bmatrix}$$

CBSE TEST PAPER-05

CLASS - XII MATHEMATICS (algebra)

Topic:- Matrices

1. Show that $A=\begin{bmatrix}0 & 1 & -1\\ -1 & 0 & 1\\ 1 & -1 & 0\end{bmatrix}$, is skew symmetric matrix. [1]

2. $A=\begin{bmatrix}2 & 4\\ 5 & 6\end{bmatrix}$, Prove that $A+A'$ is a symmetric matrix [1]

3. If $A=\begin{bmatrix}-1 & 5\\ 3 & 2\end{bmatrix}$ show that $(3A)'=3A'$ [1]

4. Solve for x and y, given that $\begin{bmatrix}x & y\\ 3y & x\end{bmatrix}\begin{bmatrix}1\\ 2\end{bmatrix}=\begin{bmatrix}3\\ 5\end{bmatrix}$ [1]

5. Given example of matrix A and B such that AB = 0 but A $\neq$ 0, B $\neq$ 0 [1]

6. If $A=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$ then prove that $A^n=\begin{bmatrix}\cos n\theta & \sin n\theta\\ -\sin n\theta & \cos n\theta\end{bmatrix}$ [4]

7. Find X and Y, if 2x + 3y = $\begin{bmatrix}2 & 3\\ 4 & 0\end{bmatrix}$ *and* $3x+2y=\begin{bmatrix}2 & -2\\ -1 & 5\end{bmatrix}$ [4]

8. If $A=\begin{bmatrix}\cos^2\alpha & \cos\alpha\sin\alpha\\ \cos\alpha\sin\alpha & \sin^2\alpha\end{bmatrix}$, $B=\begin{bmatrix}\cos^2\beta & \cos\beta\sin\beta\\ \cos\beta\sin\beta & \sin^2\beta\end{bmatrix}$

   Show that AB is a zero matrix if α and β differ by an odd multiple of $\frac{\pi}{2}$. [4]

9. If f(x) = x^2 – 5x + 7 and $A=\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}$ find f(A) [4]

10. $A=\begin{bmatrix}4 & 3\\ 2 & 5\end{bmatrix}$, find x and y such that A^2 – xA + yI = 0 [4]

CBSE TEST PAPER-05

CLASS - XII MATHEMATICS (algebra)

[ANSWERS]

Topic:- Matrices

1. $\begin{bmatrix} x & y \\ 3y & x \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix} A' = \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix}$

$A' = -\begin{bmatrix} 0 & -1 & 1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$

$A' = -A$ Prove

2. $P = A + A' = \begin{bmatrix} 2 & 4 \\ 5 & 6 \end{bmatrix} + \begin{bmatrix} 2 & 5 \\ 4 & 6 \end{bmatrix}$

$P = \begin{bmatrix} 4 & 9 \\ 9 & 12 \end{bmatrix}$

$P' = \begin{bmatrix} 4 & 9 \\ 9 & 12 \end{bmatrix}$

$P' = P$ prove

3. $3A = \begin{bmatrix} -3 & 15 \\ 9 & 6 \end{bmatrix}$

$(3A)' = \begin{bmatrix} -3 & 9 \\ 15 & 6 \end{bmatrix}$

$3A' = 3\begin{bmatrix} -1 & 3 \\ 5 & 2 \end{bmatrix}$

$= \begin{bmatrix} -3 & 9 \\ 15 & 6 \end{bmatrix}$

Prove

4. $\begin{bmatrix} x & y \\ 3y & x \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$

x + 2y = 3

$3y + 2x = 5$
$\Rightarrow 2x + 4y = 6$
$2x + 3y = 5$
$- \quad - \quad -$
$y = 1$

$x + 2(1) = 3$
$x = 1$

5. $\alpha - \beta \ A = \begin{bmatrix} 0 & -1 \\ 0 & 2 \end{bmatrix}, B = 3\begin{bmatrix} 3 & 5 \\ 0 & 0 \end{bmatrix}$

$AB = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

6. For $n = 1$

$$A' = \begin{bmatrix} \cos 1.\theta & \sin 1.\theta \\ -\sin 1.\theta & \cos 1.\theta \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

Result is true for n = 1

Let result is true for n = k

$$A^k = \begin{bmatrix} \cos k\theta & \sin k\theta \\ -\sin k\theta & \cos k\theta \end{bmatrix}$$

for $n = k + 1$

$A^{k+1} = A.A^k$

$$= \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} . \begin{bmatrix} \cos k\theta & \sin k\theta \\ -\sin k\theta & \cos k\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta.\cos k\theta - \sin\theta.\sin k\theta & \cos\theta.\sin k\theta + \sin\theta.\cos k\theta \\ \sin\theta.\cos k\theta - \cos\theta.\sin k\theta & -\sin\theta.\sin k\theta + \cos\theta.\cos k\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos(\theta + k\theta) & \sin(\theta + k\theta) \\ -\sin(\theta + k\theta) & \cos(\theta + k\theta) \end{bmatrix}$$

$$= \begin{bmatrix} \cos(k+1)\theta & \sin(k+1)\theta \\ -\sin(k+1)\theta & \cos(k+1)\theta \end{bmatrix}$$

Thus result is true for n = k + 1

Whenever result is true for n = k

7. On adding

$$5x+5y=\begin{bmatrix}2 & 3\\ 4 & 0\end{bmatrix}+\begin{bmatrix}2 & -2\\ -1 & 5\end{bmatrix}$$

$$5(x+y)=3\begin{bmatrix}4 & 1\\ 4 & 5\end{bmatrix}$$

$$(x+y)=\frac{1}{5}\begin{bmatrix}\frac{4}{5} & \frac{1}{5}\\ \frac{3}{5} & 1\end{bmatrix}$$

$$x-y=\begin{bmatrix}2 & -2\\ -1 & 5\end{bmatrix}-\begin{bmatrix}2 & 3\\ 4 & 0\end{bmatrix}$$

$$x-y=\begin{bmatrix}0 & 3\\ -5 & 5\end{bmatrix}$$

$$2x=\begin{bmatrix}\frac{4}{5} & \frac{-24}{5}\\ \frac{-22}{5} & 6\end{bmatrix}$$

$$x=\begin{bmatrix}\frac{2}{5} & \frac{-12}{5}\\ \frac{-11}{5} & 3\end{bmatrix}$$

$$x+y=\begin{bmatrix}\frac{4}{5} & \frac{1}{5}\\ \frac{3}{5} & 1\end{bmatrix}$$

$$\begin{bmatrix}\frac{2}{5} & \frac{-12}{5}\\ \frac{-11}{5} & 3\end{bmatrix}+y=\begin{bmatrix}\frac{4}{5} & \frac{1}{5}\\ \frac{3}{5} & \frac{1}{1}\end{bmatrix}$$

$$y=\begin{bmatrix}\frac{4}{5} & \frac{1}{5}\\ \frac{3}{5} & 1\end{bmatrix}+\begin{bmatrix}\frac{-2}{5} & \frac{12}{5}\\ \frac{11}{5} & -3\end{bmatrix}$$

$$=\begin{bmatrix}\frac{4}{5}-\frac{2}{5} & \frac{1}{5}+\frac{12}{5}\\ \frac{3}{5}+\frac{11}{5} & 1-3\end{bmatrix}$$

$$=\begin{bmatrix}\frac{2}{5} & \frac{13}{5}\\ \frac{14}{5} & -2\end{bmatrix}$$

8. $$AB=\begin{bmatrix}\cos^2\alpha\cos^2\beta+\cos\alpha.\sin\alpha.\cos\beta.\sin\beta & \cos^2\alpha\cos\beta.\sin\beta+\cos\alpha.\sin\alpha.\sin^2\beta\\ \cos\alpha\sin\alpha.\cos^2\beta+\sin^2\alpha\cos\beta.\sin\beta & \cos\alpha.\sin\alpha\cos\beta.\sin\beta+\sin\alpha\sin^2\beta\end{bmatrix}$$

$$=\begin{bmatrix}\cos\alpha\cos\beta.\cos(\alpha-\beta) & \cos\alpha.\sin.\cos(\alpha-\beta)\\ \sin\alpha.\cos\beta.\cos(\alpha-\beta) & \sin\alpha.\sin\beta.\cos(\alpha-\beta)\end{bmatrix}$$

$$=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$$

$\alpha-\beta$ is odd multiple of $\frac{\pi}{2}$

$$\therefore \cos(\alpha-\beta)=0$$

9. f(A) = $A^2 - 5A + 7I$

$$A^2 = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$$

f(A) = $A^2 - 5A + 7I$

$$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - 5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$f(A) = \begin{bmatrix} 8-15+7 & 5-5+0 \\ -5+5+0 & 3-10+7 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$f(A) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

10. $$A^2 = \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix} . \begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 22 & 27 \\ 18 & 31 \end{bmatrix}$$

$$\begin{bmatrix} 22 & 27 \\ 18 & 31 \end{bmatrix} - x\begin{bmatrix} 4 & 3 \\ 2 & 5 \end{bmatrix} + y\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 22-4x+y & 27-3x \\ 18-2x & 31-5x+y \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

x = 9

y = 14

CBSE TEST PAPER-06
CLASS - XII MATHEMATICS (algebra)

Topic:- Determinants

1. Find values of x for which $\begin{vmatrix} 3 & x \\ x & 1 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix}$. [1]

2. A be a square matrix of order 3×3, there $|KA|$ is equal to [1]

3. Evaluate $\Delta = \begin{vmatrix} 3 & 2 & 3 \\ 2 & 2 & 3 \\ 3 & 2 & 3 \end{vmatrix}$ [1]

4. Let $\begin{vmatrix} 4 & y \\ x & 1 \end{vmatrix} = \begin{vmatrix} 4 & 2 \\ 4 & 1 \end{vmatrix}$ find all the possible value of x and y if x and y are natural numbers.[1]

5. Show that, using proportion of let. [1]

$$\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix} = 1 + a^2 + b^2 + c^2 \text{ OR } \begin{vmatrix} a & a+b & a+2b \\ a+2b & a & a+b \\ a+b & a+2b & a \end{vmatrix} = 9b^2(a+b)$$

6. $\begin{vmatrix} (y+z)^2 & xy & zx \\ xy & (x+z)^2 & yz \\ xz & yz & (x+y)^2 \end{vmatrix} = 2xyz(x+y+z)^3$ [4]

7. Find the equation of line joining (3, 1) and (9, 3) using determinants. [4]

8. If $A = \begin{bmatrix} 2 & 3 \\ 1 & -4 \end{bmatrix}$ *and* $B = \begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix}$ then verify that $(AB)^{-1} = B^{-1} A^{-1}$ [4]

9. Using cofactors of elements of third column, evaluate $\Delta = \begin{vmatrix} 1 & x & yz \\ 1 & y & zx \\ 1 & z & xy \end{vmatrix}$ [4]

10. If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$ find A^{-1}, using A^{-1} solve the system of equations [4]

$2x - 3y + 5z = 11$
$3x + 2y - 4z = -5$
$x + y - 2z = -3$

CBSE TEST PAPER-06
CLASS - XII MATHEMATICS (algebra)
[ANSWERS]

Topic:- Determinants

1. $(3 - x)^2 = 3 - 8$

$3 - x^2 = 3 - 8$

$-x^2 = -8$

$x = \pm\sqrt{8}$

$x = \pm 2\sqrt{2}$

2. $|KA| = K^n |A|$

$n = 3$

$|KA| = K^3 |A|$

3. $\Delta = 0 [C_1 \text{ and } C_3 \text{ identical}]$

4. $4 - xy = 4 - 8$

$xy = 8$

of			
	$x = 1$	$x = 4$	$x = 8$
	$y = 8$	$y = 1$	$y = 1$

5. Multiplying R_1 R_2 and R_3 by a, b, c respectively

$$L.H.S = \frac{1}{abc}\begin{vmatrix} a^3+a & a^2b & a^2c \\ ab^2 & b^3+b & b^2c \\ c^2a & c^2b & c^3+c \end{vmatrix}$$

Taking a, b, c, common from c_1, c_2, and c_3

$$=\frac{abc}{abc}\begin{vmatrix} a^2+1 & a^2 & a^2 \\ b^2 & b^2+1 & b^2 \\ c^2 & c^2 & c^2+1 \end{vmatrix}$$

$R_1 \to R_1 + R_2 + R_3$

$$=\begin{vmatrix} 1+a^2+b^2+c^2 & 1+a^2+b^2+c^2 & 1+a^2+b^2+c^2 \\ b^2 & b^2+1 & b^2 \\ c^2 & c^2 & c^2+1 \end{vmatrix}$$

$$=\left(1+a^2+b^2+c^2\right)\begin{vmatrix} 1 & 1 & 1 \\ b^2 & b^2+1 & b^2 \\ c^2 & c^2 & c^2+1 \end{vmatrix}$$

$C_1 \to C_1 - C_3, \quad C_2 \to C_2 - C_3$

$$=\left(1+a^2+b^2+c^2\right)\begin{vmatrix} 0 & 0 & 1 \\ 0 & 1 & b^2 \\ -1 & -1 & c^2+1 \end{vmatrix}$$

Expending along R_1

$$=\left(1+a^2+b^2+c^2\right)\left[1(0+1)\right]$$

$$=1+a^2+b^2+c^2$$

$L.H.S = R.H.S$

OR $\begin{vmatrix} a & a+b & a+2b \\ a+2b & a & a+b \\ a+b & a+2b & a \end{vmatrix} = 9b^2(a+b)$ {solve it}

{hint : $C_1 \to C_1 + C_2 + C_3$}

Taking common 3 (a+b) from C_1

$R_1 \to R_1 - R_3, \quad R_2 \to R_2 - R_3$

6. $R_1 \to xR_1, \quad R_2 \to yR_2, \quad R_3 \to ZR_3$

$$\Delta = \frac{1}{xyz}\begin{vmatrix} x(y+z)^2 & x^2y & x^2z \\ xy^2 & y(x+z)^2 & y^2z \\ xz^2 & yz^2 & z(x+y)^2 \end{vmatrix}$$

$$\Delta = \frac{xyz}{xyz}\begin{vmatrix} (y+z)^2 & x^2 & x^2 \\ y^2 & (x+z)^2 & y^2 \\ z^2 & z^2 & (x+y)^2 \end{vmatrix}$$

$C_2 \to C_2 - C_1, \quad C_3 \to C_3 - C_4$

$$\Delta = \begin{vmatrix} (y+z)^2 & x^2-(y+z)^2 & x^2-(y+z)^2 \\ y^2 & (x+z)^2-y^2 & 0 \\ z^2 & 0 & (x+y)^2-z^2 \end{vmatrix}$$

Taking (x + y + z) common from c_2 and C_3

$$\Delta = (x+y+z)^2\begin{vmatrix} (y+z)^2 & x-y-z & x-y-z \\ y^2 & x+z-y & 0 \\ z^2 & 0 & x+y-z \end{vmatrix}$$

$R_1 \to R_1 - (R_2 + R_3)$

$$\Delta = (x+y+z)^2\begin{vmatrix} 2yz & -2z & -2y \\ y^2 & x+z-y & 0 \\ z^2 & 0 & x+y-z \end{vmatrix}$$

$C_2 \to C_2 + \frac{1}{y}C_1 \quad and \quad C_3 \to C_3 + \frac{1}{z}C_1$

$$\Delta = (x+y+z)^2\begin{vmatrix} 2yz & 0 & 0 \\ y^2 & x+z & \frac{y^2}{z} \\ z^2 & \frac{z^2}{y} & x+y \end{vmatrix}$$

Expending along R_1

$$= (x+y+z)^3(2xyz)$$

7. Let (x, y) be any point on the line containing (3, 1) and (9, 3)

$$\begin{vmatrix} x & y & 1 \\ 3 & 1 & 1 \\ 9 & 3 & 1 \end{vmatrix} = 0$$

$$x - 3y = 0$$

8. $$AB = \begin{bmatrix} 2 & 3 \\ 1 & -4 \end{bmatrix} \begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 5 \\ 5 & -14 \end{bmatrix}$$

$$|AB| = -11 \neq 0$$

$$(AB)^{-1} = \frac{1}{11} adj(AB)$$

$$= \frac{-1}{11} \begin{bmatrix} -14 & -5 \\ -5 & -1 \end{bmatrix}$$

$$= \frac{1}{11} \begin{bmatrix} 14 & 5 \\ 5 & 1 \end{bmatrix}$$

$$|A| = -11 \neq 0, \quad |B| = 1 \neq 0$$

$$A^{-1} = \frac{-1}{11} \begin{bmatrix} -4 & -3 \\ -1 & 2 \end{bmatrix}$$

$$B^{-1} = \frac{1}{1} \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$$

$$B^{-1}A^{-1} = \frac{-1}{11} \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} -4 & -3 \\ -1 & 2 \end{bmatrix}$$

$$= \frac{1}{-11} \begin{bmatrix} -14 & -5 \\ -5 & -1 \end{bmatrix}$$

$$=\frac{1}{11}\begin{bmatrix}14 & 5\\ 5 & 1\end{bmatrix}$$

Hence prove.

9. $\Delta = a_{13}A_{13} + a_{23}A_{23} + a_{33}A_{33}$

$= yz(z-y) + zx(x-z) + xy(y-x)$

$= yz^2 - y^2z + zx^2 - z^2x + xy^2 - x^2y$

$= zx^2 - x^2y + xy^2 - z^2x + yz^2 - y^2z$

$= x^2(z-y) + x(y^2 - z^2) + yz(z-y)$

$= (z-y)[x^2 + x(z+y) + yz]$

$= (z-y)[x^2 - xz - xy + yz]$

$= (z-y[x(x-y) - z(x-y)]$

$= (z-y)[(x-y)(x-z)]$

$= (z-y)(x-y)(x-z)$

10. $|A| = \begin{bmatrix}2 & -3 & 5\\ 3 & 2 & -4\\ 1 & 1 & -2\end{bmatrix}$

$(A) = -1 \neq 0$

$A^{-1} exists$

$A^{-1} = \frac{1}{|A|}(adjA)$

$$=\frac{1}{-1}\begin{bmatrix}0 & -1 & 2\\ 2 & -9 & 23\\ 1 & -5 & 13\end{bmatrix}$$

$$=\begin{bmatrix}0 & -1 & -2\\ -2 & 9 & -23\\ -1 & 5 & -13\end{bmatrix}$$

The given system of equation can be written is Ax = B, X = A^{-1}B

$$\begin{bmatrix}2 & -3 & 5\\ 3 & 2 & -4\\ 1 & 5 & -2\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix} = \begin{bmatrix}11\\ -5\\ 3\end{bmatrix}$$

$$\begin{bmatrix}x\\ y\\ z\end{bmatrix} = \begin{bmatrix}2 & -3 & 5\\ 3 & 2 & -4\\ 1 & 5 & -2\end{bmatrix}^{-1}\begin{bmatrix}11\\ -5\\ 3\end{bmatrix}$$

$$=\begin{bmatrix}0 & -1 & -2\\ -2 & 9 & -23\\ -1 & 5 & -13\end{bmatrix}\begin{bmatrix}11\\ -5\\ 3\end{bmatrix}$$

$$\begin{bmatrix}x\\ y\\ z\end{bmatrix} = \begin{bmatrix}1\\ 2\\ 3\end{bmatrix}$$

$x = 1$
$y = 2$
$z = 3$

CBSE TEST PAPER-07

CLASS - XII MATHEMATICS (Algebra)

Topic:- Determinants

1. Solve $\begin{vmatrix} x^2\text{-}x+1 & x+1 \\ x+1 & x+1 \end{vmatrix}$ ` [1]

2. Find minors and cofactors of all the elements of the det. $\begin{vmatrix} 1 & -2 \\ 4 & 3 \end{vmatrix}$ [1]

3. Evaluate $\begin{vmatrix} 102 & 18 & 36 \\ 1 & 3 & 4 \\ 17 & 3 & 6 \end{vmatrix}$ [1]

4. Show that $\begin{vmatrix} \sin 10^o & -\cos 10^o \\ \sin 80^o & \cos 80^o \end{vmatrix} = 1$ [1]

5. Show that, using proportion of let. [1]

$$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = \left(1+a^2+b^2\right)^3$$

6. $\Delta = \begin{vmatrix} a+bx & c+dx & p+qx \\ ax+b & cx+d & px+q \\ u & v & w \end{vmatrix} = \left(1-x^2\right)\begin{vmatrix} a & c & p \\ b & d & q \\ u & v & w \end{vmatrix}$ [4]

7. $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$ Verify that $a_{11}A_{31} + a_{12}A_{32} + a_{13}A_{33} = 0$ [4]

8. If $A=\begin{bmatrix} 3 & -4 \\ -1 & 2 \end{bmatrix}$, find matrix B such that AB = I [4]

9. Using matrices solve the following system of equation [4]

$$\frac{2}{x}+\frac{3}{y}+\frac{10}{z}=4$$

$$\frac{4}{x}-\frac{6}{y}+\frac{5}{z}=1$$

$$\frac{6}{x}+\frac{9}{y}+\frac{-20}{z}=2$$

10. Given $A=\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ *and* $B=\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ find AB and use this result in solving the following system of equation. [4]

$$x-y+z=4$$
$$x-2y-2z=9$$
$$2x+y+3z=1$$

OR

Use produce

$$\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}\begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$$

To solve the system of equations.

x – y + 2z = 1

2y – 3z = 1

3x – 2y + 4z = 2

CBSE TEST PAPER-08
CLASS - XII MATHEMATICS (Algebra)

Topic:- Determinants

1. Find value of x, if $\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$ [1]

2. Find adj A for $A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$ [1]

3. Without expanding, prove that $\Delta = \begin{vmatrix} x+y & y+z & z+x \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix} = 0$ [1]

4. If matrix $A = \begin{bmatrix} 1 & -2 & 3 \\ 1 & 2 & 1 \\ x & 2 & -3 \end{bmatrix}$ is singular, find x. [1]

5. Show that, using properties if det. $\begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix} = \left(1 - x^3\right)^2$ [1]

6. If a, b, c is in A.P, and then finds the value of $\begin{vmatrix} x+2 & x+3 & x+2a \\ x+3 & x+4 & x+2b \\ x+4 & x+5 & x+2c \end{vmatrix}$ [4]

7. $A = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$, Find the no. a and b such that $A^2 + aA + bI = 0$ Hence find A^{-1} [4]

8. Find the area of Δ whose vertices are (3, 8) (-4, 2) and (5, 1) [4]

9. Evaluate $\Delta = \begin{bmatrix} 0 & \sin\alpha & -\cos\alpha \\ -\sin\alpha & 0 & \sin\beta \\ \cos\alpha & -\sin\beta & 0 \end{bmatrix}$ [4]

10. Solve by matrix method [4]

    $x - y + z = 4$

    $2x + y - 3z = 0$

    $x + y + z = 2$

CBSE TEST PAPER-08

CLASS - XII MATHEMATICS (Algebra)

[ANSWERS]

Topic:- Determinants

1. $(2-20) = (2x^2 - 24)$

$-18 = 2x2 - 24$

$-2x^2 = -24 + 18$

$-2x^2 = 6$

$2x^2 = 6$

$x^2 = 3$

$x = \pm\sqrt{3}$

2. adJ A = $\begin{bmatrix} 4 & -3 \\ -1 & 2 \end{bmatrix}$

$$\left[\begin{matrix} \because \quad A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} \\ \textit{change sign} \qquad \text{inter-change} \end{matrix}\right]$$

3. $R_1 \to R_1 + R_2$

$$\Delta = \begin{vmatrix} x+y+z & x+y+z & x+y+z \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix}$$

$$\Delta = (x+y+z) \begin{vmatrix} 1 & 1 & 1 \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix}$$

$$= 0 \begin{bmatrix} \because R_1 \text{ and } R_3 \\ \text{area identical} \end{bmatrix}$$

4. For singular |A| = 0

1(-6 -2) + 2(-3 -x) + 3 (2 -2x) = 0

-8 – 6 – 2x + 6 – 6x = 0

-8x = + 8

x = -1

5. $C_1 \to C_1 + C_2 + C_3$

$$= \begin{vmatrix} 1+x+x^2 & x & x^2 \\ 1+x+x^2 & 1 & x \\ 1+x+x^2 & x^2 & 1 \end{vmatrix}$$

$$= \left(1+x+x^2\right) \begin{vmatrix} 1 & x & x^2 \\ 1 & 1 & x \\ 1 & x^2 & 1 \end{vmatrix}$$

$R_1 \to R_1 - R_3 \qquad R_2 \to R_2 - R_3$

$$= \left(1+x+x^2\right) \begin{vmatrix} 0 & x-x^2 & x^2-1 \\ 0 & 1-x^2 & x-1 \\ 1 & x^2 & 1 \end{vmatrix}$$

$$= \left(1+x+x^2\right) \begin{vmatrix} 0 & x(1-x) & -(1-x)(1+x) \\ 0 & (1-x)(1+x) & -(1-x) \\ 1 & x^2 & 1 \end{vmatrix}$$

Taking (1 – x) common from R_1 and R_2

$$= \left(1+x+x^2\right)(1-x)^2 \begin{vmatrix} 0 & x & -(1+x) \\ 0 & 1+x & -1 \\ 1 & x^2 & 1 \end{vmatrix}$$

Expending along C^1

$$= (1+x+x^2)(1-x)^2[-x+(1+x)^2]$$
$$= (1+x+x^2)(1-x)^2\left(-x+1+x^2+2x\right)$$

$$= (1-x)(1+x+x^2)(1-x)(1+x+x^2)$$
$$= \left(1-x^3\right)^2$$

6. $R_1 \to R_1 + R_3$

$$= \begin{vmatrix} 2x+6 & 2x+8 & 2x+2a+2c \\ x+3 & x+4 & x+2b \\ x+4 & x+5 & x+2c \end{vmatrix}$$

$$= \begin{vmatrix} 2x+6 & 2x+8 & 2x+4b \\ x+3 & x+4 & x+2b \\ x+4 & x+5 & x+2c \end{vmatrix} \quad [2b = a + c]$$

$R_1 \to R_1 - 2R_2$

$$= \begin{vmatrix} 0 & 0 & 0 \\ x+3 & x+4 & x+2b \\ x+4 & x+5 & x+2c \end{vmatrix}$$

$= 0$

7. $A^2 = \begin{bmatrix} 11 & 8 \\ 4 & 3 \end{bmatrix}$

$$A^2 + aA + bI = \begin{bmatrix} 11 & 8 \\ 4 & 3 \end{bmatrix} + a\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} + b\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 11+3a+b & 8+2a \\ 4+a & 3+a+b \end{bmatrix}$$

$$ATQ \begin{bmatrix} 11+3a+b & 8+2a \\ 4+a & 3+a+b \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$a = -4, \quad b = 1$

$A^2 - 4A + I = 0$

$A^2 - 4A = -I$

$AAA^{-1} - 4AA^{-1} = -IA^{-1}$

$A - 4I = -A^{-1}$

$A^{-1} = 4I - A$

$$= \begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix}$$

8. $\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

$$=\frac{1}{2}\begin{vmatrix}3 & 8 & 1\\ -4 & 2 & 1\\ 5 & 1 & 1\end{vmatrix}$$

$$=\frac{1}{2}\left[3(2-1)-8(-4-5)+1(-4-10)\right]$$

$$=\frac{1}{2}[3+72-14]=\frac{61}{2}$$

9. $$\Delta=0\begin{vmatrix}0 & \sin\beta\\ -\sin\beta & 0\end{vmatrix}-\sin\alpha\begin{vmatrix}-\sin\alpha & \sin\beta\\ \cos\alpha & 0\end{vmatrix}-\cos\alpha\begin{vmatrix}-\sin\alpha & 0\\ \cos\alpha & -\sin\beta\end{vmatrix}$$

$$=0$$

10. $$A=\begin{bmatrix}1 & -1 & 1\\ 2 & 1 & -3\\ 1 & 1 & 1\end{bmatrix}$$

$$|A|=\begin{bmatrix}1 & -1 & 1\\ 2 & 1 & -3\\ 1 & 1 & 1\end{bmatrix}$$

$$=10\neq 0$$

$$AdJ\ \ A=\begin{bmatrix}4 & 2 & 2\\ -5 & 0 & 5\\ 1 & -2 & 3\end{bmatrix}$$

$$A^{-1}=\frac{1}{|A|}(adJ\ \ A)$$

$$=\frac{1}{10}\begin{bmatrix}4 & 2 & 2\\ -5 & 0 & 5\\ 1 & -2 & 3\end{bmatrix}$$

System of equation can be written is

$$X=A^{-1}B$$

$$=\frac{1}{10}\begin{bmatrix}4 & 2 & 2\\ -5 & 0 & 5\\ 1 & -2 & 3\end{bmatrix}\begin{bmatrix}4\\ 0\\ 2\end{bmatrix}$$

$$\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}2\\ -1\\ 1\end{bmatrix},$$

$x=2,\quad y=-1,\quad z=1$

CBSE TEST PAPER-09
CLASS - XII MATHEMATICS (Algebra)

Topic:- Determinants

1. If $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$, than x is equal to [1]

2. $A = \begin{bmatrix} 1 & 2 \\ 1 & 4 \end{bmatrix}$ is singular or not [1]

3. Without expanding, prove that $\begin{vmatrix} a & a^2 & bc \\ b & b^2 & ca \\ c & c^2 & ab \end{vmatrix} = \begin{vmatrix} a & a^2 & a^3 \\ b & b^2 & b^3 \\ c & c^2 & c^3 \end{vmatrix}$ [1]

4. $A = \begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$, Verify that det A = det (A') [1]

5. Show that using properties of det. $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$ [1]

   $= abc + bc + ca + ab$

6. If x, y, z are different and $\Delta = \begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$ then show that 1 + xyz = 0 [4]

7. Find the equation of the line joining A (1, 30 and B (0, 0) using det. Find K if D (K, 0) is a point such then area of ΔABC is 3 square unit [4]

8. Show that the matrix $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ satisfies the equation $A^2 - 4A + I = 0$.
   Using this equation, find A^{-1} [4]

9. Solve by matrix method. [4]
   $3x - 2y + 3z = 8$
   $2x + y - z = 1$
   $4x - 3y + 2z = 4$

10. The sum of three no. is 6. Of we multiply third no. by 3 and add second no. to it, we get II. By adding first and third no. we get double of ht second no. represent it algebraically and find the no. using matrix method. [4]

CBSE TEST PAPER-10
CLASS - XII MATHEMATICS (Algebra)

Topic:- Determinants

1. If $A=\begin{bmatrix}1 & 2\\ 4 & 2\end{bmatrix}$, then show that $|2A|=4|A|$ [1]

2. A be a non – singular square matrix of order 3 ×3. Then $|aiJ\ \ A|$ is equal to [1]

3. If A is an invertible matrix of order 2, then det is equal (A^{-1}) to [1]

4. $B=[-7]$ *find* det $B=[1]$ [1]

5. Show that using properties of det. $\begin{vmatrix}x & x^2 & yz\\ y & y^2 & zx\\ z & z^2 & xy\end{vmatrix}=(x-y)(y-z)(z-x)(xy+yz+zx)$ [1]

6. $\begin{vmatrix}\alpha & \alpha^2 & \beta-\gamma\\ \beta & \beta^2 & \gamma+\alpha\\ \gamma & \gamma^2 & \alpha+\beta\end{vmatrix}=(\beta-\gamma)(\gamma-\alpha)(\alpha-\beta)(\alpha+\beta+\gamma)$ [4]

7. Find values of K is area of b is 35 square. Unit and vertices are (2, -6), (5, 4), (K, 4) [4]

8. Using cofauors of elements of second row, evaluate $\Delta=\begin{vmatrix}5 & 3 & 8\\ 2 & 0 & 1\\ 1 & 2 & 3\end{vmatrix}$ [4]

9. If $A=\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}$ Show that $A^2 - 5A + 7I = 0$. Hence find A^{-1} [4]

10. The cost of 4kg onion, 3kg wheat and 2kg rise is Rs. 60. The cost of 2kg onion, 4kg wheat and 6kg rise is Rs. 90. The cost of 6kg onion 2kg wheat and 3kg rise is Rs. 70. Find the cost of each item per kg by matrix method. [4]

CBSE TEST PAPER-10

CLASS - XII MATHEMATICS (Algebra)

[ANSWERS]

Topic:- Determinants

1. $2A = 2\begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}$

$= \begin{bmatrix} 2 & 4 \\ 8 & 4 \end{bmatrix}$

$|2A| = 8 - 32$

$= -24$

Hence Prove

$4|A| = 4 \times (2 - 8)$

$= 4 \times (-6)$

$= -24$

2. $|aiJ\ A| = |A|^{n-1}$

$n = 3$

$|aiJ\ A| = |A|^{3-1}$

$= |A|^2$

3. A is invertible $AA^{-1} = \sqrt{2}$

det (AA^{-1}) = det ($\sqrt{2}$)

det A.(det A^{-1}) = det ($\sqrt{2}$) $[|AB| = |A||B|]$

det A^{-1} = $\dfrac{1}{\det A}$ $\left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 0 \ \ i.e. |I| = 1 \right\}$

4. $|B| = -7 \qquad [\because 10] = a$

5. $R_1 \to R_1 - R_3, \qquad R_2 \to R_2 - R_3$

$$= \begin{vmatrix} (x-z) & (x^2-z^2) & yz-xy \\ y-z & y^2-z^2 & zx-xy \\ z & z^2 & xy \end{vmatrix}$$

$$= (x-z)(y-z)\begin{vmatrix} 1 & x+z & -(y) \\ 1 & y+z & -x \\ z & z^2 & xy \end{vmatrix}$$

$R_1 \to R_1 - R_2$

$$= (x-z)(y-z)\begin{vmatrix} 0 & x-y & x-y \\ 1 & y+z & -x \\ z & z^2 & xy \end{vmatrix}$$

$$= (x-z)(y-z)(x-y)\begin{vmatrix} 0 & 1 & 1 \\ 1 & y+z & -x \\ z & z^2 & xy \end{vmatrix}$$

$$= (x-z)(y-z)(x-y)\left[-1(xy+zx)+1\left(z^2-y^2-z^2\right)\right]$$

$$= (x-z)(y-z)(x-y)[-xy-zx-yz]$$

$$= (x-y)(y-z)(z-x)(xy+yz+zx)$$

6. $R_1 \to R_1 - R_3, \quad R_2 \to R_2 - R_3$

$$L.H.S = \begin{vmatrix} \alpha-\gamma & \alpha^2-\gamma^2 & \beta+\gamma-\alpha-\beta \\ \beta-\gamma & \beta^2-\gamma^2 & \gamma+\alpha-\alpha-\beta \\ \gamma & \gamma^2 & \alpha+\beta \end{vmatrix}$$

$$= \begin{vmatrix} \alpha-\gamma & (\alpha+\gamma) & (\gamma-\alpha) \\ \beta-\gamma & (\beta-\gamma)(\beta+\gamma) & \gamma-\beta \\ \gamma & \gamma^2 & \alpha+\beta \end{vmatrix}$$

$$= (\alpha-\gamma)(\beta-\gamma)\begin{vmatrix} 1 & \alpha+\gamma & -1 \\ 1 & \beta+\gamma & 1 \\ \gamma & \gamma^2 & \alpha+\beta \end{vmatrix}$$

$R_1 \to R_1 - R_2$

$$= (\alpha-\gamma)(\beta-\gamma)\begin{vmatrix} 0 & \alpha-\beta & 0 \\ 1 & \beta+\gamma & -1 \\ \gamma & \gamma^2 & \alpha+\beta \end{vmatrix}$$

Expending along R_1

$$= (\alpha-\gamma)(\beta-\gamma)\left[-(\alpha-\beta)(\alpha+\beta+\gamma)\right]$$

$$= (\beta-\gamma)(\gamma-\alpha)(\alpha-\beta)(\alpha+\beta+\gamma)$$

7. $area\ \Delta = \begin{vmatrix} 2 & -6 & 1 \\ 5 & 4 & 1 \\ K & 4 & 1 \end{vmatrix}$

$= \frac{1}{2}\left[2(4-4)+6(5-K)+1(20-4K)\right]$

$= \frac{1}{2}[50-10K]$

$= 25-5K$

$A+\theta \quad 25-5K = 35$

$K = 12$

8. $\Delta = a_{21}A_{21} + a_{22}A_{22} + a_{23}A_{23}$

$= -2(9-16)+0(15-8)+1(10-3)$

$= 14+0-7$

$= 7$

9. $A^{-1} = \frac{1}{7}\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$ $A^2 - 5A + 7I = \begin{bmatrix} 8 & 5 \\ -5 & 2 \end{bmatrix} - 5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 8-15+7 & 5-5+0 \\ -5+50 & 3-10+7 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Prove.

$A^2 - 5A + 7I = 0$ (given)

$A^2 - 5A = -7I$

$A^2A^{-1} - 5AA^{-1} = -7IA^{-1}$

$AAA^{-1} - 5AA^{-1} = -7IA^{-1}$

$A - 5I = -7A^{-1}$ $\qquad \left[AA^{-1} = I\right]$

$7A^{-1} = 5I - A$

$= 5\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

$= \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$

$A^{-1} = \frac{1}{7}\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$

10. cost of 1kg onion = x
cost of 1kg wheat = y
cost of 1kg rise = z
4x + 3y + 2z = 60

2x + 4y + 6z = 90
6x + 2y + 3z = 70

$$A = \begin{bmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3 \end{bmatrix} B = \begin{bmatrix} 60 \\ 90 \\ 70 \end{bmatrix} X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$|A| = \begin{vmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3 \end{vmatrix} = 50 \neq 0$$

$$aiJ\ A = \begin{bmatrix} 0 & \text{-5} & 10 \\ 30 & 0 & \text{-20} \\ \text{-20} & 10 & 10 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}(aiJ\ A) = \frac{1}{80}\begin{bmatrix} 0 & \text{-5} & 10 \\ 30 & 0 & \text{-20} \\ \text{-20} & 10 & 10 \end{bmatrix}$$

$$X = A^{-1}B$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \\ 8 \end{bmatrix}$$

$x = 5, \qquad y = 8, \qquad z = 8$

CBSE TEST PAPER-01

CLASS - XII MATHEMATICS (Calculus : Continuity & differentiability)

Topic:- differentiation

1. Find the values of K so that the function f is continues at the indicated print. [4]

$$f(x)=\begin{cases}\dfrac{K\cos x}{\pi-2x}, if \ \ x\neq\dfrac{\pi}{2} \\ & at \ x=\dfrac{\pi}{2} \\ 3, \text{ if } x=\dfrac{\pi}{2}\end{cases}$$

2. Differentiate the function $x^{\sin x}+(\sin x)^{\cos x}$ [4]

3. If $x=\sqrt{a^{\sin^{-1}t}}$, $y=\sqrt{a^{\cos^{-1}t}}$ show that $\dfrac{dy}{dx}=\dfrac{-y}{x}$ [4]

4. If y = $(\tan^{-1}x)^2$ show that $(x^2 + 1)^2 y_2 + 2x (x^2 + 1)y_1 = 2$ [4]

5. Verify Rolle's Theorem for the function y = x^2 +2 , [-2 , 2] [4]

6. Differentiate $\sin^{-1}\left[\dfrac{2^{x+1}}{1+4^x}\right]$ [4]

7. Differentiate $\sin^2 x$ w.r. to $e^{\cos x}$ [4]

8. If $x\sqrt{1+y}+y\sqrt{1+x}=0$ prove that $\dfrac{dy}{dx}=-\dfrac{1}{(1+x)^2}$ [4]

9. If cosy = x cos (a + y) prove that $\dfrac{dy}{dx}=\dfrac{\cos^2(a+y)}{\sin a}$ [4]

10. If x = a (cos t + t sin t) [4]

    y = a (sin t – t cos t)

    find $\dfrac{d^2y}{dx^2}$

CBSE TEST PAPER-01

CLASS - XII MATHEMATICS (Calculus : Continuity & differentiability)

Topic: - differentiation

1. $\lim_{x\to\frac{\pi}{2}^-} f(x) = \lim_{x\to\frac{\pi}{2}^-} \frac{K\cos x}{\pi - 2x}$

$= \lim_{h\to 0} \frac{K\cos\left(\frac{\pi}{2}-h\right)}{\pi - 2\left(\frac{\pi}{2}-h\right)}$

$= \lim_{h\to 0} \frac{K\sin\ h}{\pi - 2\frac{\pi}{2} + 2h}$

$= \lim_{h\to 0} \frac{k\sin\ h}{2h}$

$= \lim_{h\to 0} \frac{K}{2}.\left(\frac{\sin\ h}{h}\right)$

$= \frac{K}{2}$

$\lim_{x\to\frac{\pi}{2}^+} f(x) = \lim_{x\to\frac{\pi}{2}} \quad 3$

$= \lim_{h\to 0} 3 = 3$

$AT\theta \frac{K}{2} = \frac{3}{1}$

$\boxed{K = 6}$

2. Let y = u + v

When u = x $^{\sin x}$, v = $(\sin x)^{\cos x}$

$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} ----(1)$

$u = x^{\sin x}$

Taking log both side

log u = log $x^{\sin x}$

log u = sinx . logx

diff. both side w.r. to x

$$\frac{1}{u}\frac{du}{dx} = \sin x.\frac{1}{x} + \log x.\cos x$$

$$\frac{du}{dx} = u\left[\frac{\sin x}{x} + \log x.\cos x\right]$$

$$\frac{du}{dx} = x^{\sin x}\left[\frac{\sin x + x\log x.\cos x}{x}\right]$$

$$v = (\sin x)^{\cos x}$$

Taking log both side

log v = log $(\sin x)^{\cos x}$

$$\log v = \cos x.\log(\sin x)$$

Differentiation both side w.r. to x

$$\frac{1}{v}.\frac{dv}{dx} = \cos x.\frac{1}{\sin x}(\cos x) + \log(\sin x)(-\sin x)$$

$$\frac{dv}{dx} = v\left[\cot x.\cos x - \log(\sin x).\sin x\right]$$

$$\frac{dv}{dx} = (\sin x)^{\cos x}\left[\cot x.\cos x - \log(\sin x).\sin x\right]$$

Hence

$$\frac{dy}{dx} = x^{\sin x}\left[\frac{\sin x + x\log x.\cos x}{x}\right] + (\sin x)^{\cos x}\left[\cot x\cos x - \log(\sin x).\sin x\right]$$

3. $x = \sqrt{a^{\sin^{-1}t}}$

Square both side

$$x^2 = a^{\sin^{-1}t}$$

Differentiation

$$2x\frac{dx}{dt} = a^{\sin^{-1}t}.\log a\frac{1}{\sqrt{1-t^2}} ------(1)$$

$$y = \sqrt{a^{\cos^{-1}t}}$$

$$\Rightarrow y^2 = a^{\cos^{-1}t}$$

$$2y\frac{dy}{dt} = a^{\cos^{-1}t}\log a\frac{-1}{\sqrt{1-t^2}} - - - - - (2)$$

Dividing (2) and (1)

$$\frac{2y\frac{dy}{dt}}{2x\frac{dx}{dt}} = \frac{a^{\cos^{-1}t}\log a\frac{-1}{\sqrt{1-t^2}}}{a^{\sin^{-1}t}\log a\frac{1}{\sqrt{1-t^2}}}$$

$$\frac{y}{x}.\frac{dy}{dx} = -\frac{a^{\cos^{-1}t}}{a^{\sin^{-1}t}}$$

$$\frac{y}{x}.\frac{dy}{dx} = -\frac{y^2}{x^2}\begin{bmatrix}\because a^{\cos^{-1}t} = y^2 \\ a^{\sin^{-1}t} = x^2\end{bmatrix}$$

$$\frac{dy}{dx} = \frac{-y}{x}$$

4. $y = (\tan^{-1} x)^2$ (given)

Differentiation both side w.r. to x

$$y_1 = 2\tan^{-1}x.\frac{1}{1+x^2}$$

$$(1+x^2)y_1 = 2\tan^{-1}x$$

Again differentiation both side w.r. to

$$(1+x^2)y_2 + y_1.(2x) = 2.\frac{1}{1+x^2}$$

$$(1+x^2)y_2 + 2x(x^2+1)y_1 = 2$$

5. $y = x^2 + 2$ is continuous in [-2, 2] and differentiable in (-2, 2). Also f (-2) = f(2) = 6

Hence all the condition of Rolle's Theorem are verified hence their exist value c such that

f' (c) = 0

0 = 2c.

C = 0

Hence prove.

6. $y=\sin^{-1}\left(\frac{2^{x+1}}{1+4^x}\right)$

$=\sin^{-1}\left[\frac{2^x.2}{1+(2^x)^2}\right]$

$Put\;\; 2^x=\tan\theta$

$=\sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$

$=\sin^{-1}(\sin 2\theta)$

$=2\theta$

$y=2.\tan^{-1}2^x$

$\frac{dy}{dx}=2.\frac{1}{1+(2^x)^2}.\frac{d}{dx}(2^x)$

$=\frac{2}{1+4^x}.2^x.\log 2$

7. $u=\sin^2 x$

$\frac{du}{dx}=2\sin x.\cos x$

$v=e^{\cos x}$

$\frac{dv}{dx}=e^{\cos x}(-\sin x)$

$\frac{du}{dv}=\frac{2\sin x.\cos x}{-e^{\cos x}.\sin x}$

$=\frac{2\cos x}{e^{\cos x}}$

8. $x\sqrt{1+y}+y\sqrt{1+x}=0$

$x\sqrt{1+y}=-y\sqrt{1+x}$

Square both side

$$x^2(1+y)=y^2(1+x)$$
$$x^2+x^2y=y^2+xy^2$$
$$x^2-y^2+x^2y-xy^2=0$$
$$(x-y)(x+y)+xy(x-y)=0$$
$$(x-y)[x+y+xy]=0$$
$$x+y+xy=0$$
$$y(1+x)=-x$$
$$y=\frac{-x}{1+x}$$
$$\frac{dy}{dx}=-\left[\frac{(1+x)(1)-(x)(1)}{(1+x)^2}\right]$$
$$=-\left[\frac{1+x-x}{(1+x)^2}\right]$$
$$\frac{dy}{dx}=\frac{-1}{(1+x)^2}$$

9. $\cos y = x.\cos(a+y) \quad (given)$

$$x=\frac{\cos y}{\cos(a+y)}$$
$$\frac{dx}{dy}=\frac{\cos(a+y)(-\sin y)-\cos y.(-\sin(a+y))}{\cos^2(a+y)}$$
$$\frac{dx}{dy}=\frac{\cos y.\sin(a+y)-\sin y\cos(a+y)}{\cos^2(a+y)}$$
$$\frac{dx}{dy}=\frac{\sin(a+y-y)}{\cos^2(a+y)}$$
$$\frac{dy}{dx}=\frac{\cos^2(a+y)}{\sin a}$$

10. $x=a(\cos t+t.\sin t)$

$$\frac{dx}{dt}=a[-\sin t+t.\cos t+\sin t.1]$$
$$=a[t.\cos t]-----(1)$$

$$y = a(\sin t - t\cos t)$$

$$\frac{dy}{dt} = a\left[\cos t. - (-t\sin t + \cos t.1)\right]$$

$$= a.t.\sin t - - - - - - (2)$$

$$(2) \div (1)$$

$$\frac{dy}{dx} = \frac{a.t.\sin t}{a.t.\cos t}$$

$$\frac{dy}{dx} = \tan t$$

$$\frac{d^2 y}{dx^2} = \frac{d}{dx}(\tan t)$$

$$== \frac{d}{dt}(\tan t).\frac{dt}{dx}$$

$$= \sec^2 t.\frac{1}{at\cos t}$$

$$\frac{d^2 y}{dx^2} = \frac{\sec^3 t}{a\ t}$$

CBSE TEST PAPER-02

CLASS - XII MATHEMATICS (Calculus : Continuity and Differentiability)

Topic:- Differentiation

1. Find all points of discontinuity if [4]

$$f(x)=\begin{cases}|x|+3, & \text{if } x \le -3\\ -2x, & \text{if } -3 < x < 3\\ 6x+2, & \text{if } x \ge 3\end{cases}$$

2. Differentiate $y=\underset{I}{\cos} x^3 . \underset{B}{\sin^2}(x^5)$ [4]

3. Find $\frac{dy}{dx}$ if $x^3+x^2y+xy^2+y^3=81$ [4]

4. Differentiate xy = $e^{(x-y)}$ [4]

5. Find $\frac{dy}{dx}$ if $x=a(\cos\theta+\theta\sin 0)$ and $y=a(\sin 0-\theta\cos\theta)$ [4]

6. If y = 3 cos (log x) + 4 sin (log x). Show that $x^2y_2 + xy_1 + y = 0$ [4]

7. Verify Rolle's Theorem for the function f(x) = $x^2 + 2x - 8$, x∈ [-4, 2] [4]

8. Find $\frac{dy}{dx}$ $y=\tan^{-1}\left(\frac{\sin x}{1+\cos x}\right)$ [4]

9. If x = a (cos t + t sin t) and y = a (sin t – t cos t), find $\frac{d^2y}{dx^2}$ [4]

10. If $y=\begin{vmatrix} f(x) & g(x) & h(x)\\ l & m & n\\ a & b & c\end{vmatrix}$ Prove that $\frac{dy}{dx}=\begin{vmatrix} f'(x) & g'(x) & h'(x)\\ l & m & n\\ a & b & c\end{vmatrix}$ [4]

CBSE TEST PAPER-02

CLASS - XII MATHEMATICS (Calculus : Continuity and Differentiability)

[ANSWERS]

Topic:- Differences

1. At x = -3

f(-3) = |-3| + 3 = 3 + 3 = 6

$$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} f(x)$$

$$= \lim_{x\to -3^+} -2x$$

$$= \lim_{h\to 0} -2(-3+h)$$

$$= 6$$

$$\lim_{x\to -3^+} f(x) = \lim_{x\to -3^-} f(x) = \lim_{x\to -3} f(x) = 6$$

Hence continuous at x = -3

At x = 3

$$f(3) = 6\times 3 + 2 = 20$$

$$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} (-2x)$$

$$\lim_{h\to 0} -2(3-h)$$

$$= -6$$

$$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} (6x+2)$$

$$= \lim_{h\to 0} [6(3+h)+2]$$

$$= 20$$

$$\lim_{x\to -3^-} f(x) \neq \lim_{x\to 3^+} f(x)$$

Hence it is continuous

2. $$y = \underset{I}{\cos} x^3 . \underset{B}{\sin}^2(x^5)$$

$$\frac{dy}{dx} = \cos x^3 \frac{d}{dx}\sin^2(x^5) + \sin^2(x^5)\frac{d}{dx}\cos x^3$$

$$= \cos x^3 . 2\sin(x^5)\cos x^5 . 5x^4 + \sin^2(x^5).(-\sin x^3).3x^2$$

$$= 10x^4 \sin(x^5)\cos x^5 . \cos x^3 - 3x^2 \sin^2(x^5). \sin x^3$$

3. Differentiate both side w.r.t. to x, $x^3 + x^2y + xy^2 + y^3 = 81$

$$3x^2 + x^2.\frac{dy}{dx} + y.2x + x.2y\frac{dy}{dx} + y^2.1 + 3y^2\frac{dy}{dx} = 0$$

$$\left(x^2 + 2xy + 3y^2\right)\frac{dy}{dx} = -3x^2 - 2xy - y^2$$

$$\frac{dy}{dx} = \frac{\left(3x^2 + 2xy + y^2\right)}{x^2 + 2xy + 3y^2}$$

4. $xy = e^{x-y}$

Taking log both side

$$\log(xy) = \log e^{x-y}$$

$$l\,og(xy) = (x-y)\log e$$

$$\log x + \log y = x - y \qquad \{\log e = 1\}$$

Diff. both side w.r.t. to x

$$\frac{1}{x} + \frac{1}{y}.\frac{dy}{dx} = 1 - \frac{dy}{dx}$$

$$\frac{dy}{dx}\left(\frac{1}{y} + 1\right) = 1 - \frac{1}{x}$$

$$\frac{dy}{dx} = \frac{\frac{x-1}{x}}{\frac{1+y}{y}}$$

$$\frac{dy}{dx} = \frac{x-1}{x} \times \frac{y}{1+y}$$

$$= \frac{y(x-1)}{x(1+y)}$$

5. $x = a(\cos\theta + \theta.\sin\theta)$

$$\frac{dx}{d\theta} = a\left[-\sin\theta + \theta.\cos\theta + \sin\theta.1\right]$$

$$\frac{dy}{dx} = a \quad \theta.\cos\theta ---------- (1)$$

$$y = a(\sin\theta - \theta.\cos\theta)$$

$$\frac{dy}{dx} = a\left[\cos\theta - (-\theta\sin\theta + \cos\theta.1)\right]$$

$$= a\left[\cos\theta + \theta.\sin\theta - \cos\theta\right]$$

$= a \quad \theta . \sin\theta ------------ (2)$

$(2) \div (1)$

$$\frac{dy}{dx} = \frac{a \quad \theta.\sin\theta}{a \quad \theta.\cos\theta}$$

$= \tan\theta$

6. $y = 3\cos(\log x) + 4\sin(\log x)$

Diff. both side w.r.t. to x

$$y_1 = \frac{-3\sin(\log x)}{x} + \frac{4\cos(\log x)}{x}$$

$xy_1 = -3\sin(\log x) + 4\cos(\log x)$

Again diff.

$$xy_2 + y_1.1 = \frac{-3\cos(\log x)}{x} - \frac{4\sin(\log x)}{x}$$

$x^2y_2 + xy_1 = -3\cos(\log x) - 4\sin(\log x)$

$x^2y_2 + xy_1 = -y$

$x^2y_2 + xy_1 + = 0$

7. The function $y = x^2 + 2x - 8, x \in [-4, 2]$

Continuous in [-4, 2] and differentiable in (-4, 2)

Also $f(-4) = f(2) = 0$

Hence all the condition of all Rolle 's Theorem, is verified

Their exist a value C

Such that f' (c) = 0

f' (c) = 2c +2

0 = 2C+2

C = -1

8. $y = \tan^{-1}\left(\dfrac{\sin x}{1+\cos x}\right)$

$$y = \tan^{-1}\left(\frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}}\right)$$

$$y = \tan^{-1}\left(\tan\frac{x}{2}\right)$$

$$y = \frac{x}{2}$$

$$\frac{dy}{dx}=\frac{1}{2}$$

9. $x=a(\cos t+t.\sin t)$

$$\frac{dx}{dt}=a[-\sin t+t\cos t+\sin t.1]$$

$$\frac{dx}{dt}=a(t.\cos t)----------(1)$$

$$y=a(\sin t-t\cos t)$$

$$\frac{dy}{dt}=a[\cos t-(-t\sin t+\cos t.1)]$$

$$=a[\cos t+t\sin t-\cos t]$$

$$=a(t.\sin t)$$

$$\frac{dy}{dx}=\frac{a\ \ t.\sin t}{a\ \ t.\cos t}=\tan t$$

$$\frac{d^2y}{dx^2}=\frac{d}{dx}(\tan t)$$

$$=\frac{d}{dt}(\tan t).\frac{dt}{dx}$$

$$=\sec^2 t.\frac{1}{a\ \ t.\cos t}$$

$$=\frac{1}{a\ \ t.\cos^3 t}$$

10. $y=\begin{vmatrix} \mathrm{f(x)} & \mathrm{g(x)} & \mathrm{h(x)} \\ \mathrm{l} & \mathrm{m} & \mathrm{n} \\ \mathrm{a} & \mathrm{b} & \mathrm{c} \end{vmatrix}$

$$\frac{dy}{dx}=\begin{vmatrix} \mathrm{f'(x)} & \mathrm{g'(x)} & \mathrm{h'(x)} \\ \mathrm{l} & \mathrm{m} & \mathrm{n} \\ \mathrm{a} & \mathrm{b} & \mathrm{c} \end{vmatrix}+\begin{vmatrix} \mathrm{f(x)} & \mathrm{g(x)} & \mathrm{h(x)} \\ 0 & 0 & 0 \\ \mathrm{a} & \mathrm{b} & \mathrm{c} \end{vmatrix}+\begin{vmatrix} \mathrm{f(x)} & \mathrm{g(x)} & \mathrm{h(x)} \\ \mathrm{l} & \mathrm{m} & \mathrm{n} \\ 0 & 0 & 0 \end{vmatrix}$$

$$=\begin{vmatrix} \mathrm{f'(x)}\ . & \mathrm{g'(x)}\ . & \mathrm{h'(x)} \\ \mathrm{l} & \mathrm{m} & \mathrm{n} \\ \mathrm{a} & \mathrm{b} & \mathrm{c} \end{vmatrix}+0+0$$

CBSE TEST PAPER-03

CLASS - XII MATHEMATICS (Calculus: Continuity and Differentiability)

Topic: - Differentiations

1. Find the value of K so that function is continuous. [4]

$$f(x) = \begin{cases} Kx+1 & \text{if} \quad x \le \pi \\ \\ \cos x & \text{if} \quad x > \pi \end{cases} \quad \text{at } x = \pi$$

2. Differentiate $y = 2\sqrt{\cot x^2}$ [4]

3. Find $\frac{dy}{dx}$ *if* $\sin^2 y + \cos xy = \pi$ [4]

4. Find $\frac{dy}{dx}$ *if* $y^x + x^y + x^x = a^b$ [4]

5. Find $\frac{dy}{dx}$ when $x = a(\theta - \sin\theta)$, $y = a(1+\cos\theta)$ [4]

6. If $y = 3e^{2x} + 2e^{3x}$ Prove that $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$ [4]

7. If $y = e^{a^{\cos^{-1} x}}$ Show that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2 y = 0$ [4]

8. If $(x-a)^2 + (y-b)^2 = c^2$ Prove $\dfrac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$ is a constant independent of a & b. [4]

9. Find $\frac{dy}{dx}$, if $y = \sin^{-1} x + \sin^{-1}\sqrt{1-x^2}$ [4]

10. $y = (\sin x - \cos x)^{(\sin x - \cos x)}$ Find $\frac{dy}{dx}$, [4]

CBSE TEST PAPER-03

CLASS - XII MATHEMATICS (Calculus : Continuity and Differentiability)

[ANSWERS]

Topic: - Differentiability

1. $\lim\limits_{x \to \pi^-} f(x) = \lim\limits_{x \to \pi^-} (Kx+1)$

$= \lim\limits_{h \to 0} \left[K(\pi - h) + 1 \right]$

$= K\pi + 1$

$\lim\limits_{x \to \pi^+} f(x) = \lim\limits_{x \to \pi^+} \cos x$

$= \lim\limits_{h \to 0} -\cos(\pi + \text{h}) = \lim\limits_{h \to 0} -\cos\text{h}$

$= -\cos 0 = 1$

$AT\theta$

$K\pi + 1 = -1$

$K = \dfrac{-2}{\pi}$

2. $y = 2\sqrt{\cot x^2}$

$\dfrac{dy}{dx} = 2.\dfrac{1}{2}\left(\cot x^2\right)^{-\frac{1}{2}}.\dfrac{d}{dx}\left(\cot x^2\right)$

$= \dfrac{1}{\sqrt{\cot x^2}} - \cos ec^2 x^2.2x$

$= \dfrac{-2x.\cos ec^2 x^2}{\sqrt{\cot x^2}}$

3. $\sin^2 y + \cos xy = \pi$

$diff.$

$2\sin y \cos y \dfrac{dy}{dx} - \sin xy(x\dfrac{dy}{dx} + y.1) = 0$

$2\sin y.\cos y \dfrac{dy}{dx} - x.\sin xy \dfrac{dy}{dx} - y.\sin xy = 0$

$\dfrac{dy}{dx}(\sin 2y - x.\sin xy) = y \sin xy$

$\dfrac{dy}{dx} = \dfrac{y.\sin xy}{\sin 2y - x.\sin xy}$

4. Let $u = y^x$, $\text{v} = x^y$, $\text{w} = x^x$

$u + v + w = a^b$

Therefore $\frac{du}{dx}+\frac{dw}{dx}+\frac{dv}{dx}=0$

$u = y^x$

Taking log both side

$\log u = \log y^x$

$\log u = x.\log y$

Differentiate both side w.r.t. to x

$$\frac{1}{u}.\frac{du}{dx} = x.\frac{1}{y}.\frac{dy}{dx}+\log y.1$$

$$\frac{du}{dx} = u\left[\frac{x}{y}.\frac{dy}{dx}+\log y\right]$$

$$\frac{du}{dx} = y^x\left(\frac{x}{y}.\frac{dy}{dx}+\log y\right)$$

$v = x^y$

Taking log both side

$\log v = \log x^y$

$\log v = y.\log x$

$$\frac{1}{v}.\frac{dv}{dx} = y.\frac{1}{x}+\log x.\frac{dy}{dx}$$

$$\frac{dv}{dx} = v\left[\frac{y}{x}+\log x.\frac{dy}{dx}\right]$$

$$\frac{dv}{dx} = x^y\left[\frac{y}{x}+\log x.\frac{dy}{dx}\right]$$

$w = x^x$

Taking log both side

$\log w = \log x^x$

$\log w = x\log x$

$$\frac{1}{w}.\frac{dw}{dx} = x.\frac{1}{x}+\log x.1$$

$$\frac{1}{w}.\frac{dw}{dx} = 1+\log x$$

$$\frac{dw}{dx} = w(1+\log x)$$

$$\frac{dw}{dx} = x^x(1+\log x)$$

$$\frac{dw}{dx} = \frac{-x^x(1+\log x) - y.x^{y-1} - y^x \log y}{x.y^{x-1} + x^y \log x.}$$

5. $$\frac{dx}{d\theta} = a[1-\cos\theta], \quad \frac{dy}{d\theta} = a[0-\cos\theta]$$

$$\frac{dy}{d\theta} = \frac{-a\sin\theta}{a(1-\cos\theta)}$$

$$= \frac{-2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\sin^2\frac{\theta}{2}} = -\tan\frac{\theta}{2}$$

6. $$y = 3e^{2x} + 2e^{3x}$$

$$\frac{dy}{dx} = 3e^{2x}.2 + 2e^{3x}.3$$

$$= \frac{d^2y}{dx^2} = 6e^{2x}.2 + 6e^{3x}.3$$

$$= 12e^{2x} + 18e^{3x}$$

$$\text{LHS} = \frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y$$

$$= \left(12e^{2x} + 18e^{3x}\right) - 5\left(6e^{2x} + 6e^{3x}\right) + 6(3e^{2x} + 2e^{3x})$$

$$= 12e^{2x} + 18e^{3x} - 30e^{2x} - 30e^{3x} + 18e^{2x} + 12e^{3x}$$

$$= 0$$

7. $$y = e^{a^{\cos^{-1}x}}$$

$$\frac{dy}{dx} = -a.e^{a^{\cos^{-1}x}}.\frac{1}{\sqrt{1-x^2}}$$

$$\sqrt{1-x^2}\,\frac{dy}{dx} = -e^{a^{\cos^{-1}x}}.a$$

$$\sqrt{1-x^2}\,\frac{d^2y}{dx^2} + \frac{dy}{dx}\frac{1(-2)}{2\sqrt{1-x^2}} = -ae^{a^{\cos^{-1}x}}a\frac{(-1)}{\sqrt{1-x^2}}$$

$$\sqrt{1-x^2}\,\frac{d^2y}{dx^2}-\frac{1}{\sqrt{1-x^2}}\cdot\frac{dy}{dx}=\frac{a^2e^{a^{\cos^{-1}x}}}{\sqrt{1-x^2}}$$

$$\left(1-x^2\right)\frac{d^2y}{dx^2}-\frac{dy}{dx}=a^2e^{a^{\cos^{-1}x}}$$

$$\left(1-x^2\right)\frac{d^2y}{dx^2}-\frac{dy}{dx}-a^2y=0$$

8. $(x-a)^2+(y-b)^2=C^2 \quad (Given)--------(1)$

Diff. both side w.r.t. to x

$$2(x-a)+(y-b)y_1=0$$

$$(x-a)+(y-b)y_1=0--------------(2)$$

Again diff. both side

$$(1-0)+(y-b)y_2+y_1.y_1=0$$

$$1+(y-b)y_2+y_1^2=0$$

$$y-b=\frac{-1-y_1^2}{y_2}$$

Put (y-b) in equation (1)

$$(x-a)-\left(\frac{1+y_1^2}{y_2}\right)y_1=0$$

$$x-a=\left(\frac{1+y_1^2}{y_2}\right).y_1$$

Put the value of (x-a) and (y-b) in equation (1)

$$\left[\left(\frac{1+y_1^2}{y_2}\right)y_1\right]^2+\left(\frac{1+y_1^2}{y_2}\right)^2=C^2$$

$$\frac{\left(1+y_1^2\right)^2y_1^2}{y_2^2}+\frac{\left(1+y_1^2\right)^2}{y_2^2}=C^2$$

$$\frac{\left(1+y_1^2\right)\left(y_1^2+1\right)}{y_2^2}=C^2$$

$$\pm\sqrt{\frac{\left(1+y_1^2\right)^3}{y_2^2}}=C$$

$$\pm\frac{\left(1+y_1^2\right)^{3/2}}{y_2^{2\,3/2}}=C$$

$$\frac{\left[\left(1+\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}=C$$

Hence prove

9. $y=\sin^{-1}x+\sin^{-1}\sqrt{1-x^2}$

Differentiate both side w.r.t. x

$$\frac{dy}{dx}=\frac{1}{\sqrt{1-x^2}}+\frac{1}{\sqrt{1-\left(\sqrt{1-x^2}\right)^2}}\frac{d}{dx}\left(\sqrt{1-x^2}\right)$$

$$\frac{1}{\sqrt{1-x^2}}+\frac{1}{\sqrt{1-(1-x^2)}.}\frac{1(-2x)}{2\sqrt{1-x^2}}$$

$$=\frac{1}{\sqrt{1-x^2}}+\frac{1}{\sqrt{1-\left(\sqrt{1-x^2}\right)^2}}.\left(\frac{-x}{\sqrt{1-x^2}}\right)$$

$$=\frac{1}{\sqrt{1-x^2}}+\frac{1}{\sqrt{x^2}}\left(\frac{-x}{\sqrt{1-x^2}}\right)$$

$$=\frac{1}{\sqrt{1-x^2}}+\frac{1}{x}\left(\frac{-x}{\sqrt{1-x^2}}\right)$$

$$=\frac{1}{\sqrt{1-x^2}}-\frac{1}{\sqrt{1-x^2}}=0$$

10. $y=(\sin x-\cos x)^{\sin x-\cos x}$

Taking log both side

$$\log y=\log(\sin x-\cos x)^{\sin x-\cos x}$$

$$\log y=(\sin x-\cos x).\log(\sin x-\cos x)$$

Differentiate both side w.r.t. x

$$\frac{1}{y}.\frac{dy}{dx}=(\sin x-\cos x).\frac{1}{(\sin x-\cos x)}(\cos x+\sin x)+\log(\sin x-\cos x).(\cos x+\sin x)$$

$$\frac{dy}{dx}=y\left[((\cos x+\sin x))+\log((\sin x-\cos x)).(\cos x+\sin x)\right]$$

CBSE TEST PAPER-04

CLASS - XII MATHEMATICS (Calculus: Continuity and Differentiability)

Topic: - Differentiation

1. Discuss the continuity of the function [4]

$$f(x)=\begin{cases}-2, & \text{if } x \le -1 \\ 2x, & \text{if } -1 < x \le 1 \\ 2, & \text{if } x > 1\end{cases}$$

2. Find $\frac{dy}{dx}$ if $y=\frac{\sin(ax+b)}{\cos(cx+d)}$ [4]

3. Find $\frac{dy}{dx}$ if $y=\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$ [4]

4. Find $\frac{dy}{dx}$, if y= $(x\cos x)^x+(x\sin x)^{\frac{1}{x}}$ [4]

5. $x=a\left(\cos t+\log\tan\frac{t}{2}\right)$, $y=a\sin t$ find $\frac{dy}{dx}$ [4]

6. If $e^y(x+1)=1$ show that $\frac{d^2y}{dx^2}=\left(\frac{dy}{dx}\right)^2$ [4]

7. $y=a^{t+\frac{t}{2}}$ *and* $x=\left(t+\frac{t}{2}\right)^a$ Find $\frac{dy}{dx}$ [4]

8. $y=\cot^{-1}\left[\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right]$ [4]

9. If $y=\sqrt{\cos x+\sqrt{\cos x+\sqrt{\cos x+........}}}$ Prove that $(1-2y)\frac{dy}{dx}=\sin x$ [4]

10. $y=\log\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)$ Show that $\frac{dy}{dx}-\sec x=0$ [4]

CBSE TEST PAPER-04

CLASS - XII MATHEMATICS (Calculus: Continuity and Differentiability)

[ANSWERS]

Topic: - Differentiation

1. At x = -1

f(-1) = -2

$\lim_{x\to -1^-} f(x) = \lim_{x\to -1^-} (-2)$

$\lim_{x\to 0} -2$

$= -2$

$\lim_{x\to -1^+} f(x) = \lim_{x\to -1^+} 2x$

$\lim_{h\to 0} 2(-1+h)$

$= -2$

Hence continuous at x = -1

$x = 1$

$f(1) = 2\times 1 = 2$

$\lim_{x\to 1-} f(x) = \lim_{x\to 1-} 2x$

$\lim_{h\to 0} 2(1-h)$

$= 2$

$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} 2$

$\lim_{x\to 0} 2$

$= 2$

Continuous

2. $y = \dfrac{\sin(ax+b)}{\cos(cx+d)}$

$$\frac{dy}{dx} = \frac{\cos(cx+d)\frac{d}{dx}\sin(ax+b) - \sin(ax+b)\frac{d}{dx}\cos(cx+d)}{\cos^2(cx+d)}$$

$$\frac{dy}{dx} = \frac{\cos(cx+d)\cos(ax+b).a + \sin(ax+b)\sin(cx+d).c}{\cos^2(cx+d)}$$

3. $y=\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$

$Put\ x=\tan\theta$

$$y=\tan^{-1}\left(\frac{3.a\tan\theta-\tan^3\theta}{1-3\,a^2.\tan^2\theta}\right)$$

$$y=\tan^{-1}[\tan 3\theta]$$

$$y=3\theta$$

$$y=3.\tan^{-1}x$$

Diff.

$$\frac{dy}{dx}=3\left(\frac{1}{1+x^2}\right)$$

4. Let $y=u+v$

Where $u=(x\cos x)^x, v=(x.\sin x)^{\frac{1}{x}}$

$$u=(x\cos x)^x$$

Taking log both side

$$\log u=\log(x\cos x)^x$$

$$\log u=x.\log(x.\cos x)$$

Differentiate

$$\frac{1}{u}.\frac{du}{dx}=x.\frac{1}{x\cos x}(-x\sin x+\cos x.1)+\log(x\cos x).1$$

$$\frac{du}{dx}=u[-x\tan x+1+\log(x.\cos x)]$$

$$v=(x.\sin x)^{\frac{1}{x}}$$

Taking log both side

$$\log v=\log(x.\sin x)^{\frac{1}{x}}$$

$$\log v=\frac{1}{x}.\log(x.\sin x)$$

Differentiate

$$\frac{1}{v}.\frac{dv}{dx}=\frac{1}{x}.\frac{1}{x.\sin x}(x\cos x+\sin x.1)+\log(x.\sin x)\left(-\frac{1}{x^2}\right)$$

$$\frac{dv}{dx}=v\left[\frac{\cot x}{x}+\frac{1}{x^2}-\frac{\log(x\sin x)}{x^2}\right]$$

$$\frac{dy}{dx}=\frac{du}{dx}+\frac{dv}{dx}$$

$$=(x\cos x)^x\left[-x.\tan x+1+\log\left(x.\cos x\right)\right]+(x.\sin x)^{\frac{1}{x}}\left[\frac{\cot x}{x}+\frac{1}{x^2}-\frac{\log(x.\sin x)}{x^2}\right]$$

5. $$x=a\left(\cos t+\log\tan\frac{t}{2}\right)$$

$$\frac{dx}{dt}=a\left[-\sin t+\frac{1}{\tan\frac{t}{2}}.\sec^2\frac{t}{2}.\frac{1}{2}\right]$$

$$=a\left[-\sin t+\frac{1}{\frac{\sin\frac{t}{2}}{\cos\frac{t}{2}}}\times\frac{1}{\cos^2\frac{t}{2}}\times\frac{1}{2}\right]$$

$$=a\left[-\sin t+\frac{1}{2\sin\frac{t}{2}.\cos\frac{t}{2}}\right]$$

$$=a\left[-\sin t+\frac{1}{\sin t}\right]$$

$$=a\left[\frac{-\sin^2 t+1}{\sin t}\right]$$

$$=a\left[\frac{\cos^2 t}{\sin t}\right]$$

$$\frac{dx}{dt}=a\frac{\cos^2 t}{\sin t}$$

$$y=a\sin t$$

$$\frac{dy}{dt}=a\cos t$$

$$\frac{dy}{dx}=\frac{a\cos t}{a\dfrac{\cos^2 t}{\sin t}}$$

$$=\frac{\sin t}{\cos t}$$

$$\frac{dy}{dx}=\tan t$$

6. $e^y(x+1)=1 \quad (Given)$

Differentiate

$$e^y(1)+(x+1)e^y.\frac{dy}{dx}=0$$

$$e^y\left[1+(x+1)\frac{dy}{dx}\right]=0$$

$$1+(x+1)\frac{dy}{dx}=0$$

$$\frac{dy}{dx}=\frac{-1}{x+1}$$

$$\frac{d^2y}{dx^2}=-\left[\frac{(x+1)(0)-(1)(1)}{(x+1)^2}\right]$$

$$=\left[\frac{1}{(x+1)^2}\right]$$

$$\frac{d^2y}{dx^2}=\frac{1}{(x+1)^2}$$

$$\frac{d^2y}{dx^2}=\left(\frac{1}{x+1}\right)^2$$

$$\frac{d^2y}{dx^2}=\left(\frac{dy}{dx}\right)^2$$

7. $y=a^{t+\frac{1}{t}}$

$$\frac{dy}{dt}=a^{t+\frac{1}{t}}.\log a.\left(1-\frac{1}{t^2}\right)$$

$$x=\left(t+\frac{1}{t}\right)^a$$

$$\frac{dx}{dt} = a\left(t+\frac{1}{t}\right)^{a-1}.\left(1-\frac{1}{t^2}\right)$$

$$\frac{dy}{dx} = \frac{a^{t+\frac{t}{t}}.\log a\left(1-\frac{1}{t^2}\right)}{a\left(t+\frac{1}{t}\right)^{a-1}\left(1-\frac{1}{t^2}\right)}$$

$$= \frac{a^{t+\frac{t}{t}}.\log a}{a\left(t+\frac{1}{t}\right)^{a-1}}$$

8. $$\sqrt{1\pm\sin x} = \sqrt{\left(\cos\frac{x}{2}\pm\sin\frac{x}{2}\right)^2}$$

$$y = \cot^{-1}\left[\frac{\sqrt{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2}+\sqrt{\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)^2}}{\sqrt{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2}-\sqrt{\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)^2}}\right]$$

$$= \cot^{-1}\frac{\cos\frac{x}{2}+\sin\frac{x}{2}+\cos\frac{x}{2}-\sin\frac{x}{2}}{\cos\frac{x}{2}+\sin\frac{x}{2}-\cos\frac{x}{2}+\sin\frac{x}{2}}$$

$$= \cot^{-1}\left(\frac{2\cos\frac{x}{2}}{2\sin\frac{x}{2}}\right)$$

$$= \cot^{-1}\left(\cot\frac{x}{2}\right)$$

$$= \frac{x}{2}$$

$$y = \frac{x}{2}$$

$$\frac{dy}{dx} = \frac{1}{2}$$

9. Let $y = \sqrt{\cos x + \sqrt{\cos x + \sqrt{\cos x............}}}$

$$y = \sqrt{\cos x + y}$$

Squaring both side

$$y^2 = \cos x + y$$

Differentiate

$$2y\frac{dy}{dx} = -\sin x + \frac{dy}{dx}$$

$$\frac{dy}{dx}(2y-1) = -\sin x$$

$$(1-2y)\frac{dy}{dx} = +\sin x$$

$$(1-2y)\frac{dy}{dx} = \sin x$$

10. $y = \log\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)$

$$\frac{dy}{dx} = \frac{1}{\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)}.\sec^2\left(\frac{\pi}{4}+\frac{x}{2}\right).\left(\frac{1}{2}\right)$$

$$= \frac{1}{\frac{\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)}{\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)}} \times \frac{1}{\cos^2\left(\frac{\pi}{4}+\frac{x}{2}\right)} \times \frac{1}{2}$$

$$= \frac{1}{2\sin\left(\frac{\pi}{4}+\frac{x}{2}\right).\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)}$$

$$= \frac{1}{\sin\left(2\left(\frac{\pi}{4}+\frac{x}{2}\right)\right)}$$

$$= \frac{1}{\sin\left(\frac{\pi}{2}+x\right)}$$

$$= \frac{1}{\cos x}$$

$\frac{dy}{dx} = \sec x$, hence $\frac{dy}{dx} - \sec x = 0$

CBSE TEST PAPER-05
CLASS - XII MATHEMATICS (Calculus: Continuity and Differentiability)

Topic: - Differentiation

1. For what value of K is the following function continuous at x = 2? [4]

$$f(x)=\begin{cases} 2x+1 & ; \quad x<2 \\ K & ; \quad x=2 \\ 3x-1 & ; \quad x>2 \end{cases}$$

2. Differentiate the following w.r.t. to x $\tan^{-1}\left(\dfrac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)$ [4]

3. If $y=\sin^{-1}\left(\dfrac{5x+12\sqrt{1-x^2}}{13}\right)$ find $\dfrac{dy}{dx}$ [4]

4. Discuss the continuity of the following function at x = 0 [4]

$$f(x)=\begin{cases} \dfrac{x^4+2x^3+x^2}{\tan^{-1}x} & , \quad x \neq 0 \\ 0 & , \quad x=0 \end{cases}$$

5. Verify L.M.V theorem for the following function $f(x) = x^2 + 2x + 3$, for [4, 6] [4]

6. If $f(x)=\sqrt{\dfrac{\sec x-1}{\sec x+1}}$, find $f'(x)$ also find $f'\left(\dfrac{\pi}{2}\right)$ [4]

7. If $x^p y^q=(x+y)^{p+q}$ prove that $\dfrac{dy}{dx}=\dfrac{y}{x}$ [4]

8. If $x=a\sin pt,\ y=b\cos pt$ find the value of $\dfrac{d^2y}{dx^2}$ at t = 0 [4]

9. If $y\sqrt{x^2+1}=\log\left(\sqrt{x^2+1}-x\right)$ prove that $\left(x^2+1\right)\dfrac{dy}{dx}+xy+1=0$ [4]

10. If $y=\sqrt{\sin x+\sqrt{\sin x+\sqrt{\sin x+\ldots\ldots\ldots\ldots+\infty}}}$ prove that $\dfrac{dy}{dx}=\dfrac{\cos x}{2y-1}$ [4]

OR

If $\sin y = x\sin(a+y)$ prove that $\dfrac{dy}{dx}=\dfrac{\sin^2(a+y)}{\sin a}$

CBSE TEST PAPER-05

CLASS - XII MATHEMATICS (Calculus: Continuity and Differentiability)

[ANSWERS]

Topic: - Differentiability

1. $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} (2x-1)$

$= \lim_{h\to 0} [2(2-h)+1]$

$= 5$

$= \lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} (3x-1)$

$= \lim_{h\to 0} 3(2+h)-1$

$= 5$

$at\ x = 2$

$f(2) = K$

A T θ

$5 = K = 5$

$K = 5$

2. $y = \tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)$

$Put\ x = \cos 2\theta$

$y = \tan^{-1}\left(\frac{\sqrt{1+\cos 2\theta}-\sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta}+\sqrt{1-\cos 2\theta}}\right)$

$= \tan^{-1}\left(\frac{\sqrt{2\cos^2\theta}-\sqrt{2\cos^2\theta}}{\sqrt{\cos^2\theta}+\sqrt{2\cos^2\theta}}\right)$

$= \tan^{-1}\left[\frac{\sqrt{2}(\cos\theta-\sin\theta)}{\sqrt{2}(\cos\theta+\sin\theta)}\right]$

$= \tan^{-1}\left[\frac{\frac{\cos\theta}{\cos\theta}-\frac{\sin\theta}{\cos\theta}}{\frac{\cos\theta}{\cos\theta}+\frac{\sin\theta}{\cos\theta}}\right]$

$= \tan^{-1}\left[\frac{1-\tan\theta}{1+\tan\theta}\right]$

$$= \tan^{-1}\left[\frac{\tan\frac{\pi}{4} - \tan\theta}{1 + \tan\frac{\pi}{4}.\tan\theta}\right]$$

$$= \tan^{-1}\left[\tan\left(\frac{\pi}{4} - \theta\right)\right]$$

$$= \frac{\pi}{4} - \theta$$

$$y = \frac{\pi}{4} - \frac{1}{2}\cos^{-1} x$$

$$\frac{dy}{dx} = 0 - \frac{1}{2}\left(\frac{-1}{\sqrt{1-x^2}}\right)$$

$$= \frac{1}{2\sqrt{1-x^2}}$$

3. $$y = \sin^{-1}\left(\frac{5x + 12\sqrt{1-x^2}}{13}\right)$$

$Put\ x = \sin\theta$

$$y = \sin^{-1}\left(\frac{5x + 12\sqrt{1-\sin^2\theta}}{13}\right)$$

$$= \sin^{-1}\left[\frac{5\sin\theta + 12\cos\theta}{13}\right]$$

$$= \sin^{-1}\left[\frac{5}{13}\sin\theta + \frac{12}{13}\cos\theta\right]$$

$$Let\ \frac{5}{13} = r\cos\alpha$$

$$\frac{12}{13} = r\sin\alpha$$

$$\tan\alpha = \frac{12}{5}$$

Squaring and adding

$$\frac{25}{169} + \frac{144}{169} = r^2(\sin^2\alpha + \cos^2\alpha)$$

$1 = r^2$

$r = 1$

$\frac{5}{13} = \cos\alpha, \ \frac{12}{13} = \sin\alpha$

$\Rightarrow y = \sin^{-1}[\cos\alpha.\sin\theta + \sin\alpha.\cos\theta]$

$= \sin^{-1}\left[\sin\left(\theta+\alpha\right)\right]$

$= \theta + \alpha$

$= \sin^{-1} x + \tan^{-1}(12/5)$

$= \frac{1}{\sqrt{1-x^2}} + 0$

$= \frac{1}{\sqrt{1-x^2}}$

4. $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \left(\frac{x^4+2x^3+x^2}{\tan^{-1} x}\right)$

$= \lim_{x\to 0^-} \left[\frac{x\left(x^4+2x^3+x\right)}{\tan^{-1} x}\right]$

$= \lim_{h\to 0} \frac{h}{\tan^{-1} h} \times \lim_{h\to 0} \left(h^3+2h^2+h\right)$

$= 1 \times 0$

$= 0$

$\lim_{x\to 0^+} f(x) = 0$

Hence continuous

5. Since f(x) is polynomial hence continuous in the interval [4, 6] thus f(x) is differentiable in (4, 6) both condition of L.M.V theorem are satisfied.

$\therefore f'(c) = \frac{f(b)-t(a)}{b-a}$

$2c + 2 = \frac{f(6)-f(4)}{6-4}$

$2c + 2 = \frac{51-27}{2}$

$c = 5 \ \in (4,6)$

6. $f(x)=\sqrt{\frac{\sec x-1}{\sec x+1}}$

$f(x)=\sqrt{\frac{1-\cos x}{1+\cos x}}$

$=\sqrt{\frac{2\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}}}$

$=\sqrt{\tan^2\frac{x}{2}}$

$f(x)=\tan\frac{x}{2}$

$f'(x)=\sec^2\frac{x}{2}.\frac{1}{2}$

$f'\left(\frac{\pi}{2}\right)=\sec^2\frac{\pi}{4}.\frac{1}{2}$

$=2.\frac{1}{2}$

$=1$

7. $x^p y^q=(x+y)^{p+q}$

Taking log both side

$p\log x+q\log y=(p+q)\log(x+y)$

Differentiate both side w.r.t. to x

$\frac{p}{x}+\frac{q}{y}.\frac{dy}{dx}=(p+q).\frac{1}{(x+y)}\left(1+\frac{dy}{dx}\right)+\log(x+y).(0)$

$\frac{p}{x}+\frac{q}{y}.\frac{dy}{dx}=\frac{p+q}{x+y}\left(1+\frac{dy}{dx}\right)$

$\frac{dy}{dx}\left(\frac{q}{y}-\frac{p+q}{x+y}\right)=\frac{p+q}{x+y}-\frac{p}{x}$

$\frac{dy}{dx}\left[\frac{qx+qy-py-qy}{y(x+y)}\right]=\frac{px+qx-px-py}{(x+y).x}$

$\frac{dy}{dx}\left[\frac{qx-py}{y(x+y)}\right]=\frac{qx-py}{(x+y).x}$

$$\frac{dy}{dx}=\frac{y}{x}$$

8. $x = a\sin pt$

$$\frac{dy}{dx}=a\cos pt.p$$

$$y=b\cos pt$$

$$\frac{dy}{dx}=-b\sin pt.p$$

$$\frac{dy}{dx}=\frac{-b}{a}\tan pt$$

$$\frac{d^2y}{dx^2}=\frac{-b}{a}.\frac{d}{dt}(\tan pt).\frac{dt}{dx}$$

$$=\frac{-b}{a}.\sec^2 pt.p.\frac{1}{a\cos pt.p}$$

$$=\frac{-b}{a^2}\sec^3 pt$$

$$\left.\frac{d^2y}{dx^2}\right]_{t=0}=\frac{-b}{a^2}\sec^3(p.0)$$

$$=\frac{-b}{a^2}(1)$$

$$=\frac{-b}{a^2}$$

9. $y\sqrt{x^2+1}=\log\left(\sqrt{x^2+1}-x\right)$

$$y.\frac{1}{2\sqrt{x^2+1}}(2x)+\sqrt{x^2+1}.\frac{dy}{dx}=\frac{1}{\sqrt{x^2+1}-x}\left[\frac{1(2x)}{2\sqrt{x^2+1}}-1\right]$$

$$\frac{xy}{\sqrt{x^2+1}}+\sqrt{x^2+1}.\frac{dy}{dx}=\frac{1}{\sqrt{x^2+1}-x}\left[\frac{x-\sqrt{x^2+1}}{\sqrt{x^2+1}}\right]$$

$$\frac{xy+\left(x^2+1\right)}{\sqrt{x^2+1}}\frac{dy}{dx}=\frac{-\left(\sqrt{x^2+1}-x\right)}{-\left(\sqrt{x^2+1}-x\right)\sqrt{x^2+1}}$$

$$xy+\left(x^2+1\right)\frac{dy}{dx}=-1$$

$$\left(x^2+1\right)\frac{dy}{dx}+xy+1=0$$

10. Let $y=\sqrt{\sin x+\sqrt{\sin x+\sqrt{\sin x+.............+\infty}}}$

$$y=\sqrt{\sin x+y}$$

Squaring both side

$$y^2=\sin x+y$$

Differentiate both side w.r.t. to x

$$2y\frac{dy}{dx}=\cos x+\frac{dy}{dx}$$

$$\frac{dy}{dx}\left(2y-1\right)=\cos x$$

$$\frac{dy}{dx}=\frac{\cos x}{2y-1}$$

OR

$$\sin y=x\sin(a+y)$$

$$x=\frac{\sin y}{\sin(a+y)}$$

Differentiate both side w.r.t. to x

$$\frac{dy}{dx}=\frac{\sin(a+y)\cos y-\sin y\cos(a+y)}{\sin^2(a+y)}$$

$$\frac{dy}{dx}=\frac{\sin(a+y-y)}{\sin^2(a+y)}$$

$$\frac{dy}{dx}=\frac{\sin^2(a+y)}{\sin a}$$

CBSE TEST PAPER-01

CLASS - XII MATHEMATICS (Calculus: Application of Derivatives)

Topic: - Application of Derivatives

1. The length x of a rectangle is decreasing at the rate of 3 cm/ mint and the width y is increasing at the rate of 2cm/min. when x = 10cm and y = 6cm, find the ratio of change of (a) the perimeter (b) the area of the rectangle. [4]
2. Find the interval in which the function of given by $f(x) = 4x^3 - 6x^2 - 72x + 30$ is (a) strictly increasing (b) strictly decreasing. [4]
3. Find point on the curve $\frac{x^2}{4} + \frac{y^2}{25} = 1$ at which the tangents are (i) parallel to x – axis (ii) parallel to y – axis [4]
4. Use differentiate to approximate $(25)^{\frac{1}{3}}$ [4]
5. Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere. [6]
6. The volume of a cube is increasing at a rate of $9cm^3$/s. How fast is the surface area increasing when the length of on edge is 10cm? [4]
7. Find the interval in which the function is strictly increasing and decreasing. $(x+1)^3 (x-3)^3$ [4]
8. Find the equations of the tangent and normal to curve $x^{2/3} + y^{2/3} = 2$ at (1, 1) [4]
9. IF the radius of a sphere is measured as 9cm with an error of 0.03cm, then find the approximate error in calculating its volume. [4]
10. A wire of length 28m is to be cut into two pieces. One of the pieces is to be made into a square and the other into a circle. What should be the length of the two pieces to that the combined areas of the square and the circle is minimum. [6]

CBSE TEST PAPER-01

CLASS - XII MATHEMATICS (Calculus: Application of Derivatives)

[ANSWERS]

Topic: - application of derivatives

1. $\frac{dx}{dt} = -3cm / \text{mint}, \frac{dx}{dt} = 2cm / \text{mint}$

(a) Let P be the perimeter

$P = 2(x + y)$

$\frac{dP}{dx} = 2\left(\frac{dx}{dt} + \frac{dy}{dt}\right)$

$= 2(-3 + 2)$

$= -2cm / \text{mint}$

(b) $A = xy$

$\frac{dA}{dt} = x\frac{dy}{dt} + y.\frac{dx}{dt}$

$= 10(2) + 6(-3)$

$= 20 - 18$

$= 2cm^2 / \text{mint}$

2. $f(x) = 4x^3 - 6x^2 - 72x + 30$

$f'(x) = 12x^2 - 12x - 72$

$= 12(x^2 - x - 6)$

$= 12[x^2 - 3x + 2x - 6]$

$= 12[x(x - 3) + 2(x - 3)]$

$= 12(x - 3)(x + 2)$

$Put\ f'(x) = 0$

$x = -2, 3$

int	Sign of f'(x)	Result
$(-\infty,-2)$	+ tive	Increase
$(-2,3)$	+ tive	Decrease
$(3,\infty)$	+ tive	increase

Hence function is increasing in $(-\infty,-2)$ and $(3,\infty)$ decreasing in (-2, 3)

3. $\frac{x^2}{4}+\frac{y^2}{25}=1----------(1)$

Differentiate side w.r.t. to x

$\frac{2x}{4}+\frac{2y}{25}\frac{dy}{dx}=0$

$\frac{dy}{dx}=\frac{-25}{4}.\frac{x}{y}$

For tangent || to x – axis the slope of tangent is zero

$\frac{0}{1}\times\frac{-25x}{4y}$

$x=0$

Put x = 0 in equation (1)

$y=\pm5$

Points are (0, 5) and (0, -5) now is tangent is || is to y – axis

4. Let $x=27,\ \Delta x=-2$ $y=x^{\frac{1}{3}}$

Let $x=27,\ \Delta x=-2$

Then $\Delta y=(x+\Delta x)^{\frac{1}{3}}-(x)^{\frac{1}{3}}$

$\Delta y=(25)^{\frac{1}{3}}-(27)^{\frac{1}{3}}$

$(25)^{\frac{1}{3}}=\Delta y+(27)^{\frac{1}{3}}$

$(25)^{\frac{1}{3}}=\Delta y+3---------(1)$

$dy\sim\Delta y$

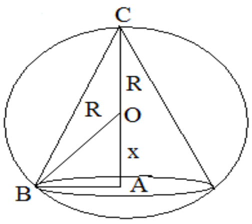
$dy=\left(\frac{dy}{dx}\right).\Delta x$ $\quad [\because \Delta x=-2]$

$$=\frac{1}{3}x^{\frac{-2}{3}}.(-2)$$

$$=-0.074 \qquad [x=27]$$

Put the value of dy in equation (1)

$$(25)^{\frac{1}{3}}=0.074+3$$
$$=2.926$$

5. 

$$V=\frac{1}{3}\pi r^2 h \qquad \left[r^2=\sqrt{R^2-x^2}\right]$$

$$V=\frac{1}{3}\pi.\left(R^2-x^2\right).\left(R+x\right)$$

$$\frac{dv}{dx}=\frac{1}{3}\pi\left[\left(R^2-x^2\right)(1)+\left(R+x\right)\left(-2x\right)\right]$$

$$=\frac{1}{3}\pi\left[\left(R+x\right)\left(R-x\right)-2x\left(R+x\right)\right]$$

$$=\frac{1}{3}\pi\left(R+x\right)\left[R-x-2x\right]$$

$$=\frac{1}{3}\pi\left(R+x\right)\left(R-3x\right)---------(1)$$

$$put \ \frac{dv}{dr}=0$$

$$R=-x \ (neglect)$$
$$R=3x$$
$$\frac{R}{3}=x$$

On again differentiate equation (1)

$$\frac{d^2v}{dx^2}=\frac{1}{3}\pi\left[\left(R+x\right)\left(-3\right)+\left(R-3x\right)(1)\right]$$

$$\left.\frac{d^2v}{dx^2}\right]_{x=\frac{R}{3}}=\frac{1}{3}\pi\left[\left(R+\frac{R}{3}\right)\left(-3\right)+\left(R-3.\frac{R}{3}\right)\right]$$

$$=\frac{1}{3}\pi\left[\frac{4R}{3}\times -3+0\right]$$

$$=\frac{-1}{3}\pi 4R$$

$\frac{d^2v}{dx^2}<0$ Hence maximum

Now $v=\frac{1}{3}\pi\left[\left(R^2-x^2\right)\left(R+x\right)\right]$ $\left[x=\frac{R}{3}\right]$

$$v=\frac{1}{3}\pi\left[\left(R^2-\left(\frac{R}{3}\right)^2\right)\left(R+\left(\frac{R}{3}\right)\right)\right]$$

$$=\frac{1}{3}\pi\left[\frac{8R^2}{9}\times\frac{4R}{3}\right]$$

$$v=\frac{8}{27}\left(\frac{4}{3}\right)\pi R^3$$

$v=\frac{8}{27}$ Value of sphere

Value of cone $==\frac{8}{27}$ of value of sphere.

6. Let x be the length, V be the value and S be the surface area of cube

$$\frac{dv}{dt}=9cm^3/s$$
$$v=x^3$$
$$\frac{dv}{dt}=3x^2\frac{dx}{dt}$$
$$9=3x^2\frac{dx}{dt}$$
$$\frac{3}{x^2}=\frac{dx}{dt}$$
$$s=6x^2$$
$$\frac{ds}{dt}=12x\frac{dx}{dt}$$
$$=12x\times\frac{3}{x^2}$$
$$\frac{ds}{dt}=\frac{36}{x}$$

$$\left.\frac{ds}{dt}\right]_{x=10} = \frac{36}{10}$$

$= 3.6cm^2 / \sec$

7. $f(x) = (x+1)^3 (x-3)^3$

$f'(x) = (x+1)^3 .3(x-3)^2 + (x-3)^3 .3(x+1)^2$

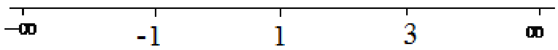
$= 3(x+1)^2 (x-3)^2 [x+1+x-3]$

$= 3(x+1)^2 (x-3)^2 [2x-2]$

$= 6(x+1)^2 (x-3)^2 (x-1)$

$put\ f'(x) = 0$

$x = -1,\ 3,\ 1$



int	Singh of f'(x)	Result
$(-\infty, -1)$	-tive	Decrease
$(-1, 1)$	-tive	Decrease
$(1, 3)$	+tive	Increase
$(3, \infty)$	+tive	Increase

8. $x^{2/3} + y^{2/3} = 2$

Differentiate both side w.r.t to x

$\frac{2}{3} x^{\frac{-1}{3}} + \frac{2}{3} y^{\frac{-1}{3}} \frac{dy}{dx} = 0$

$\frac{dy}{dx} = -\left(\frac{y}{x}\right)^{\frac{1}{3}}$

$\left.\frac{dy}{dx}\right]_{(1,1)} = -(1)^{\frac{1}{3}} = -1$

Slope of tangent = -1

Slope of normal $= \frac{-1}{\frac{dy}{dx}}$

$= \frac{-1}{(-1)}$

$= 1$

9. Let r be radius and Δr be error

$$r = 9cm$$
$$\Delta r = 0.03cm$$
$$v = \frac{4}{3}\pi r^3$$
$$dv = \left(\frac{dv}{dr}\right).\Delta r$$
$$= \frac{4}{3}.\pi 3r^3.\Delta r$$
$$= 4\pi(9)^2 \times 0.03$$
$$= 9.72\pi cm^3$$

10. Let 1st length = x

2nd length = 28-x

$$2\pi r = x$$
$$r = \frac{x}{2\pi}$$
$$4a = 28 - x$$
$$a = 7 - \frac{x}{4}$$
$$AT\theta$$

A = area A circle + area of spare

$$\pi\left(\frac{x}{2\pi}\right)^2 + \left(7 - \frac{x}{4}\right)^2$$
$$A = \pi.\frac{x^2}{4\pi^2} + \left(7 - \frac{x}{4}\right)^2$$
$$\frac{dA}{dx} = \frac{2x}{4\pi} + 2\left(7 - \frac{x}{4}\right)\left(-\frac{1}{4}\right)$$
$$\frac{dA}{dx} = 0$$

$$\frac{1}{2}\left(7-\frac{x}{4}\right)=\frac{x}{2\pi}$$

$$7-\frac{x}{4}=\frac{x}{\pi}$$

$$7=\frac{x}{\pi}+\frac{x}{4}$$

$$7=x\left(\frac{4+\pi}{4\pi}\right)$$

$$\frac{28\pi}{4+\pi}=x$$

$$\frac{d^2A}{dx^2}=\frac{1}{2\pi}-\frac{1}{2}\left(\frac{-1}{4}\right)$$

$$=\frac{1}{2\pi}+\frac{1}{8}$$

+tive hence minimum

1st length $=\frac{28\pi}{4+\pi}$

2nd length $=\frac{28}{1}-\frac{28\pi}{4+\pi}$

$$=.28\left[\frac{4+\pi-\pi}{4+\pi}\right]$$

$$=\frac{112}{4+\pi}$$

CBSE TEST PAPER-02

CLASS - XII MATHEMATICS (Calculus: Application of Derivatives)

Topic: - application of derivatives

1. A ladder 5m long is leaning against a wall. The bottom of the ladder is pulled along the ground away from the wall, at the rate 2cm/s. how fast is its height on the wall decreasing when the foot of the ladder is 4m away from the wall. [4]
2. A particle moves along the curve $6y = x^3 + 2$. Find the points on the curve at which the y-coordinate is changing 8 times as fast as the x – coordinate. [4]
3. Find the interval in which increase/decrease. $f(x) = \sin 3x,\ x \in \left[0, \frac{\pi}{2}\right]$ [4]
4. Find the intervals in which the function f given by $f(x) = \sin x + \cos x,\ 0 \le x \le 2\pi$ is strictly increasing or decreasing. [4]
5. Find the equation, of the tangent line to the curve $y = x^2 - 2x + 7$ which is [4]
   (a) Parallels to the line $2x - y + 9 = 0$
   (b) Perpendicular to the line $5y - 15x = 13$
6. Find the equation of the tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at the point (x_0, y_0). [4]
7. Find the approximate value of $(0.0037)^{1/2}$ [4]
8. Using differentiates find the approximate value of $(3.968)^{3/2}$ [4]
9. Show that the right circular cone of least curved surface area and given volume has an altitude equal to $\sqrt{2}$ time the radius of the base. [6]
10. Show that semi – vertical angle of right circular cone of given surface area and maximum volume is $\sin^{-1}\left(\frac{1}{3}\right)$ [6]

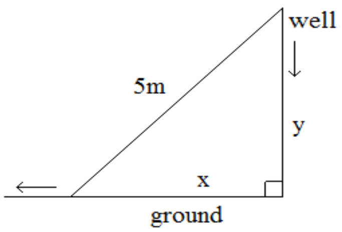
CBSE TEST PAPER-02

CLASS - XII MATHEMATICS (Calculus: Application of Derivatives)

[ANSWERS]

Topic: - application of derivatives

1. $\frac{dx}{dt} = 2cm/s$



$\frac{dx}{dt} = 0.02m/\sec$

$x^2 + y^2 = 5^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

When $x = 4$

$y = \sqrt{5^2 - 4^2}$

$= 3$

$2\times4(0.02) + 2\times3\frac{dy}{dt} = 0$

$\frac{dy}{dt} = \frac{2\times4\times0.02}{2\times3}$

$= -\frac{0.08}{3}\times\frac{100}{100}$

$-\frac{8}{3}cm/\sec$

2. $6y = x^3 + 2 ------(1)$

$6\frac{dy}{dt} = 3x^2\frac{dx}{dt}$

$6\times8\frac{dx}{dt} = 3x^2\frac{dx}{dt}\left[\because \frac{dy}{dt} = 8\frac{dx}{dt}\right]$

$16 = x^2$

$x = \pm4$

Put the value of x in equation (1)

$when\ x = 4$

$6y = (4)^3 + 2$

$= 64 + 2$

$y = \frac{68}{6}$

$(4,11)$

$when\ x = -4$

$6y = (-4)^3 + 2$

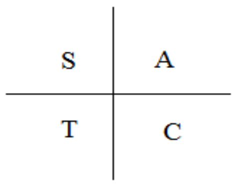
$= -64 + 2$

$y = \frac{-62}{6}$

$\left(-4, \frac{-62}{6}\right)$

$\left(-4, \frac{-31}{3}\right)$

3. $f(x) = \sin 3x$



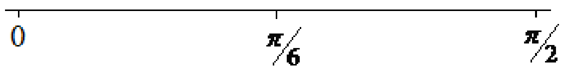
$f'(x) = 3\cos 3x$

$f'(x) = 0$

$\cos 3x = 0$

$3x = \frac{\pi}{2}$

$x = \frac{\pi}{6}$



int.	Sign of f'(x)	Result
$\left[0, \frac{\pi}{6}\right)$	+tive	increase
$\left(\frac{\pi}{6}, \frac{\pi}{2}\right]$	-tive	Decrease

Hence, f(x) is increasing on $\left(0, \frac{\pi}{6}\right)$ and decreasing on $\left(\frac{\pi}{6}, \frac{\pi}{2}\right)$

4. $f(x) = \sin x + \cos x$

$f'(x) = \cos x - \sin x$

$f'(x) = 0$

$cox - \sin x = 0$

$\frac{\sin x}{\cos x} = \frac{\cos x}{\cos x}$

$\tan x = 1$

$x = \frac{\pi}{4}, \frac{5\pi}{4} \; as$

$0 \leq x \leq 2\pi$

int	Singh of f'(x)	Result
$\left(0, \frac{\pi}{4}\right)$	+tive	Increase
$\left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$	-tive	Decrease
$\left(\frac{5\pi}{4}, 2\pi\right)$	+tive	increase

5. Let (x, y) be the point a

(a) $y = x^2 - 2x + 7$ ---------- (1)

$\frac{dy}{dx} = 2x - 2$

$\left.\frac{dy}{dx}\right]_{x,y} = 2x_1 - 2$

Slope of line = 2

$AT\theta \; 2x_1 - 2 = 2$

$x_1 = 2$

$y_1 = x_1^2 - 2x_1 + 7 \quad [\, from\ (1)]$

$y_1 = 4 - 4 + 7$

$= 7$

Equation of tangent

$$y - y_1 = \frac{dy}{dx}(x - x,)$$

$$y - 7 = 2x - 4$$

$$2x - y + 3 = 0$$

(b) $5y - 15x = 13$

Slope of Line = $\dfrac{-(-15)}{5} = 3$

$AT\theta\ (2x, -2) \times 3 = -1, \ \ x = 5/6$

Put x_1 in equation (1)

$$y = \left(\frac{5}{6}\right)^2 - 2\left(\frac{5}{6}\right) + 7$$

$$= \frac{25}{36} - \frac{10}{6} + 7$$

$$= \frac{25 - 60 + 7 \times 36}{36}$$

$$= \frac{-35 + 252}{36}$$

$$= \frac{217}{36}$$

Equation of tangent

$$y - y_1 = \frac{dy}{dx}(x - x_1)$$

$$y - \frac{217}{36} = \frac{-1}{3}\left(x - \frac{5}{6}\right)$$

$$\frac{36y - 217}{36} = \frac{-1}{3}\left(\frac{x}{1} - \frac{5}{6}\right)$$

$$\frac{36y - 217}{36} = \frac{-1}{3}\left(\frac{6x - 5}{6}\right)$$

$$12x + 36y - 217 = 0$$

6. $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

$$\frac{2x}{a^2}-\frac{2y}{b^2}\frac{dy}{dx}=0$$

$$\frac{dy}{dx}=\frac{\frac{-2x}{a^2}}{\frac{-2y}{b^2}}$$

$$=\frac{-2x}{a^2}\times\frac{b^2}{-2y}$$

$$\left.\frac{dy}{dx}\right]_{x_o y_o}=\frac{x}{y}.\frac{b^2}{a^2}$$

$$=\frac{x_o}{y_o}.\frac{b^2}{a^2}$$

Equation

$$y-y_1=\frac{dy}{dx}(x-x_1)\Rightarrow y-y_0=\frac{x_0}{y_0}\frac{b^2}{a^2}(x-x_0)$$

$$yy_oa^2-y_o{}^2a^2=xx_ob^2-x_o{}^2b^2$$

$$x_o{}^2b^2-y_o{}^2a^2=xx_ob^2-yy_oa^2$$

Dividing by a^2b^2

$$\frac{x_o{}^2b^2}{a^2b^2}-\frac{y_o{}^2a^2}{a^2b^2}=\frac{xx_ob^2}{a^2b^2}-\frac{yy_oa^2}{a^2b^2}$$

$$\frac{x_o{}^2}{a^2}-\frac{y_o{}^2}{b^2}=\frac{xx_o}{a^2}-\frac{yy_o}{b^2}$$

$1=a\frac{xx_o}{a^2}-\frac{yy_o}{b^2}$ From (1)

7. Let $y=x^{\frac{1}{2}}$

$$x=0.0036,\ \Delta x=0.0001$$

$$y+\Delta y=(x+\Delta x)^{\frac{1}{2}}$$

$$\Delta y=(x+\Delta x)^{\frac{1}{2}}-y$$

$$\left(\frac{dy}{dx}\right).\Delta x=(x+\Delta x)^{\frac{1}{2}}-(x)^{\frac{1}{2}}$$

$$\frac{1}{2\sqrt{x}}.\Delta x=\left(x+\Delta x\right)^{\frac{1}{2}}-\left(x\right)^{\frac{1}{2}}$$

We get

$$\left(x+\Delta x\right)^{\frac{1}{2}}=0.0608$$

$$(0.0037)^{\frac{1}{2}}=0.0608$$

8. $y=x^{3/2}$

$$x=4,\ \Delta x=-0.032$$

$$y+\Delta y=\left(x+\Delta x\right)^{3/2}$$

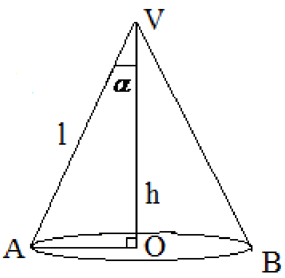
$$\Delta y=\left(x+\Delta x\right)^{3/2}-y$$

$$\left(\frac{dy}{dx}\right).\Delta x=\left(x+\Delta x\right)^{3/2}-x^{3/2}$$

$$\frac{3}{2}\left(x\right)^{1/2}.\Delta x=\left(x+\Delta x\right)^{3/2}-x^{3/2}$$

$$\left(3.968\right)^{3/2}=7.904$$

9. $\frac{vo}{l}-\cos\alpha$



$$vo=l\cos\alpha$$

$$\frac{OA}{l}=\sin\alpha$$

$$OA=l\sin\alpha$$

$$V=\frac{1}{3}\pi\left(OA\right)^{2}.vo$$

$$=\frac{1}{3}\pi\left(l\sin\alpha\right)^{2}.\left(l\cos\alpha\right)$$

$$=\frac{1}{3}\pi l^{2}.\sin^{2}\alpha.l\cos\alpha$$

$$=\frac{1}{3}\pi l^{3}\sin^{2}\alpha.\cos\alpha$$

$$\frac{dv}{d\alpha}=\frac{\pi l^{3}}{3}\left[-\sin^{3}\alpha+2\sin\alpha.\cos^{2}\alpha\right]$$

For maximum/minimum

$$\frac{dv}{d\alpha}=0$$

$$\sin^{3}\alpha=2\sin\alpha.\cos^{2}\alpha$$

$\tan\alpha = \sqrt{2},\ \alpha = \tan^{-1}\sqrt{2}$

$\cos\alpha = \dfrac{1}{\sqrt{1+\tan^2\alpha}} = \dfrac{1}{\sqrt{3}}$

$\dfrac{d^2v}{d\alpha^2} = \dfrac{\pi l^3}{3}\left[-3\sin^2\alpha.\cos\alpha + 2\left(\sin\alpha.2\cos\alpha(-\sin\alpha) + \cos^2\alpha.\cos\alpha\right)\right]$

$= \dfrac{\pi l^3}{3}\left[-3\sin^3\alpha.\cos\alpha - 4\sin^2\alpha.\cos\alpha + 2\cos^3\alpha\right]$

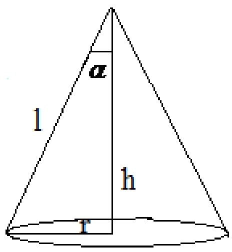
$= \dfrac{\pi l^3}{3}\left[-7\sin^2\alpha.\cos\alpha + 2\cos\alpha\right]$

-tive maximum

$\alpha = \tan^{-1}\sqrt{2}$

10. $s = \pi r^2 + \pi rl \qquad (Given)$

$l = \dfrac{s-\pi r^2}{\pi r}$



Let v be the volume

$v = \dfrac{1}{3}\pi r^2 h$

$v^2 = \dfrac{1}{9}\pi^2 r^4 h^2 \qquad \left[h^2 = l^2 - r^2\right]$

$v^2 = \dfrac{1}{9}\pi^2 r^4\left(l^2 - r^2\right)$

$v^2 = \dfrac{1}{9}\pi^2 r^4\left[\left(\dfrac{s-\pi r^2}{\pi r}\right)^2 - r^2\right]$

$= \dfrac{1}{9}\pi^2 r^4\left[\dfrac{\left(s-\pi r^2\right)^2}{\pi^2 r^2} - \dfrac{r^2}{1}\right]$

$= \dfrac{1}{9}r^2\left[\left(s-\pi r^2\right)^2 - \pi^2 r^4\right]$

$= \dfrac{1}{9}r^2\left[s^2 + \pi^2 r^4 - 2s\pi r^2 - \pi^2 r^4\right]$

$= \dfrac{1}{9}r^2\left[s^2 - 2s\pi r^2\right]$

$z = \dfrac{1}{9}\left[s^2 r^2 - 2s\pi r^4\right]$

$\left[\because v^2 = z\right]$

$$\frac{dz}{dr} = \frac{1}{9}\left[2rs^2 - 8s\pi r^3\right]$$

$$0 = \frac{1}{9}\left[2rs^2 - 8s\pi r^3\right]$$

$$48s\pi r^3 = 2rs^2$$

$$4\pi r^2 = s$$

$$\frac{d^2z}{dr^2} = \frac{1}{9}\left[2s^2 - 24s\pi r^2\right]$$

$$\left.\frac{d^2z}{dr^2}\right]_{r^2=\frac{s}{4\pi}} = \frac{1}{9}\left[25^2 - 24\pi.\frac{5}{4\pi}\right]$$

$$= +tive$$

Hence minimum

Now $s = 4\pi r^2$

$$s = \pi rl + \pi r^2$$

$$4\pi r^2 = \pi rl + \pi r^2$$

$$3\pi r^2 = \pi rl$$

$$3r - l$$

$$\frac{r}{l} = \frac{1}{3}$$

$$\sin\alpha = \frac{1}{3}$$

$$\alpha = \sin^{-1}\left(\frac{1}{3}\right)$$

CBSE TEST PAPER-03

CLASS - XII MATHEMATICS (Calculus: Application of Derivatives)

Topic: - application of derivatives

1. Sand is pouring from a pipe at the rate of 12cm^3/s. the falling sand forms a cone on the ground in much a way that the height of the cone is always one – sixth of the radius of the here. How fast is the height of the sand cone increasing when the height in 4cm. [4]

2. The total revenue in RS received from the sale of x units of the product is given by R (x) = 13x^2 + 26x + 15 find MR when 17 unit are produce. [4]

3. Prove that $y = \frac{4\sin\theta}{2+\cos\theta} - \theta$ is an increasing function = x of θ in $\left[\theta, \frac{\pi}{2}\right]$ [4]

4. Prove that the function of given by f(x) = log sinx is strictly increasing on $\left[0, \frac{\pi}{2}\right]$ and strictly decreasing on $\left(\frac{\pi}{2}, \pi\right)$ [4]

5. Find a point on the curve y = (x – 2)^2 at which the tangent is parallel to the chord joining the points (2, 0) and (4, 4) [4]

6. Find the equation of tangent to the curve given by $x = a\sin^3 t,\ \ y = b\cos^3 t$ at a point where $t = \pi/2$ [4]

7. Find the approximate value of f(3.02) where f(x) = 3x^2 + 5x + 3. [4]

8. Find the approximate value of $(32.15)^{1/5}$ [4]

9. A square piece of tin of side 18cm is to be made into a box without top by cutting a square from each corner and folding of the flaps to form the box. What should be the side of the square to be cut off so that the volume of the box is the maximum possible? [6]

10. Show that the right circular cylinder of given surface and maximum volume is much that its height is equal to the diameter of the base. [6]

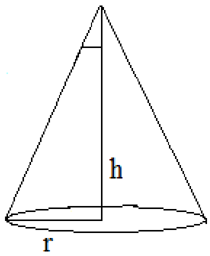
CBSE TEST PAPER-03

CLASS - XII MATHEMATICS (Calculus: Application of Derivatives)

[ANSWERS]

Topic: - application of derivatives

1.



$$\frac{dv}{dt} = 12cm^3 / s \qquad (Given)$$

$$h = \frac{1}{6}r \qquad (Given)$$

$$v = \frac{1}{3}\pi r^2 h$$

$$v = \frac{1}{3}.\pi.(6h)^2.h \qquad \left[\because h = \frac{1}{6}r\right]$$

$$v = \frac{1}{3}.\pi.36.h^3$$

$$v=12\pi h^3$$

$$\frac{dv}{dt} = 12\pi.3h^3.\frac{dh}{dt}$$

$$12 = 12\pi.3(4)^2.\frac{dh}{dt} \qquad [h = 4cm]$$

$$\frac{12}{12\pi.3.16} = \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{1}{48\pi}cm / s$$

2. $MR = \frac{d}{dx}\left(R(x)\right) = 26x + 26$

$$MR\big]_{x=17} = 25 \times 17 + 26$$

$$= 442$$

3. $y = \frac{4\sin\theta}{2+\cos\theta} - \theta$

$f'(\theta) = \frac{(2+\cos\theta)(4\cos\theta)+4\sin^2\theta}{(2+\cos\theta)^2} - 1$

$= \frac{8\cos\theta+4}{(2+\cos\theta)^2} - 1$

$= \frac{8\cos\theta+4-\left(4+\cos^2\theta+4\cos\theta\right)}{(2+\cos\theta)^2}$

$f'(\theta) = \frac{\cos\theta(4-\cos\theta)}{2+\cos\theta}$

$\Rightarrow f'(\theta) = \frac{\cos\theta(4-\cos\theta)}{2+\cos\theta} > 0 \forall \theta \in \left(0, \frac{\pi}{2}\right)$

4. $f'(x) = \frac{1}{\sin x}.\cos x$

$f'(x) = \cot x$

$f'(x) > 0 \forall x \in \left(0, \frac{\pi}{2}\right)$

and

$f'(x) < 0 \forall x \in \left(\frac{\pi}{2}, \pi\right)$

Hence f(x) = log sinx is strictly increasing on $\left(0, \frac{\pi}{2}\right)$ and decreasing on $\left(\frac{\pi}{2}, \pi\right)$

5. $y = (x-2)^2 ------(1)$

Slope of tangent to curve

$\frac{dy}{dx} = 2(x-2)$

Slope of chord $= \frac{4-0}{4-2}$ $\left[m = \frac{y_2 - y_1}{x_2 - x_1}\right]$

$AT\theta \quad 2(x-2) = 2$

$x = 3$

Put x = 3 in equation (1)

$y = 1$

Points (3, 1)

6. $\frac{dx}{dt} = 3a\sin^2 t.\cos t dt$

$\frac{dy}{dt} = -3b\cos^2 t \sin t dt$

$\frac{dy}{dx} = \frac{-b}{a}\cot t$

$\left.\frac{dy}{dx}\right]_{t=\pi/2} = \frac{-b}{a} \times 0 = 0$

When $t = \pi/2,\ x - 9,\ and\ y = 0$

Equation of tangent

$y - y_1 = \frac{dy}{dx}(x - x_1),\ y - 0 = 0(x - a)$

$y = 0$

7. $x = 3,\ \Delta x = 0.02$

$f(x + \Delta x) = f(x) + f'(x)\Delta x$

$f(x + \Delta x) = (3x^2 + 5x + 3) + (6x + 5) \times 0.02$

Put $x = 3,\ \Delta x = 0.02$

$f(3.02) = 45.46$

8. $y = x^{1/5}$

$x = 32,\ \Delta x = 0.15$

$y + \Delta y = (x + \Delta x)^{1/5}$

$\Delta y = (x + \Delta x)^{1/5} - y$

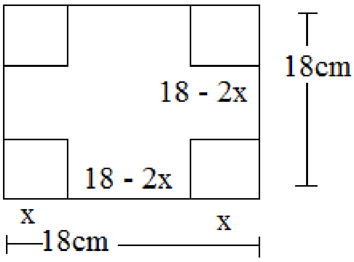
$= (x + \Delta x)^{1/5} - x^{1/5}$

$\left(\frac{dy}{dx}\right).\Delta x = (x + \Delta x)^{1/5} - x^{1/5}$

$\frac{1}{5}(x)^{-4/5}.\Delta x = (32.15)^{1/5} - (32)^{1/5}$

$(32.15)^{1/5} = 2.0018$

9. $l=(18-2x)cm$



$b=(18-2x)cm$

$h=xcm$

$v=l\times b\times h$

$v=(18-2x)\times(18-2x)\times x$

$v=(18-2x)^2\times x$

$$\frac{dv}{dx}=(18-2x)^2.(1)+(x).2(18-2x)(-2)$$

$$\frac{dv}{dx}=(18-2x)[(18-2x)-4x]$$

For maximum/minimum

$0=(18-2x)(18-6x)$

$x=9\ \ (neglect)$

$x=3$

$$\frac{d^2v}{dx^2}=(18-2x)(-6)+(18-6x)(-2)$$

$$\left.\frac{d^2v}{dx^2}\right]_{x=3}=-12\times 6$$

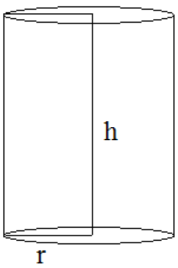
$=-72$ maximum

$l=18-2\times 3=12cm$

$b=12cm$

$h=3cm$

10.



$s=2\pi rh+2\pi r^2$

$$\frac{s-2\pi r^2}{2\pi r}=h$$

$v=\pi r^2 h$

$$v = \pi . r^2 \left(\frac{s - 2\pi r^2}{2\pi r} \right)$$

$$v = \frac{1}{2}\left[sr - 2\pi r^3 \right]$$

$$\frac{dv}{dr} = \frac{1}{2}\left[s - 6\pi r^2 \right]$$

$$\frac{d^2v}{dr^2} = \frac{1}{2}\left[0 - 12\pi r \right]$$

For maximum/minimum

$$s = 6\pi r^2$$

$$\left.\frac{d^2v}{dr^2}\right]_{r^2 = \frac{s}{6\pi}} = \frac{1}{2}\left[s - 12\pi . \frac{s}{6\pi} \right]$$

$= -$tive maximum

$$s = 2\pi rh + 2\pi r^2$$

$$s = 6\pi r^2$$

$$2\pi rh + 2\pi r^2 = 6\pi r^2$$

$$2\pi rh = 4\pi r^2$$

$$2h = 4r$$

$$h = 2r$$

CBSE TEST PAPER-04
CLASS - XII MATHEMATICS (Calculus: Application of Derivatives)

Topic: - application of derivatives

1. A balloon, which always remains spherical on inflation, is being inflated by pumping in 900cm^3/s. find the rate at which the radius of the balloon increase when the radius is 15cm. [4]

2. A circular disc of radius 3cm is being heated. Due to expansion, their radius increase at the rate of 0.05 cm/s. find the rate at which its area is increasing when radius is 3.2cm. [4]

3. Find the intervals in which the function f given by $f(x) = \frac{4\sin x - 2x - x\cos x}{2 + \cos x}$ is (i) increasing (ii) decreasing [4]

4. Find the interval in which the function f given by $f(x) = x^3 + \frac{1}{x^3}$, $x \neq 0$ is (i) increasing (ii) decreasing. [4]

5. Find the equation of the normal to the curve $x^2 = 4y$ which passes through the point (1, 2) [4]

6. Show that the normal at any point θ to the curve $x = a\cos\theta + a\theta.\sin\theta$, $y = a\sin\theta - a\theta.\cos\theta$ is at a constant distance from origin. [4]

7. Show that the altitude of the right circular cone of maximum volume that can be inscribed in a sphere of radius r is 4r/3. [4]

8. Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$ Also find the maximum volume. [4]

9. Show that the height of the cylinder of greatest volume which can be inscribed in a right circular cone of height h and having semi-vertical angle α is one third that of the cone and the greatest volume of cylinder is $\frac{4}{27}\pi^3 h \tan\alpha.$ [6]

10. Show that the right circular cone of least curved surface and given volume has an altitude equal to $\sqrt{2}$ times the radius of the base. [6]

CBSE TEST PAPER-04

CLASS - XII MATHEMATICS (Calculus: Application of Derivatives)

[ANSWERS]

Topic: - application of derivatives

1. Let V be the volume of sphere

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dv}{dt} = 900cm^3 / s$$

$$\frac{dv}{dt} = \frac{4}{3}.\pi.3r^2.\frac{dr}{dt}$$

$$900 = 4\pi.r^2.\frac{dr}{dt}$$

$$900 = 4\pi\times(15)^2.\frac{dr}{dt} \qquad [r = 15cm]$$

$$\frac{900}{4\pi\times 225} = \frac{dr}{dt}$$

$$\frac{1}{\pi}cm / s = \frac{dr}{dt}$$

2. $A = \pi r^2$

$$\frac{dA}{dt} = 2\pi r.\frac{dr}{dt}$$

$$= 2\pi(3.2)\times 0.05$$

$$= 0.320\pi cm^2 / s$$

3. $f(x) = \dfrac{4\sin x - 2x - x\cos x}{2 + \cos x}$

$$= \frac{4\sin x - x(2+\cos x)}{2+\cos x}$$

$$= \frac{4\sin x}{2+\cos x} - \frac{x(2+\cos x)}{(2+\cos x)}$$

$$f(x) = \frac{4\sin x}{2+\cos x} - x$$

$$f'(x) = \frac{\cos x(4-\cos x)}{(2+\cos x)^2}$$

$[\because - \leq \cos x \leq 1]$

Hence

$$\frac{\cos x(4-\cos x)}{(2+\cos x)^2} > 0 \forall x \in \left(0, \frac{\pi}{2}\right) and \left(\frac{3\pi}{2}, 2\pi\right)$$

$$\frac{\cos x(4-\cos x)}{(2+\cos x)^2} < 0 \forall x \in \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$$

4. $f'(x) = 3x^2 - \frac{3}{x^4}$

$$= \frac{3}{x^4}(x^6 - 1)$$

$$= \frac{3}{x^4}(x^2-1)(x^4+x^2+1)$$

For increasing

$$f'(x) = 0, \Rightarrow 3\left(\frac{x^4+x^2+1}{x^4}\right)(x^2-1) > 0$$

$$\Rightarrow (x^2-1) > 0$$

$$(x-1)(x+1) > 0$$

So f(x) is increase on $(-\infty, -1)$ and $(1, \infty)$

For decreasing

$$f'(x) < 0$$

$$3\left(\frac{x^4+x^2+1}{x^4}\right)(x^2-1) < 0$$

$$\left[\because 3\left(\frac{x^4+x^2+1}{x^4}\right) < 0\right.$$

$$(x^2-1) < 0$$

$$(x-1)(x+1) < 0$$

f(x) is decrease on (-1, 0) (0, 1)

5. $x^2 = 4y$

$$\frac{dy}{dx} = \frac{x}{2}$$

Let (x_1 y_1) be the point

$$\left.\frac{dy}{dx}\right]_{(x,y)} = \frac{x_1}{2}$$

Slope of normal $= \frac{-1}{\frac{dy}{dx}} = \frac{-1}{\frac{x_1}{2}} = \frac{-2}{x_1}$

Equation $y - y_1 = \frac{-1}{\frac{dy}{dx}}(x - x_1)$

$$y - y_1 = \frac{-2}{x_1}(x - x_1)$$

Passes through (1, 1)

$$2 - y_1 = \frac{-2}{x_1}(1 - x_1) ---------- (1)$$

$$x^2 = 4y$$

$(x_1 y_1)$ Passes through it

$$x_1^2 = 4y_1$$

$$2 - \frac{x_1^2}{4} = \frac{-2}{x_1}(1 - x_1)$$

$$8 - x_1^2 = \frac{-8 + 8x_1}{x_1}$$

$$x_1^3 = 8$$

$$x_1 = 2$$

$y_1 = 1$ $\left[\because x_1^2 = 4y_1\right]$

Now repeat equation

$$y - y_1 = \frac{-1}{\frac{dy}{dx}}(x - x_1)$$

$$y - 1 = \frac{-2}{2}(x - 2)$$

$$y-1=-1(x-2)$$
$$x+y=3$$

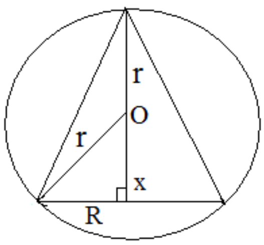
6. $$\frac{dx}{d\theta}=-a\sin\theta+a(\theta.\cos\theta+\sin\theta)$$

$$\frac{dx}{d\theta}=a\theta.\cos\theta$$
$$\frac{dy}{d\theta}=a\theta.\sin\theta$$
$$\frac{dy}{dx}=\tan\theta$$

Slope of normal $=\frac{-1}{\tan\theta}$

Equation of normal $y-y_1=\frac{-1}{\frac{dy}{dx}}(x-x_1)$

$$y-(a\sin\theta-a.\theta\cos\theta)=\frac{-\cos\theta}{\sin\theta}\left[x-\left(a\cos\theta-a.\theta\sin\theta\right)\right]$$
$$x\cos\theta+y\sin\theta=a$$
length of from $\perp$ origin
$$\frac{\left|0\cos+0\sin\theta-a\right|}{\sqrt{\cos^2\theta+\sin^2\theta}}=a \text{ Prove.}$$



7. $$V=\frac{1}{3}\pi R^2H$$

$$=\frac{1}{3}\pi R^2.(r+x)$$
$$=\frac{1}{3}\pi.\left(r^2-x^2\right)(r+x) \qquad \left[\because R^2=r^2-x^2\right]$$
$$\frac{dv}{dx}=\frac{1}{3}\pi\left[\left(r^2-x^2\right)(1)+(r+x)(0-2x)\right]$$
$$\frac{dv}{dx}=\frac{1}{3}\pi\left[r^2-x^2-2rx-2x^2\right]$$

$$=\frac{1}{3}\pi\left[r^2-2rx-3x^2\right]$$

$$=\frac{1}{3}\pi\left[r^2-3rx+rx-3x^2\right]$$

$$=\frac{1}{3}\pi\left[r\left(r-3rx\right)+x\left(r-3x\right)\right]$$

$$=\frac{1}{3}\pi\left(r-3x\right)\left(r+x\right)$$

$$\frac{dv}{dx}=0$$

$$r=3x$$

$$\frac{r}{3}=x$$

$$\frac{d^2v}{dx^2}=\frac{1}{3}\pi[0-2r-6x]$$

$$\left.\frac{d^2v}{dx^2}\right]_{x=\frac{r}{3}}=\frac{1}{3}\pi\left[-2r-6\times\frac{r}{3}\right]$$

$$=\frac{1}{3}\pi[-4r]$$

= - tive maximum

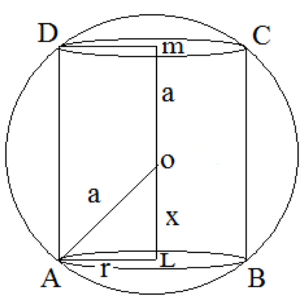
Altitude $=r+x$

$$=\frac{r}{3}+r$$

$=\frac{4r}{3}$ Prove.

8. $V=\pi r^2.2x$ $\left[\begin{array}{l}\because OL=x\\ LM=2x\end{array}\right]$

$$=\pi.\left(a^2-x^2\right).2x$$



$$V=2\pi\left(a^2x-x^3\right)$$

$$\frac{dv}{dx}=2\pi\left(a^2-3x^2\right)$$

$$\frac{d^2v}{dx^2}=2\pi[0-6x]$$

$$=-12\pi x$$

For maximum/minimum

$$\frac{dv}{dx} = 0$$

$$2\pi\left[a^2 - 3x^2\right] = 0$$

$$a^2 = 3x^2 \Rightarrow \sqrt{\frac{a^2}{3}} = x$$

$$\Rightarrow x = \frac{a}{\sqrt{3}}$$

$$\left.\frac{d^2v}{dx^2}\right]_{x=\frac{a}{\sqrt{3}}} = -12\pi.\frac{a}{\sqrt{3}}$$

= - tive maximum

Height of cylinder

$$= 2x$$

$$= 2 \times \frac{a}{\sqrt{3}}$$

$$= \frac{2a}{\sqrt{3}}$$

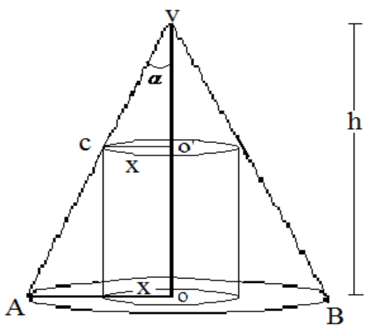
9. $$\frac{vo'}{x} = \cot\alpha$$

$$vo' = x\cot\alpha$$

$$oo' = h - x\cot\alpha$$

$$V = \pi x^2.(h - x\cot\alpha)$$

$$V = \pi x^2 h - \pi x^3 \cot\alpha$$



$$\frac{dv}{dx} = 2\pi h - 3\pi x^2 \cot\alpha$$

for maximum/minimum

$$\frac{dv}{dx} = 0$$

$$2\pi xh - 3\pi x^2 \cot\alpha = 0$$

$$x = \frac{2h}{3}\tan\alpha$$

$$\frac{d^2v}{dx^2} = 2\pi h - 6\pi x\cot\alpha$$

$$\left.\frac{d^2v}{dx^2}\right]_{x=\frac{2h}{3}\tan\alpha} = \pi(2h-4h)$$

- tive maximum

$$V = \pi.x^2(h - x\cot\alpha)$$

$$= \pi\left(\frac{2h}{3}\tan\alpha\right)^2\left[h - \frac{2h}{3}\tan\alpha\cot\alpha\right]$$

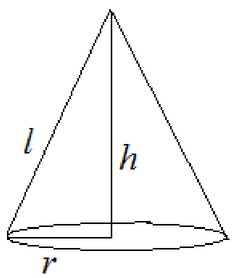
$$= \pi.\frac{4h^2}{9}\tan^2\alpha.\frac{h}{3}$$

$$V = \frac{4}{27}\pi h^3\tan^2\alpha$$

10. $v = \frac{1}{3}.\pi r^2 h$

$Let\ r^2h = k$

When K is constant



$$r^2h = k$$

$$h = \frac{k}{r^2}$$

$$s = \pi rl$$

$$s^2 = \pi^2r^2l^2$$

$$= \pi^2.r^2(r^2 + h^2)$$

$$= \pi^2r^2\left[r^2 + \frac{k^2}{r^4}\right]\left[\because h = \frac{k}{r^2}\right]$$

$$z = \pi^2r^4 + \pi^2k^2r^{-2} \qquad [s^2 = z]$$

$$z = \pi^2r^4 + \pi^2k^2r^{-2}$$

$$\frac{dz}{dr} = 4\pi^2r^3 - 2\pi^2k^2r^{-3}$$

$$\frac{d^2z}{dr^2} = 12\pi^2r^2 + 6\pi^2k^2r^{-4}$$

$$\frac{dz}{dr} = 0$$

$$= 4\pi^2r^3 - \frac{2\pi^2kr}{r^3}$$

$$0 = \frac{4\pi^2 r^6 - 2\pi^2 kr}{r^3}$$

$$2\pi^2 k^2 = 4\pi^2 r^6$$

$$k^2 = 2r^6$$

$$\left.\frac{d^2 z}{dr^2}\right]_{r^6=\frac{k^2}{2}} = \text{-tive maximum}$$

$$k^2 = 2r^6$$

$$k = r^2 h$$

$$\Rightarrow k^2 = r^4 h^2$$

$$\Rightarrow 2r^6 = r^4 h^2$$

$$2r^2 = h^2$$

$$\sqrt{2r^2} = h$$

$$h = \sqrt{2}r$$

CBSE TEST PAPER-05
CLASS - XII MATHEMATICS (Calculus: Application of Derivatives)

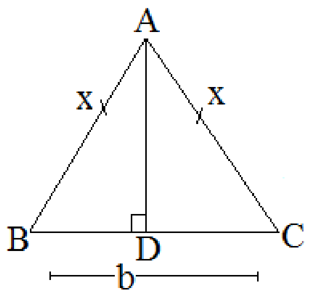
Topic: - application of derivatives

1. The two equal side of on isosceles Δ with fixed base b are decreasing at the rate of 3cm/s. How fast is the area decreasing when the two equal sides are equal to the base? [4]
2. A men of height 2m walks at a uniform speed of 5km/h away from a lamp, past which is 6m high. Find the rate at which the lengths of his shadow increase. [4]
3. A water tank has the shape of an inverted right circular cone with its axis vertical and vertex lower most. Its semi vertical angle is $\tan^{-1}$ (0.5) water is poured into it at a constant rate of $5cm^3$/hr. Find the rate at which the level of the water is rising at the instant when the depth of water in the tank is 4m. [4]
4. Find the interval in which the function given by $f(x)=\frac{3}{10}x^4-\frac{4}{5}x^3-3x^2+\frac{36}{5}x+11$ is (a) Strictly increasing (b) Strictly decreasing [4]
5. Show that $f(x)=\tan^{-1}(\sin x+\cos x)$ is always an increasing function in $\left(0,\frac{\pi}{4}\right)$ [4]
6. For the curve $y = 4x^3 - 2x^5$, find all the point at which the tangent passes through the origin. [4]
7. Prove that the curves $x = y^2$, and $xy = K$ cut at right angles if $8k^2 = 1$ [4]
8. Find the maximum area of an isosceles Δ inscribed in the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ with its vertex at one end of the major axis. [4]
9. A tank with rectangular base and rectangular sides, open at the top is to be constructed so that its depth is 2m and volume is $8m^3$. If building of tank costs Rs 70 per sq. metres for the base and Rs 45 per sq. metres for sides what is the cost of least expansive tank? [6]
10. The sum of the perimeter of a circle and square is k, where K is some constant. Prove that the sum of their area is least when the side of square is double the radius of circle. [6]
11. A window is the form of a rectangle surmounted by a semi circular opening the total perimeter of the window is 10m. Find the dimensions of the window to admit maximum light through the whole opening. [6]
12. A point on the hypotenuse of a triangle is at distance a and b from the sides of the triangle. Show that the minimum length of the hypotenuse is $\left(a^{2/3}+b^{2/3}\right)^{3/2}$. [6]

CBSE TEST PAPER-05
CLASS - XII MATHEMATICS (Calculus: Application of Derivatives)
[ANSWERS]

Topic: - application of derivatives

1. $\frac{dx}{dt} = 3cm/s$



Let A be area of Δ

$$A = \frac{1}{2} \times b \times AD$$

$$\left[AD = \sqrt{x^2 - \frac{b^2}{4}} = \frac{\sqrt{4x^2 - b^2}}{2} \right]$$

$$= \frac{1}{2} \times b \times \frac{\sqrt{4x^2 - b^2}}{2}$$

$$\frac{dA}{dt} = \frac{1}{4} b . \frac{1}{2\sqrt{4x^2 - b^2}} .8x. \frac{dx}{dt}$$

$$\frac{dA}{dt} = \frac{b}{\sqrt{4 \times b^2 - b^2}} \times b.3 \qquad [x = b]$$

$$= \frac{3b^2}{\sqrt{3}b}$$

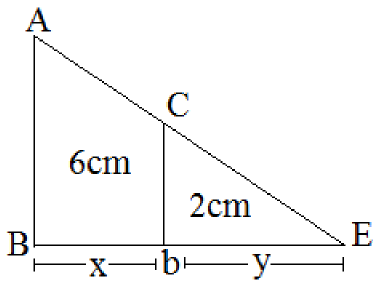
$$= \frac{\sqrt{3}.\sqrt{3}}{\sqrt{3}}.b$$

$$\frac{dA}{dt} = \sqrt{3}bcm^2 / \sec$$

2. AB is lamp post DC is man

$\frac{dx}{dt} = 5km/h, \quad \frac{dy}{dt} = ?$

$\Delta DEC \sim \Delta BEA$



$$\frac{2}{6} = \frac{y}{x+y}$$

$$x + y = 3y$$

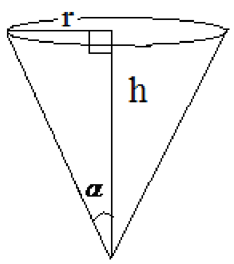
$$x = 2y$$

$$\frac{dx}{dt} = 2\frac{dy}{dt}$$

$$5 = 2\frac{dy}{dt}$$

$$\frac{5}{2} km/h = \frac{dy}{dt}$$

3. $$\tan\alpha = \frac{r}{h}$$



$$\tan^{-1}(0.5) = \alpha$$

$$\tan\alpha = 0.5$$

$$\frac{r}{h} = \frac{0.5}{10}$$

$$h = 2r$$

$$V = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}\pi.\left(\frac{h}{2}\right)^2.h$$

$$= \frac{1}{3}\pi\frac{h^3}{4}$$

$$\frac{dv}{dt} = \frac{1}{12}\pi.3h^2\frac{dh}{dt}$$

$$5 = \frac{1}{12}\pi.3(4)^2.\frac{dh}{dt}$$

$$\frac{dh}{dt} = 35/88\ m/h$$

4. $$f(x) = \frac{3}{10}x^4 - \frac{4}{5}x^3 - 3x^2 + \frac{36}{5}x + 11$$

$$f'(x) = \frac{3}{10}.4x^3 - \frac{4}{5}.3x^2 - 6x + \frac{36}{5}$$

$$= \frac{6}{5}x^3 - \frac{12}{5}x^2 - \frac{6x}{1} + \frac{36}{5}$$

$$f'(x) = \frac{6x^3 - 12x^2 - 30x + 36}{5}$$

$$= \frac{6}{5}\left[x^3 - 2x^2 - 5x + 6\right]$$

$$P(x) = x^3 - 3x^2 - 5x + 6$$

$$Put\ x = 1$$

$$P(1) = 1 - 2 - 5 + 6 = 0$$

$$f'(x) = \frac{6}{5}(x-1)\left(x^2 - x - 6\right)$$

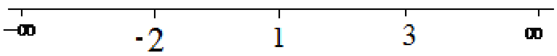
$$= \frac{6}{5}(x-1)\left[x^2 - 3x + 2x - 6\right]$$

$$= \frac{6}{5}(x-1)\left[x(x-3) + 2(x-2)\right]$$

$$= \frac{6}{5}(x-1)(x-3)(x+2)$$

$Put\ f'(x) = 0$

$x = 1,\ x = 3,\ x = -2$



int	Sign of f'(x)	Result
$(-\infty, -2)$	-tive	Decrease
$(-2, 1)$	+tive	Increase
$(1, 3)$	-tive	Decrease
$(3, \infty)$	+tive	increase

5. $f(x) = \tan^{-1}(\sin x + \cos x)$

$$f'(x) = \frac{1}{1 + (\sin x + \cos x)^2}.(\cos x - \sin x)$$

$$= \frac{\cos x(1 - \tan x)}{1 + (\sin x + \cos x)^2}$$

$$\left[\because \tan x < 1 \forall x \in \left(0, \frac{\pi}{4}\right)\right]$$

$$f'(x) < 0 \forall x \in \left(0, \frac{\pi}{4}\right)\left[0 < x < \frac{\pi}{4}\right]$$

Hence f(x) is strictly increasing on $\left(0, \frac{\pi}{4}\right)$

6. $y = 4x^3 - 2x^5$

$y_1 = 4x_1^3 - 2x_1^5$ ----------(1)

$$\frac{dy}{dt} = 12x^2 - 10x^4$$

$$\left.\frac{dy}{dt}\right]_{x_1 y_1} = 12x_1^2 - 10x_1^4$$

Equation $y - y_1 = \frac{dy}{dx}\left(x - x_1\right)$

$0 - y_1 = \left(12x_1^2 - 10x_1^4\right)\left(0 - x_1\right)$ [Passes through (0, 0)]

$-y_1 = -12x_1^3 + 10x_1^5$

$y_1 = 12x_1^3 - 10x_1^5$

$but\ y_1 = 4x_1^3 - 2x_1^5 \quad \left[from\ (1)\right]$

$-12x_1^3 - 10x_1^5 = 4x_1^3 - 2x_1^5$

$8x_1^3 = 8x_1^5$

$x_1^3 - x_1^5 = 0$

$x_1^3\left(1 - x_1^2\right) = 0$

$x_1 = 1$

$x_1 = -1$

$x_1 = 0$

$When\ x_1 = 1 \quad When\ x_1 = -1 \quad When\ x_1 = 0$

$y_1 = 2 \quad y_1 = -2 \quad y_1 = 0$

$(1, 2), (-1, -2), (0, 0)$

7. $x = y^2 ----------(1)$

$xy = K ----------(2)$

on solving 1 and 2

$x = K^{2/3}, y = K^{1/3}$

$\frac{dy}{dx} = \frac{1}{2y} \quad \left[from(1)\right]$

$$\left.\frac{dy}{dx}\right]_{\left(K^{2/3}, K^{1/3}\right)} = \frac{1}{2K^{1/3}}$$

$\frac{dy}{dx} = \frac{-y}{x}\left[from(2)\right]$

$= -\frac{K^{1/3}}{K^{2/3}} = -K^{\frac{1}{3}}.K^{\frac{-2}{3}}$

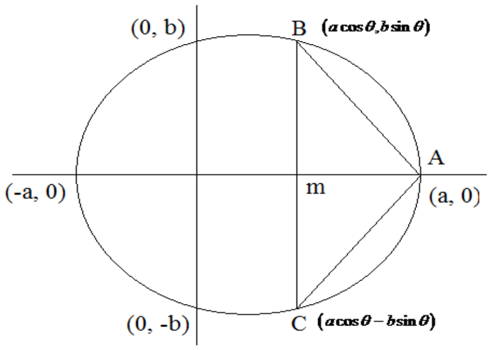
$$= -K^{\frac{1}{3}}$$

$$AT\theta$$

$$= \frac{1}{2K^{1/3}} \times \frac{-1}{(K)^{1/3}} = -1$$

$$1 = 8K^2$$

8.



Let A be the area of Δ ABC

$$A = \frac{1}{2}(2b\sin\theta) \times (a - a\cos\theta)$$

$$A = ab(\sin\theta - \sin\theta.\cos\theta)$$

$$\frac{dA}{d\theta} = ab\left[\cos\theta - \cos^2\theta + \sin^2\theta\right]$$

$$= ab[\cos\theta - \cos 2\theta]$$

For maximum/minimum

$$\frac{dA}{d\theta} = \theta$$

$$ab(\cos\theta - \cos 2\theta) = 0$$

$$\cos\theta = \cos 2\theta$$

$$\cos\theta = \cos(2\pi - 2\theta)$$

$$\theta = 2\pi - 2\theta$$

$$3\theta = 2\pi$$

$$\theta = \frac{2\pi}{3}$$

$$\frac{d^2A}{d\theta^2} = ab(-\sin\theta + 2\sin 2\theta)$$

$$\left.\frac{d^2A}{d\theta^2}\right]_{\theta=\frac{2\pi}{3}} = ab\left[-\sin\frac{2\pi}{3} + 2\sin 2.\frac{2\pi}{3}\right]$$

$$= ab\left[-\frac{\sqrt{3}}{2} - 2\times\frac{\sqrt{3}}{2}\right] < 0$$

$$A = ab\left[\sin\frac{2\pi}{3} - \sin\ \frac{2\pi}{3}\cos\ \frac{2\pi}{3}\right]$$

$$= ab\left[\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}\times\frac{1}{2}\right]$$

$$= \frac{3\sqrt{3}}{4}ab.$$

9. Let x and y be the length and width of rectangular base, v be the volume.

$$v = 8 \qquad (Given)$$

$$v = 2xy$$

$$8 = 2xy$$

$$y = \frac{4}{x}$$

$$s = (xy)\times 70 + 2(x+y)\times 2\times 45$$

$$= x\times\frac{4}{x}\times 70 + 180\left(x + \frac{4}{x}\right)$$

$$= 280 + 180\left(x + \frac{4}{x}\right)$$

$$\frac{ds}{dx} = 0 + 180\left(1 - \frac{4}{x^2}\right)$$

$$\frac{d^2s}{dx^2} = 180\left(0 + \frac{8}{x^3}\right)$$

For maximum/minimum

$$\frac{ds}{dx} = 0$$

$$x = 2$$

$$\left(\frac{d^2s}{dx^2}\right)_{x=2} \ \frac{1440}{2^3} > 0$$

Minimum

$$\cos\alpha = 280 + 180\left(2 + \frac{4}{2}\right)$$

$$= 280 + 180(4)$$

$$= 1000$$

10. r is the radius of circle and x be side of sq.

$$2\pi r + 4x = K$$

$$s = \pi r^2 + x^2$$

$$s = \pi r^2 + \left(\frac{K - 2\pi r}{4}\right)^2$$

$$\frac{ds}{dr} = 2\pi r + 2\left(\frac{K - 2\pi r}{4}\right).\left(0 - \frac{2\pi}{4}\right)$$

$$\frac{ds}{dr} = 2\pi r - \frac{\pi}{4}(K - 2\pi r)$$

$$\frac{d^2 s}{dr^2} = 2\pi - \frac{\pi}{4}(0 - 2\pi)$$

$$= 2\pi + \frac{2\pi^2}{4}$$

For maximum/minimum

$$\frac{ds}{dr} = 0$$

$$\frac{\pi}{4}(K - 2\pi r) = 2\pi r$$

$$K - 2\pi r = 8r$$

$$K = 8r + 2\pi r$$

$$\frac{K}{2(4+\pi)} = r$$

$$Now \frac{d^2 s}{dr^2} > 0$$

Hence maximum

$$2\pi r + 4x = K$$

$$x = \frac{K - 2\pi r}{4}$$

$$x = \frac{K - 2\pi \dfrac{K}{2(4+\pi)}}{4}$$

$$= \frac{4K + K\pi - K\pi}{4(4+\pi)} = \frac{4K}{4(4+\pi)}$$

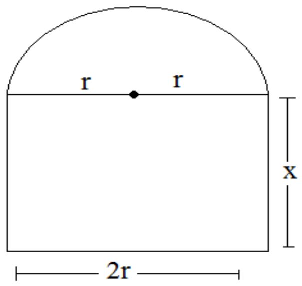
$$x = 2r$$

11. Let P be the perimeter of window

$$P = 2x + 2r + \frac{1}{2} \times 2\pi r$$

$$10 = 2x + 2r + \pi r \qquad [P = 10]$$

$$x = \frac{10 - 2r - \pi r}{2}$$



Let A be area of window

$$A = 2rx + \frac{1}{2}\pi r^2$$

$$= 2r\left[\frac{10 - 2r - \pi r}{2}\right] + \frac{1}{2}\pi r^2$$

$$= 10r - 2r^2 - \pi r^2 + \frac{1}{2}\pi r^2$$

$$= 10r - 2r^2 - \frac{\pi r^2}{2}$$

$$\frac{dA}{dr} = 10 - 4r - \pi r$$

$$\frac{dA}{dr} = 0$$

$$\frac{d^2A}{dr^2} = -(\pi + 4)$$

$$\frac{dA}{dr} = 0$$

$$r = \frac{10}{\pi + 4}$$

$$\frac{d^2A}{dr^2} < 0 \text{ maximum}$$

$$x = \frac{10 - 2r - \pi r}{2}$$

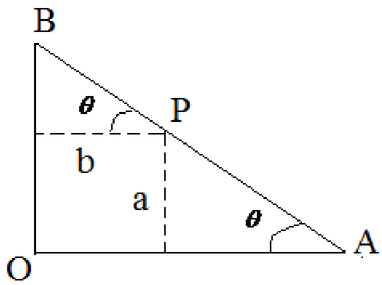
$$x = \frac{10}{\pi + 4}$$

Length of rectangle $= 2r$

$$= \frac{20}{\pi + 4}$$

Width $= \dfrac{10}{\pi + 4}$

12. $AP = a\cos ec\theta$



$BP = b\sec\theta$

$l = AP + BP$

$l = a\cos ec\theta + b\sec\theta$

$$\frac{dl}{d\theta} = -a\cos ec\theta\cot\theta + b\sec\theta.\tan\theta$$

$$\frac{d^2l}{d\theta^2} = a\cos ec^3\theta + a\cos ec\theta\cot^2\theta + b - \sec^3\theta + b\sec\theta.\tan^2\theta$$

For maximum/minimum

$$\frac{dl}{d\theta} = 0$$

$$\tan^3\theta = \frac{a}{b}$$

$$\tan\theta = \left(\frac{a}{b}\right)^{1/3}$$

$$\sin\theta = \frac{a^{1/3}}{\sqrt{a^{2/3} + b^{2/3}}}, \cos\theta = \frac{b^{1/3}}{\sqrt{a^{2/3} + b^{2/3}}}$$

$$\frac{d^2l}{d\theta^2} > 0 \; for \; \tan\theta = \left(\frac{a}{b}\right)^{1/3}$$

L is minimum

$l = a\cos ec\theta + b\sec\theta$

$l = \left(a^{2/3} + b^{2/3}\right)^{2/3}$

CBSE TEST PAPER-01

CLASS - XII MATHEMATICS (Calculus: Integrals)

Topic: - Integrals

1. $\int \frac{x^3-1}{x^2}dx$ [1]

2. $\int \frac{\sec^2 x}{\cos ec^2 x}dx$ [1]

3. $\int \frac{x^3-x^2+x-1}{x-1}dx$ [1]

4. $\int \frac{\tan^4 \sqrt{x} \sec^2 \sqrt{x}}{\sqrt{x}}dx$ [1]

5. Prove $\int \sec x dx = \log|\sec x + \tan x| + c$ [1]

6. $\int_{-1}^{2} |x^3 - x| dx$ [4]

7. $\int_{o}^{\pi} \frac{x \tan x}{\sec x + \tan x}dx$ [4]

8. $\int \frac{\sin^8 x - \cos^8 x}{1 - 2\sin^2 x.\cos^2 x}dx$ [4]

9. $\int \frac{dx}{\sqrt{\sin^3 x \sin(x+\alpha)}}$ [6]

10. $\int \left(\sqrt{\cot x} + \sqrt{\tan x}\right)dx$ [6]

Topic: - Integrals (ANSWERS)

1. $$\int \frac{x^3-1}{x^2}dx = \int\left(\frac{x^3}{x^2}-\frac{1}{x^2}\right)dx$$

$$= \int\left(x - x^{-2}\right)dx$$

$$= \frac{x^2}{2} - \frac{x^{-1}}{-1} + c$$

$$= \frac{x^2}{2} + \frac{1}{x} + c$$

2. $$\int \frac{\sec^2 x}{\cos ec^2 x}dx$$

$$\int \frac{\sec^2 x}{\cos ec^2 x}dx = \int \frac{1}{\cos^2 x} \times \frac{\sin^2 x}{1}dx$$

$$\int \tan^2 x\ dx = \int\left(\sec^2 x - 1\right)dx$$

$$= \tan x - x + c$$

3. $$\int \frac{x^2\left(x-1\right)+1\left(x-1\right)}{\left(x-1\right)}dx$$

$$= \int \frac{\left(x-1\right)\left(x^2+1\right)}{\left(x-1\right)}dx$$

$$= \frac{x^3}{3} + x + c$$

4. Put $\tan\sqrt{x} = t$

$$\sec^2\sqrt{x}.\frac{1}{2\sqrt{x}}dx = dt$$

$$\frac{\sec\sqrt{x}}{\sqrt{x}}dx = 2dt$$

$$= 2\int t^4 dt = 2\frac{t^5}{5} + c$$

$$= \frac{2}{5}\tan^5\sqrt{x} + c$$

5. $\int \sec x \; dx = \int \frac{\sec x(\sec x + \tan x)}{(\sec x + \tan x)} dx$

$Put \;\; \sec x + \tan x = t$

$(\sec x . \tan x . + \sec^2 x) dx = dt$

$\sec x(\tan x + \sec x) dx = dt$

$= \int \frac{dt}{t}$

$= \log | t | + c$

$= \log | \sec x + \tan x | + c$

6. $\int_{-1}^{2} | x^3 - x | \, dx = \int_{-1}^{o} \left(x^3 - x \right) + \int_{o}^{1} - \left(x^3 - x \right) dx + \int_{1}^{2} \left(x^3 - x \right) dx$

$= \int_{-1}^{o} \left(x^3 - x \right) + \int_{o}^{1} \;\; \left(x - x^3 \right) dx + \int_{1}^{2} \left(x^3 - x \right) dx$

$= \frac{11}{4}$

7. $I = \int_{o}^{\pi} \frac{x \tan x}{\sec x + \tan x} dx - - - - - - -(1)$

$I = \int_{o}^{\pi} \frac{(\pi - x) \tan (\pi - x)}{\sec (\pi - x) + \tan (\pi - x)} dx$

$= \int_{o}^{\pi} \frac{(\pi - x) \tan x}{\sec x + \tan x} dx$

$= \int_{o}^{\pi} \frac{\pi \tan x}{\sec x + \tan x} dx - \int_{o}^{\pi} \frac{x \tan x}{\sec x + \tan x} dx$

$I = \int_{o}^{\pi} \frac{\pi \tan x}{\sec x + \tan x} dx - I$

$2I = \pi \int_{o}^{\pi} \frac{\frac{\sin x}{\cos x}}{\frac{1}{\cos x} + \frac{\sin x}{\cos x}} dx$

$= \pi \int_{o}^{\pi} \frac{\sin x}{1 + \sin x} dx$

$= \pi \int_{o}^{\pi} \frac{1 + \sin x - 1}{1 + \sin x} dx$

$$=\pi\int_o^{\pi}\left[\frac{1+\sin x}{1+\sin x}-\frac{1}{1+\sin x}\times\frac{1-\sin x}{1-\sin x}\right]dx$$

$$=\pi\int_o^{\pi}1\,dx-\pi\int_o^{\pi}\frac{1-\sin x}{\cos^2 x}dx$$

$$=\pi[x]_o^{\pi}-\pi\int_o^{\pi}\left(\sec^2 x-\tan x\sec x\right)dx$$

$$=\pi[\pi-o]-\pi[\tan x-\sec x]_o^{\pi}$$

$$=\pi^2-\pi\left[(\tan\pi-\sec\pi)-(\tan o-\sec o)\right]$$

$$=\pi^2-\pi\left[\left(0-(-1)\right)-(o-1)\right]$$

$$2I=\pi^2-\pi 2$$

$$=\pi^2-2\pi$$

$$=\pi(\pi-2)$$

$$I=\frac{\pi}{2}(\pi-2)$$

8. $$\int\frac{\left(\sin^4 x\right)^2-\left(\cos^4 x\right)^2}{1-2\sin^2 x.\cos^2 x}dx$$

$$\int\frac{\left(\sin^4 x+\cos^4 x\right)\left(\sin^2 x+\cos^2 x\right)\left(\sin^2 x-\cos^2 x\right)}{1-2\sin^2 x.\cos^2 x}dx$$

$$\int\frac{\left(1-2\sin^2 x.\cos^2 x\right)\left(\sin^2 x-\cos^2 x\right)}{\left(1-2\sin^2 x.\cos^2 x\right)}dx$$

$$\int-\left(\sin^2 x-\cos^2 x\right)$$

$$\int-\cos 2x dx$$

$$-\frac{\sin 2x}{2}+c$$

9. $$\int\frac{dx}{\sqrt{\sin^3 x.\sin(x+\alpha)}}$$

$$\int\frac{dx}{\sqrt{\sin^4 x.\frac{\sin(x+\alpha)}{\sin x}}}$$

$$\int \frac{dx}{\sin^2 x\sqrt{\frac{\sin(x+\alpha)}{\sin x}}} = \int \frac{\cos ec^2 dx}{\sqrt{\frac{\sin(x+\alpha)}{\sin x}}}$$

$$= \int \frac{\cos ec^2 dx}{\sqrt{\frac{\sin x.\cos\alpha + \cos x.\sin\alpha}{\sin x}}}$$

$$= \int \frac{\cos ec^2 dx}{\sqrt{\cos\alpha + \cot x.\sin\alpha}}$$

$put\ \cos\alpha + \cot x.\sin\alpha = t$

$0 - \cos ec^2 x.\sin\alpha dx = dt$

$$= \int -\frac{dt}{\sqrt{t}} = -\frac{1}{\sin\alpha}.\frac{t^{1/2}}{1/2} + c$$

$$= \frac{-2}{\sin\alpha}\sqrt{t} + c$$

10. $I = \int\left(\sqrt{\cot x} + \sqrt{\tan x}\right)dx$

$$I = \int\left(\frac{1}{\sqrt{\tan x}} + \frac{\sqrt{\tan x}}{1}\right)dx$$

$$I = \int \frac{1+\tan x}{\sqrt{\tan x}}dx$$

$put\ \sqrt{\tan x} = t$

$\tan x = t^2$

$\sec^2 x\ dx = 2t\ dt$

$$dx = \frac{2t\ dt}{\sec^2 x}$$

$$= \frac{2t\ dt}{1+\tan^2 x}$$

$$= \frac{2t}{1+t^4}$$

$$= \int \frac{1+t^2}{t} \times \frac{2t}{1+t^4}dt$$

$$= 2\int \frac{t^2+1}{t^4+1}dt = 2\int \frac{t^2\left(1+\frac{1}{t^2}\right)}{t^2\left(1+\frac{1}{t^2}\right)}dt$$

$$2\int \frac{1+\frac{1}{t^2}}{t^2+\frac{1}{t^2}-2+2}dt$$

$$2\int \frac{1+1/t^2}{\left(t-\frac{1}{t}\right)^2+\left(\sqrt{2}\right)^2}dt$$

$$put\ \ t-\frac{1}{t}=u$$

$$\left(1-\frac{1}{t^2}\right)dt=du$$

$$=2\int \frac{du}{(u)^2+\left(\sqrt{2}\right)^2}$$

$$=2.\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{u}{\sqrt{2}}\right)+c$$

$$=\sqrt{2}\tan^{-1}\frac{\left(t-\frac{1}{t}\right)}{\sqrt{2}}+c$$

$$=\sqrt{2}\tan^{-1}\left(\frac{t^2-1}{\sqrt{2}t}\right)+c$$

$$=\sqrt{2}\tan^{-1}\left(\frac{\tan x-1}{\sqrt{2\tan x}}\right)+c$$

CBSE TEST PAPER-02

CLASS - XII MATHEMATICS (Calculus: Integrals)

Topic: - Integrals

1. $\int \cos^3 x . e^{\log \sin x} dx$ [1]

2. $\int f'(ax+b)[f(ax+b)]^x dx$ [1]

3. $\int_o^1 xe^x dx$ [1]

4. $\int_{-1}^1 \sin^5 x . \cos^4 x \ dx$ [1]

5. $\int \frac{dx}{x + x\log x}$ [1]

6. $\int_o^\pi \frac{x \sin x}{1+\cos^2 x} dx$ [4]

7. $\int_o^{\pi/4} \frac{\sin x + \cos x}{9+16\sin 2x} dx$ [4]

8. $\int \frac{5x}{(x+1)(x^2+9)} dx$ [4]

9. $\int \frac{\sin^{-1}\sqrt{x} - \cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x}} dx$ [6]

10. $\int \left[\log(\log x) + \frac{1}{(\log x)^2} \right] dx$ [6]

CBSE TEST PAPER-02

CLASS - XII MATHEMATICS (Calculus: Integrals)

Topic: - Integral (ANSWERS)

1. $\because e^{\log\theta} = \theta$

$\therefore e^{\log\sin x} = \sin x$

$= \int \cos^3 x.\sin x \; dx$

$put \;\; \cos x = t$

$-\sin x \; dx = dt$

$\sin x \; dx = -dt$

$= \int -t^3 dt$

$= -\frac{t^4}{4} + c = -\frac{\cos^4 x}{4} + c$

2. Put $f(ax+b) = t$

$f'(ax+b).a \; dx = dt$

$= \int \frac{1}{a} t^n dt = \frac{1}{a}.\frac{t^{n+1}}{n+1} + c$

$= \frac{1}{a}.\frac{\left[f(ax+b)\right]^{n+1}}{n+1} + c$

3. $\int \underset{I}{x} \underset{II}{e^x} dx = xe^x - \int 1.e^x dx$

$= xe^x - e^x + c$

$\int_o^1 xe^x dx = \left[xe^x - e^x\right]_o^1$

$= (1.e^1 - e^1) - (0.e^0 - e^0) \qquad \left[\because e^0 = 1\right]$

$= 0 - (0 - 1)$

$=1$

4. Let $f(x) = \sin^5 x.\cos^4 x$

$f(-x) = \sin^5(-x).\cos^4(-x)$

$= -\sin^5 x.\cos^4 x$

$= -f(x)$

f is odd function

$$\therefore \int_{-1}^{1} \sin^5 x.\cos^4 x \, dx = 0$$

5. $$\int \frac{dx}{x + x\log x} = \int \frac{dx}{x(1+\log x)}$$

$put \ 1+\log x = t$

$$0 + \frac{1}{x} dx = dt$$

$$= \int \frac{dt}{t}$$

$= \log|t| + c$

$= \log|1+\log x| + c$

6. $$I = \int_o^{\pi} \frac{x\sin x}{1+\cos^2 x} dx ------(1)$$

$$= \int_o^{\pi} \frac{(\pi - x)\sin(\pi - x)}{1+\cos^2(\pi - x)} dx \qquad [byP_4$$

$$= \int_o^{\pi} \frac{(\pi - x).\sin x}{1+\cos^2 x} dx$$

$$I = \int_o^{\pi} \frac{\pi.\sin x}{1+\cos^2 x} dx - I$$

$$2I = \pi \int_o^{\pi} \frac{\sin x}{1+\cos^2 x} dx$$

$put \ \cos x = t$

$\sin x \ dx = -dt$

$when \ x = 0$

$t \to 1$

$when \ x = \pi$

$t \to -1$

$$= \int_1^{-1} \frac{dt}{1+t^2}$$

$$= \pi \left[\tan^{-1} t \right]_1^{-1}$$

$$= \frac{\pi^2}{4}$$

7. Let $I = \int_o^{\pi/4} \frac{\sin x + \cos x}{9 + 16\sin 2x} dx$

$put \;\; \sin x - \cos x = t$

$(\cos x + \sin x)dx = dt$

$\Rightarrow (\sin x - \cos)^2 = t^2$

$1 - \sin 2x = t^2$

$1 - t^2 = \sin 2x$

$= \int_{-1}^{o} \frac{dx}{9 + 16(1 - t^2)}$

$\int_{-1}^{o} \frac{dt}{9 + 16 - 16t^2}$

$= \int_{-1}^{o} \frac{dx}{25 - 16t^2} == \int_{-1}^{o} \frac{dx}{16\left[\frac{25}{16} - t^2\right]}$

$= \frac{1}{16} \int_{-1}^{o} \frac{dx}{\left(\frac{5}{4}\right)^2 - t^2}$

$= \frac{1}{16} \times \frac{1}{2 \times \frac{5}{4}} \log \left[\frac{\frac{5}{4} + t}{\frac{5}{4} - t}\right]_{-1}^{o}$

$= \frac{1}{40} \log 9.$

8. Let $\frac{5x}{(x+1)(x^2+9)} = \frac{A}{x+1} + \frac{Bx + c}{x^2 + 9}$

$5x = A(x^2 + 9) + (Bx + c)(x + 1)$

On comparing coeff. Of x^2 and x and constant.

$0 = A + B$

$5 = C + B$

$O = 9A + C$

$$\Rightarrow \begin{array}{l} A - \cancel{C} = -5 \\ \underline{9A + \cancel{C} = O} \\ 10A = -5 \end{array}$$

$$A = -\frac{1}{2}$$

$$B = 1/2$$

$$C = \frac{9}{2}$$

$$\int \frac{5x}{(x+1)(x^2+9)} = \int \frac{-1/2}{(x+1)} \int \frac{\frac{1}{2}x + \frac{9}{2}}{x^2+9} dx$$

$$= -\frac{1}{2}\int \frac{dx}{x+1} + \frac{1}{2}\int \frac{x}{x^2+9} dx + \frac{9}{2}\int \frac{dx}{x^2+9}$$

$$= -\frac{1}{2}\int \frac{dx}{x+1} + \frac{1}{4}\int \frac{2x}{x^2+9} dx + \frac{9}{2}\int \frac{dx}{x^2+3^2}$$

$$= -\frac{1}{2}\log(x+1) + \frac{1}{4}\log(x^2+9) + \frac{9}{2}.\frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right) + c$$

$$= -\frac{1}{2}\log(x+1) + \frac{1}{4}\log(x^2+9) + \frac{3}{2}\tan^{-1}\left(\frac{x}{3}\right) + c$$

9. $$\int \frac{\sin^{-1}\sqrt{x} - \cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x}} dx$$

$$\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x} = \frac{\pi}{2}$$

$$\cos^{-1}\sqrt{x} = \frac{\pi}{2} - \sin^{-1}\sqrt{x}$$

$$= \int \frac{\sin^{-1}\sqrt{x} - \left(\frac{\pi}{2} - \sin^{-1}\sqrt{x}\right)}{\pi/2} dx$$

$$= \int \frac{2\sin^{-1}\sqrt{x} - \frac{\pi}{2}}{\pi/2} dx$$

$$= \frac{4}{\pi}\int \sin^{-1}\sqrt{x}\ dx - \int 1\ dx$$

$$= \frac{4}{\pi} I - x + c \qquad (i)$$

$I = \int \sin^{-1}\sqrt{x}\ dx$

$put\ \ \sin^{-1}\sqrt{x} = t$

$\sin t = \sqrt{x}$

$\sin^2 t = x$

$2\sin t\cos t = dx$

$= \int t.2\sin t\cos t\ dt$

$= \int \underset{1}{t}.\underset{11}{\sin} 2t\,dt$

$= -t\dfrac{\cos 2t}{2} - \int 1.\left(-\dfrac{\cos 2t}{2}\right)dt$

$= \dfrac{-t\cos 2t}{2} + \dfrac{1}{2}.\dfrac{\sin 2t}{2} + c$

$= -t.\dfrac{(1-2\sin 2t)}{2} + \dfrac{1}{4}.\dfrac{\sin 2t}{2} + c$

$= \dfrac{-t(1-2\sin 2t)}{2} + \dfrac{1}{2}.\sin t\sqrt{1-\sin^2 t}$

$= \dfrac{-\sin^{-1}\sqrt{x}(1-2x)}{2} + \dfrac{1}{2}.\sqrt{x}\sqrt{1-x} + c$

$= \dfrac{\sin^{-1}\sqrt{x}(2x-1)}{2} + \dfrac{1}{2}\sqrt{x-x^2} + c$

From (i)

$$\int \frac{\sin^{-1}\sqrt{x} - \cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x}}dx = \frac{4}{\pi}\left(\frac{\sin^{-1}\sqrt{x}\,(2x-1)}{2} + \frac{1}{2}\sqrt{x-x^2}\right) - x + c$$

10. $= \int \underset{I}{\log(\log x)} \times \underset{II}{1} dx + \int \dfrac{1}{(\log x)^2}dx$

$= \log(\log x).x - \int \dfrac{1}{\log x}.\dfrac{1}{\cancel{x}}.\cancel{x}\,dx + \int \dfrac{1}{(\log x)^2}dx$

$= \log(\log x).x - \dfrac{x}{\log x} - \int \underset{I}{\dfrac{1}{\log x}} \times \underset{II}{1}\,dx + \int \dfrac{1}{(\log x)^2}dx$

$= \log(\log x).x - \dfrac{x}{\log x} - \int \dfrac{dx}{(\log x)^2} + \int \dfrac{dx}{(\log x)^2}$

$= x.\log(\log x) - \dfrac{x}{\log x} + c$

CBSE TEST PAPER-03

CLASS - XII MATHEMATICS (Calculus: Integrals)

Topic: - Integrals

1. $\int \frac{10x^9 + 10^x . \log e10}{x^{10} + 10^x} dx$ [1]

2. $\int \frac{\cos 2x + 2\sin^2 x}{\cos^2 x} dx$ [1]

3. $\int_{-\frac{\pi}{4}}^{\pi/4} \sin^2 x \ dx$ [1]

4. $\int \frac{e^{5\log x} - e^{4\log x}}{e^{3\log x} - e^{2\log x}} dx$ [1]

5. $\int \frac{e^{2x} - 1}{e^{2x} + 1} dx$ [1]

6. $\int_{\pi/6}^{\pi/3} \frac{dx}{1 + \sqrt{\tan x}}$ [4]

7. $\int_o^{\pi/4} \frac{\sin x \cos x}{\cos^4 x + \sin^4 x} dx$ [4]

8. $\int \frac{dx}{\cos(x+a)\cos(x+b)}$ [4]

9. $\int_o^{\pi/2} \frac{\cos^2 x}{\cos^2 x + 4\sin^2 x} dx$ [6]

10. $\int_o^{\pi/2} \log \sin x \ dx$ [6]

Topic: - Integrals (ANSWERS)

1. Put $x^{10}+10^{x}=t$

$(10x^{9}+10^{x}.\log_{e}10\ )dx=dt$

$=\int\frac{dt}{t}$

$=\log|t|+c$

$=\log(x^{10}+10^{x})+c$

2. $=\int\frac{\cos 2x+2\sin^{2}x}{\cos^{2}x}dx=\int\frac{1-2\sin^{2}x+2\sin^{2}x}{\cos^{2}x}dx$

$=\int\frac{1}{\cos^{2}x}dx=\int\sec^{2}x\ dx$

$=\tan x+c$

3. Let $f(x)=\sin^{2}x$

$f(-x)=\sin^{2}(-x)$

$=\sin^{2}x$

$=f(x)$

$\therefore$ function is even

$\therefore\int_{-\pi/4}^{\pi/4}\sin^{2}x\ dx=2\int_{0}^{\pi/4}\sin^{2}x\ dx$

$=\int 2\left(\frac{1-\cos 2x}{2}\right)dx$

$=\left[x-\frac{\sin 2x}{2}\right]_{o}^{\pi/4}$

$=\frac{\pi}{4}-\frac{1}{2}$

4. $\int\left(\dfrac{x^5-x^4}{x^3-x^2}\right)dx$ $\qquad \left[\because e^{\log\theta}=\theta\right.$

$=\int \dfrac{x^4\,\cancel{(x-1)}}{x^2\,\cancel{(x-1)}}dx$

$=\int x^2 dx$

$=\dfrac{x^3}{3}+c$

5. $=\int \dfrac{\cancel{e^x}\left(e^x-e^{-x}\right)}{\cancel{e^x}\left(e^x+e^{-x}\right)}dx$

$put\ e^x+e^{-x}=t$

$\left(e^x-e^{-x}\right)dx=dt$

$=\int \dfrac{dt}{t}=\log|t|+c$

$=\log\left(e^x+e^{-x}\right)+c$

6. $I=\int_{\pi/6}^{\pi/3}\dfrac{dx}{1+\sqrt{\dfrac{\sin x}{\cos x}}}=\int_{\pi/6}^{\pi/3}\dfrac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}dx--------(1)$

$$I=\int_{\pi/6}^{\pi/3}\frac{\sqrt{\cos\left(\frac{\pi}{3}+\frac{\pi}{6}-x\right)}}{\sqrt{\cos\left(\frac{\pi}{3}+\frac{\pi}{6}-x\right)}+\sqrt{\sin\left(\frac{\pi}{3}+\frac{\pi}{6}-x\right)}}dx$$

$\left[\because \int_a^b f(x)dx=\int_a^b f\left(a+b-x\right)dx\right.$

$I=\int_{\pi/6}^{\pi/3}\dfrac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\sin x}}dx--------(2)$

$(1)+(2)$

$2I=\int_{\pi/6}^{\pi/3}1dx$

$=[x]_{\pi/6}^{\pi/3}$

$=\dfrac{\pi}{3}-\dfrac{\pi}{6}=\dfrac{\pi}{6}$

$I=\dfrac{\pi}{12}$

7. $I=\int_o^{\pi/4}\frac{\sin x.\cos x}{\cos^4 x+\sin^4 x}dx$

Dividing N and D by $\cos^4$x

$$=\int_o^{\pi/4}\frac{\frac{\sin x.\cos x}{\cos^4 x}}{\frac{\cos^4 x}{\cos^4 x}+\frac{\sin^4 x}{\cos^4 x}}dx$$

$$=\int_o^{\pi/4}\frac{\tan x.\sec^2 x}{1+\tan^4 x}dx$$

$$=\int_o^{\pi/4}\frac{\tan x.\sec^2 x}{1+(\tan^2 x)^2}dx$$

$put\ \ \tan^2 x=t$

$2\tan x.\sec^2 x\ dx=dt$

$$=\frac{1}{2}\int_o^1\frac{dt}{1+t^2}$$

$$=\frac{1}{2}\left[\tan^{-1}t\right]_o^1$$

$$=\frac{1}{2}.\frac{\pi}{4}=\frac{\pi}{8}$$

8. $I=\int\frac{dx}{\cos(x+a).\cos(x+b)}$

$$=\frac{1}{\sin(a-b)}\int\frac{\sin[(x+a)-(x+b)]}{\cos(x+a).\cos(x+b)}$$

$$=\frac{1}{\sin(a-b)}\int\left[\frac{\sin(x+a).\cos(x+b)-\cos(x+a).\sin(x+b)}{\cos(x+a).\cos(x+b)}\right]$$

$$=\frac{1}{\sin(a-b)}\int\left[\tan(x+a)-\tan(x+b)\right]dx$$

$$=\frac{1}{\sin(a-b)}\left[\log\sec(x+a)-\log\sec(x+b)\right]+c$$

$$=\frac{1}{\sin(a-b)}\left[\log\frac{\sec(x+a)}{\sec(x+b)}\right]+c$$

9. $$I=\int_o^{\pi/2}\frac{\cos^2 x}{\cos^2 x+4\sin^2 x}dx$$

$$=\int_o^{\pi/2}\frac{\cos^2 x}{\cos^2 x+4\left(1-\cos^2 x\right)}dx$$

$$=\int_o^{\pi/2}\frac{\cos^2 x}{\cos^2 x+4-4\cos^2 x}dx$$

$$=\int_o^{\pi/2}\frac{\cos^2 x}{4-3\cos^2 x}dx$$

$$=\frac{-1}{3}\int_o^{\pi/2}\frac{-3\cos^2 x}{4-3\cos^2 x}dx$$

$$=\frac{-1}{3}\int_o^{\pi/2}\frac{4-3\cos^2 x-4}{4-3\cos^2 x}dx$$

$$=\frac{-1}{3}\int_o^{\pi/2}\left(1-\frac{4}{4-3\cos^2 x}\right)dx$$

$$=\frac{-1}{3}\int_o^{\pi/2}1dx+\frac{4}{3}\int_o^{\pi/2}\frac{dx}{4-3\cos^2 x}$$

$$=\frac{-1}{3}[x]_o^{\pi/2}+\frac{4}{3}\int_o^{\pi/2}\frac{dx/\cos^2 x}{\frac{4}{\cos^2 x}-\frac{3\cos^2 x}{\cos^2 x}}$$

$$=\frac{-1}{3}\left[\frac{\pi}{2}-o\right]+\frac{4}{3}\int_o^{\pi/2}\frac{\sec^2 x\ dx}{4\sec^2 x-3}$$

$$=\frac{-\pi}{6}+\frac{4}{3}\int_o^{\pi/2}\frac{\sec^2 x}{4(1+\tan^2 x)-3}dx$$

put $\tan x=t$

$\sec^2 x\ dx=dt$

$$=\frac{-\pi}{6}+\int_0^{\infty}\frac{dt}{4(1+t^2)-3}$$

$$=\frac{-\pi}{6}+\frac{1}{\cancel{4}}.\frac{\cancel{4}}{3}\int_0^{\infty}\frac{dt}{t^2+\frac{1}{4}}$$

$$=\frac{-\pi}{6}+\frac{1}{3}.\frac{1}{2}\left[\tan^{-1}2t\right]_o^{\infty}$$

$$=\frac{-\pi}{6}+\frac{2}{3}\left(\tan^{-1}\infty-\tan^{-1}o\right)$$

$$= \frac{-\pi}{6} + \frac{2}{3}\left(\frac{\pi}{2} - o\right)$$

$$= \frac{-\pi}{6} + \frac{\pi}{3}$$

$$= \pi/6$$

10. $$I = \int_o^{\pi/2} \log \sin x \, dx ----------(1)$$

$$I = \int_o^{\pi/2} \log \sin\left(\frac{\pi}{2} - x\right) dx \qquad [by \text{ p4}$$

$$I = \int_o^{\pi/2} \log \cos x \, dx ----------(2)$$

$$(1) + (2)$$

$$2I = \int_o^{\pi/2} (\log \sin x + \log \cos x) dx$$

$$= \int_o^{\pi/2} \left(\underline{\log \sin x . \cos x + \log 2} - \log 2\right) dx$$

$$= \int_o^{\pi/2} \log \sin^2 x \, dx - \int_o^{\pi/2} \log 2 dx$$

$$put \;\; 2x = t$$

$$dx = \frac{dt}{2}$$

$$2I = \frac{1}{2}\int_o^{\pi} \log \sin t \, dt - \frac{\pi}{2}\log 2$$

$$= \frac{\cancel{2}}{\cancel{2}}\int_o^{\pi/2} \log \sin t \, dt - \frac{\pi}{2}\log 2 \qquad [by P_6$$

$$= \int_o^{\pi/2} \log \sin x \, dx - \frac{\pi}{2}\log 2 \qquad [by P_o$$

$$2I = I - \frac{\pi}{2}\log 2$$

$$I = -\frac{\pi}{2}\log 2$$

CBSE TEST PAPER-04

CLASS - XII MATHEMATICS (Calculus: Integrals)

Topic: - Integrals

1. $\int \frac{\sqrt{\tan x}}{\sin x.\cos x} dx$ [1]

2. $\int \frac{(x+1)(x+\log x)^2}{x} dx$ [1]

3. $\int \frac{\cos 2x - \cos 2\alpha}{\cos x - \cos \alpha} dx$ [1]

4. $\int \frac{dx}{x^2 - 16}$ [1]

5. $\int e^x \left(\tan^{-1} x + \frac{1}{1+x^2} \right) dx$ [1]

6. $\int_o^{\pi/4} \log(1+\tan x)\, dx$ [4]

7. $\int_{\pi/6}^{\pi/3} \frac{\sin x + \cos x}{\sqrt{\sin 2x}} dx$ [4]

8. $\int_1^4 [|x-1| + |x-2| + |x-3|]\, dx$ [4]

9. $\int_o^{\pi} \frac{x\, dx}{a^2 \cos^2 x + b^2 \sin^2 x}$ [6]

10. Find is sum of limit $\int_o^4 \left(x + e^{2x} \right) dx$ [6]

CBSE TEST PAPER-04

CLASS - XII MATHEMATICS (Calculus: Integrals)

Topic: - Integrals (ANSWERS)

1. $$= \int \frac{\sqrt{\tan x}}{\frac{\sin x}{\cos x}.\cos^2 x} dx$$

$$= \int \frac{\sqrt{\tan x}}{\tan}.\sec^2 x \ dx$$

$put \ \ \tan x = t$

$\sec^2 dx = dt$

$$= \int \frac{\sqrt{t}}{t} dt$$

$$= \int \left(t^{\frac{1}{2}}.t^{-1} \right) dt$$

$$= \int t^{-1/2} dt$$

$$= \frac{t^{1/2}}{1/2} + c = 2\sqrt{\tan x} + c$$

2. Put $x + \log x = t$

$$\left(1 + \frac{1}{x}\right) dx = dt$$

$$\left(\frac{x+1}{x}\right) dx = dt$$

$$= \int t^2 dt$$

$$= \frac{t^3}{3} + c$$

$$= \frac{(x + \log x)^3}{3} + c$$

3. $$= \int \frac{(2\cos^2 x - 1) - (2\cos^2 \alpha - 1)}{(\cos x - \cos\alpha)} dx$$

$$= \int \frac{2(\cos x + \cos\alpha)\cancel{(\cos x + \cos\alpha)}}{\cancel{(\cos x - \cos\alpha)}} dx$$

$$= 2(\sin x + \cos\alpha . x) + c$$

4. $$= \int \frac{dx}{x^2 - (4)^2} = \frac{1}{8}\log\left|\frac{x-4}{x+4}\right| + c$$

5. $$f(x) = \tan^{-1} x$$

$$f'(x) = \frac{1}{1+x^2}$$

$$\int e^x \left(\tan^{-1} x + \frac{1}{1+x^2} \right) dx = e^x \tan^{-1} x + c$$

$$\left[\because \int e^x f(x) + f'(x) = e^x f(x) + c \right.$$

6. $$I = \int_o^{\pi/4} \log(1 + \tan x)\, dx ------(1)$$

$$= \int_o^{\pi/4} \log\left[1 + \tan\left(\frac{\pi}{4} - x\right)\right] dx \qquad [by\ \text{p4}$$

$$= \int_o^{\pi/4} \log\left[1 + \frac{\tan\frac{\pi}{4} - \tan x}{1 + \tan\frac{\pi}{4} . \tan x}\right] dx$$

$$= \int_o^{\pi/4} \log\left[1 + \frac{1 - \tan x}{1 + \tan x}\right) dx$$

$$= \int_o^{\pi/4} \log\left[\frac{1 + \cancel{\tan x} + 1 - \cancel{\tan x}}{1 + \tan x}\right] dx$$

$$= \int_o^{\pi/4} \log\left[\frac{2}{1 + \tan x}\right] dx$$

$$= \int_o^{\pi/4} \log 2 dx - \int_o^{\pi/4} \log(1 + \tan x) dx$$

$$= \log 2[x]_o^{\pi/4} - I$$

$$2I = \log 2\left[\frac{\pi}{4} - o\right]$$

$$I = \log 2.\frac{\pi}{8}$$

7. $I = \int_{\pi/6}^{\pi/3} \frac{\sin x + \cos x}{\sqrt{\sin 2x}} dx$ $\qquad$ *put* $\sin x - \cos x = t$

$$(\cos x + \sin x)dx = dt$$

$$(\sin x - \cos x)^2 = t^2$$

$$\sin 2x = 1 - t^2$$

when $x = \pi/6$

$$t \to \frac{1}{2} - \frac{\sqrt{3}}{2}$$

when $x \to \pi/3$

$$t \to \frac{\sqrt{3}}{2} - \frac{1}{2}$$

$$\int_{\frac{1-\sqrt{3}}{2}}^{\frac{\sqrt{3}-1}{2}} \frac{dt}{\sqrt{1-t^2}}$$

$$= \left[\sin^{-1} t\right]_{\frac{1-\sqrt{3}}{2}}^{\frac{\sqrt{3}-1}{2}}$$

$$= \sin^{-1}\left(\frac{\sqrt{3}-1}{2}\right) - \sin^{-1}\left(\frac{1-\sqrt{3}}{2}\right)$$

$$= \sin^{-1}\left(\frac{\sqrt{3}-1}{2}\right) + \sin^{-1}\left(\frac{1-\sqrt{3}}{2}\right)$$

$$= 2\sin^{-1}\left(\frac{\sqrt{3}-1}{2}\right)$$

8. $I = \int_1^4 [|x-1| + |x-2| + |x-3|]dx$

$$=\int_1^4 |x-1|\,dx=\int_1^4 (x-1)dx=\frac{9}{2}$$

$$=\int_1^4 |x-2|\,dx=\int_1^2 -(x-2)dx+\int_2^4 (x-2)dx=\frac{5}{2}$$

$$\int_1^4 (x-3)=\int_1^3 -(x-3)dx+\int_3^4 (x-3)dx=\frac{5}{2}$$

$$I=\frac{9}{\cancel{2}}+\frac{5}{\cancel{2}}+\frac{5}{2}=\frac{19}{2}$$

9. $$I=\int_o^\pi \frac{x\ dx}{a^2\cos^2 x+b^2\sin^2 x}(1)$$

$$I=\int_o^\pi \frac{(\pi-x)}{a^2\cos^2(\pi-x)+b^2\sin^2(\pi-x)}dx$$

$$I=\int_o^\pi \frac{(\pi-x)}{a^2\cos^2 x+b^2\sin^2 x}dx$$

$$I=\int_o^\pi \frac{\pi\ dx}{a^2\cos^2 x+b^2\sin^2 x}-\int_o^\pi \frac{x\ dx}{a^2\cos^2 x+b^2\sin^2 x}$$

$$I=\pi\int_o^\pi \frac{dx}{a^2\cos^2 x+b^2\sin^2 x}-I$$

$$2I=\int_o^\pi \frac{dx}{a^2\cos^2 x+b^2\sin^2 x}$$

$$I=\frac{\pi}{2}\int_o^\pi \frac{dx}{a^2\cos^2 x+b^2\sin^2 x}$$

$$=\frac{\pi}{\cancel{2}}.\cancel{2}\int_o^{\pi/2} \frac{dx}{a^2\cos^2 x+b^2\sin^2 x} \qquad \left[\because p_6^{by}\right.$$

Dividing N and D by $\cos^2 x$

$$=\pi\int_o^{\pi/2} \frac{\sec^2 x dx}{a^2+b^2\tan^2 x}$$

$put\ b\tan x=t$

$b\sec^2 x\ dx=dt$

$$=\frac{\pi}{6}\int_o^\infty \frac{dt}{a^2+t^2}$$

$$=\frac{\pi}{6}.\frac{1}{a}\left[\tan^{-1}\frac{t}{a}\right]_o^\infty$$

$$=\frac{\pi}{ab}\left(\frac{\pi}{2}\right)$$
$$=\frac{\pi^2}{2ab}$$

10. $f(x)=x+e^{2x}$

$$a=0,\ b=4,\ h=\frac{b-a}{n}$$
$$\int_o^4\left(x+e^{2x}\right)dx=\lim_{h\to o} h\left[f(o)+f(o+h)+f(o+2h)+---+f(o+(n-1)h)\right]$$
$$=\lim_{h\to o} h\left[(e^o)+(h+e^{2h})+(h+e^{3h})+------+((n-1)h+e^{2(n-1)h})\right]$$
$$=\lim_{h\to o} h\left[h[1+2+----+(n-1)]+(1+e^{2h}+e^{4h}+---+(e^{2(n-1)h}\right]$$
$$=\lim_{h\to o} h\left[h\frac{n(n-1)}{2}+1.\frac{\left(e^{2nh}-1\right)}{\frac{e^{2h}-1}{2h}\times 2h}\right]$$
$$=\lim_{n\to o} h\left[h\frac{n(n-1)}{2}+\frac{e^{2nh}-1}{2h}\right]$$
$$=\lim_{n\to\infty}\frac{4}{n}\left[\frac{4}{n}.\frac{n(n-1)}{2}+\frac{e^{2n.\frac{4}{n}}-1}{2\times\frac{4}{n}}\right]$$
$$=\lim_{n\to\infty}\left[\frac{16}{\cancel{n}}\cancel{n}.n\frac{\left(1-\frac{1}{n}\right)}{2}+\frac{4}{\cancel{n}}\frac{e^8-1}{\frac{8}{\cancel{n}}}\right]$$
$$=\cancel{16}\times\frac{1}{\cancel{2}}+\frac{e^8-1}{2}$$
$$=8+\frac{e^8-1}{2}$$
$$=\frac{16+e^8-1}{2}$$
$$=\frac{e^8+15}{2}$$

CBSE TEST PAPER-05

CLASS - XII MATHEMATICS (Calculus: Integrals)

Topic: - Integrals

1. $\int_o^1 \tan^{-1}\left(\frac{2x-1}{1+x-x^2}\right)dx$ [1]

2. If $f(a+b-x)=f(x)$ then $\int_a^b (x)f(x)dx=?$ [1]

3. $\int_o^{\pi/2}\left(\frac{4+3\sin x}{4+3\cos x}\right)dx$ [1]

4. $\int_{-\pi/2}^{\pi/2}\left(x^3+x\cos x+\tan^5 x+1\right)dx$ [1]

5. Show that $\int_o^a f(x).g(x)dx = 2\int_o^a f(x)dx$ [1]

   If $f(x)=f(a-x)$ and $g(x)+g(a-x)=4$

6. $\int_o^{\pi/2}\left(2\log\sin x-\log\sin 2x\right)dx$ [4]

7. $\int \frac{2+\sin 2x}{1+\cos x}e^x dx$ [4]

8. $\int \tan^{-1}\sqrt{\frac{1-x}{1+x}}dx$ [4]

9. $\int_{-1}^{3/2} |x\sin(\pi x)|\,dx$ [6]

10. $\int \frac{dx}{3x^2+13x-10}$ [6]

CBSE TEST PAPER-05

CLASS - XII MATHEMATICS (Calculus: Integrals)

Topic: - Integrals (ANSWERS)

1. $I = \int_o^1 \tan^{-1}\left(\frac{x+x-1}{1-x(x-1)}\right)dx$

$I = \int_o^1 \left[\tan^{-1}(x) + \tan^{-1}(x-1)\right]dx ------(1)$

$I = \int_o^1 \left[\tan^{-1}(1-x) + \tan^{-1}(\cancel{1}-x-\cancel{1})\right]dx \qquad [\because P_4$

$I = \int_o^1 \left[\tan^{-1}(1-x) - \tan^{-1}(x)\right]dx ------(2)$

$(1)+(2)$

$2I = o$

$I = o$

2. $I = \int_a^b (x).f(x)dx ------(1)$

$I = \int_a^b (a+b-x).f(a+b-x)dx$

$I = \int_a^b (a+b-x).f(x)dx \qquad [\because f(a+b-x) = f(x)$

$= \int_a^b \left[(a+b).f(x) - x\,f(x)\right]dx$

$= \int_a^b (a+b)\,f(x)\;dx - \int_a^b x\,f(x)\;dx$

$I = (a+b)\int_a^b f(x)\;dx - I$

$I = \frac{a+b}{2}\int_a^b f(x)\;dx$

3. $I = \int_o^{\pi/2}\left(\frac{4+3\sin x}{4+3\cos x}\right)dx -----(1)$

$I = \int_o^{\pi/2}\left[\frac{4+3\sin\left(\frac{\pi}{2}-x\right)}{4+3\cos\left(\frac{\pi}{2}-x\right)}\right]dx\left[by P_4\right.$

$I = \int_o^{\pi/2}\left(\frac{4+3\cos x}{4+3\sin x}\right)dx$

$$= -\int_o^{\pi/2} \left(\frac{4+3\sin x}{4+3\cos x} \right) dx$$

$I = -I$

$2I = 0$

$I = 0$

4. $I = \int_{-\pi/2}^{\pi/2} \left(x^3 + x\cos x + \tan^5 x \right) dx + \int_{-\pi/2}^{\pi/2} 1\, dx$

$let\ f(x) = x^3 + x\cos x + \tan^5 x$

$f(-x) = -x^3 - x\cos x - \tan^5 x$

$= -(x^3 + x\cos x + \tan^5 x)$

$= -f(x)$

Hence odd function

$I = o + [x]_{-\pi/2}^{\pi/2}$

$= \frac{\pi}{2} + \frac{\pi}{2} = \pi$

5. $I = \int_o^a f(x).g(x)\, dx$

$= \int_o^a f(a-x).g(a-x)\, dx \qquad [by P_4$

$= \int_o^a f(x).[4-g(x)dx \qquad$ [From given]

$= \int_o^a 4f(x)dx - \int_o^a f(x).g(x)\, dx$

$I = 4\int_o^a f(x)\, dx - I$

$I = 2\int_o^a f(x)\, dx$

6. $I = \int_o^{\pi/2} [2\log\sin x - (\log 2\sin x.\cos.)]dx$

$I = \int_o^{\pi/2} \left(2\underline{\log\sin x} - \log 2 - \underline{\log\sin x} - \log\cos x \right) dx$

$I = \int_o^{\pi/2} \left(\log\sin x - \log 2 - \log\cos x \right) dx$

$I = \int_o^{\pi/2} \log\sin x\, dx - \int_o^{\pi/2} \log 2\, dx - \int_o^{\pi/2} \log\cos x dx$

$I = \int_o^{\pi/2} \log\sin x\, dx - \int_o^{\pi/2} \log 2\, dx - \int_o^{\pi/2} \log\cos\left(\frac{\pi}{2} - x \right) dx$

$I = \cancel{\int_o^{\pi/2} \log\sin x}\ dx - \log 2\int_o^{\pi/2} 1\, dx - \cancel{\int_o^{\pi/2} \log\sin x}\ dx$

$$= -\log 2[x]_o^{\pi/2}$$

$$= -\log 2\left(\frac{\pi}{2} - o\right) = -\frac{\pi}{2}.\log 2$$

7. $$I = \int\left(\frac{2+2\sin x.\cos x}{2\cos^2 x}\right)e^x dx$$

$$\int\left(\frac{\cancel{2}}{\cancel{2}\cos^2 x} + \frac{\cancel{2}\sin x.\cancel{\cos x}}{\cancel{2}\cos^2 x}\right)e^x dx$$

$$= \int\left(\sec^2 x + \tan x\right)e^x dx$$

$$let\ f(x) = \tan x$$

$$f'(x) = \sec^2 x$$

$\because$ We know that

$$\int e^x[f(x) + f'(x)]dx = e^x f(x) + c$$

$$\therefore \int(\sec 2x + \tan x)e^x dx$$

$$= e^x.\tan x + c$$

8. $$I = \int \tan^{-1}\sqrt{\frac{1-x}{1+x}}dx$$

$$put x = \cos\theta$$

$$dx = -\sin\theta d\theta$$

$$= \int \tan^{-1}\sqrt{\left(\frac{1-\cos\theta}{1+\cos\theta}\right)} \times -\sin\theta d\theta$$

$$= \int \tan^{-1}\sqrt{\left(\frac{2\sin^2\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}}\right)} \times -\sin\theta d\theta$$

$$= \int \tan^{-1}\left(\tan\frac{\theta}{2}\right)(-\sin\theta)\,d\theta$$

$$= -\int\frac{\theta}{2}\sin\theta d\theta = \frac{-1}{2}\int \underset{I}{\theta}.\underset{II}{\sin}\theta d\theta$$

$$= \frac{-1}{2}\left[\theta.(-\cos\theta) - \int 1\times(-\sin\theta)d\theta\right]$$

$$= \frac{-1}{2}[-\theta.\cos\theta - \sin\theta] + c$$

$$= \frac{-1}{2}\left[-\theta.\cos\theta - \sqrt{1-\cos^2\theta}\right] + c$$

$$= \frac{-1}{2}\left[-x.\cos^{-1}x - \sqrt{1-x^2}\right] + c$$

9. $$\int_{-1}^{3/2} |x.\sin(\pi x)|\,dx = \int_{-1}^{1} \underset{I}{x}\,\underset{II}{\sin}\,\pi x\;dx + \int_{-1}^{3/2} -x\sin\pi x\;dx$$

$$= \left[\frac{-x\cos\pi x}{\pi} + \frac{\sin\pi x}{\pi^2}\right]_{-1}^{1} - \left[\frac{-x\cos\pi x}{\pi} + \frac{\sin\pi x}{\pi^2}\right]_{-1}^{3/2}$$

$$= \frac{2}{\pi} - \left[-\frac{1}{\pi^2} - \frac{1}{\pi}\right]$$

$$= \frac{3}{\pi} + \frac{1}{\pi^2}$$

10. $$= \int \frac{dx}{3\left[x^3 + \frac{13}{3}x - \frac{10}{3}\right]}$$

$$= \frac{1}{3}\int \frac{dx}{\left(x+\frac{13}{6}\right)^2 - \left(\frac{17}{6}\right)^2}$$

$$put\ x + \frac{13}{6} = t$$

$$dx = dt$$

$$= \frac{1}{3}\int \frac{dt}{t^2 - \left(\frac{17}{6}\right)^2}$$

$$= \frac{1}{\cancel{3}\times\cancel{2}\times\frac{17}{\cancel{6}}}\log\left|\frac{t-\frac{17}{6}}{t+\frac{17}{6}}\right| + c$$

$$= \frac{1}{17}\log\left|\frac{6x-4}{6x+30}\right| + c$$

$$= \frac{1}{17}\log\left|\frac{3x-2}{x+5}\right| + c$$

CBSE TEST PAPER-01
CLASS - XII MATHEMATICS (Calculus : Application of Integrals)

Topic: - Application of Integrals

Note : Each Question carries 6 marks.

1. Find the area of the region bounded by the curve $y^2 = x$ and the lines $x = 1$, $x = 4$ and x – axis.

2. Find the area of the region bounded by $y^2 = 9x$, $x = 2$, $x = 4$ and the x – axis in the first quadrant.

3. Find the area of the region bounded by the parabola $y = x^2 + 1$ and the lines $y = x$, $x = 0$ and $x = 2$.

4. Find area of the region bounded $x^2 = 4y$, $y = 2$, $y = 4$ and the y – axis in the first quadrant.

5. Find the area of the region bounded by the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$.

6. Find the area of the region bounded by the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$.

7. Prove the area of a circle of radius r is πr^2 square units.

8. Find the area of the region in the first quadrant enclosed by x – axis and $x = \sqrt{3}y$ by the circle $x^2 + y^2 = 4$.

9. Draw the graph of the curve $y = \sqrt{9 - x^2}$ and find the area bounded by this curve and the coordinate axis.

10. Find the area of the smaller part of the circle $x^2 + y^2 = a^2$ cut off by the line $x = \frac{a}{\sqrt{2}}$.

CBSE TEST PAPER-01

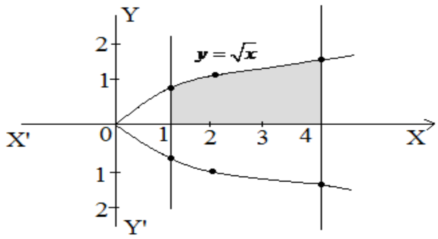
CLASS - XII MATHEMATICS (Calculus : Application of Integrals)

Topic: - Application of Integrals (ANSWERS)

Ans1. $y^2 = x$ is the equation of parabola and $x = 1$, $x = 4$ and x – axis

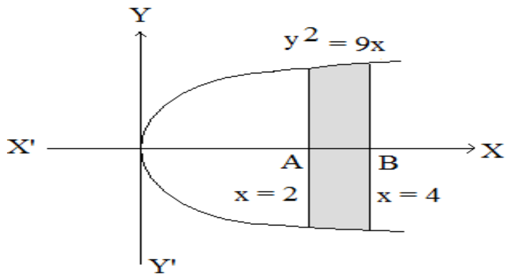
Req. area = $\int_1^4 \sqrt{x}\,dx$

$= \frac{14}{3} sq\ unit$



Ans2. $y^2 = 9x$, $x = 2$, $x = 4$, x – axis in the first quadrant.

$= \int_2^4 \sqrt{9x}\,dx = \left(16 - 4\sqrt{2}\right) sq\ unit$



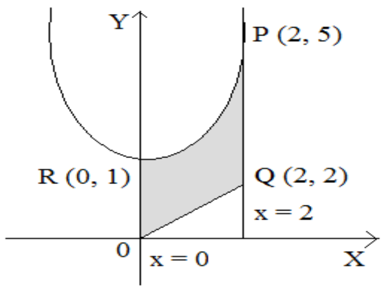
Ans3. $y = x^2 + 1$

$y = x$

$x = 0$

$x = 2$

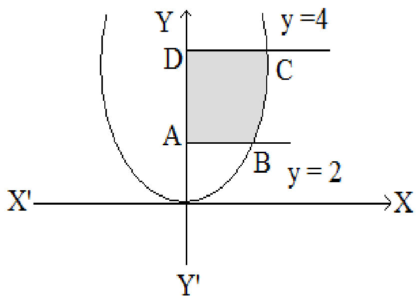
$= \int_o^2 \left(x^2 + 1\right) dx - \int_o^2 x\,dx$



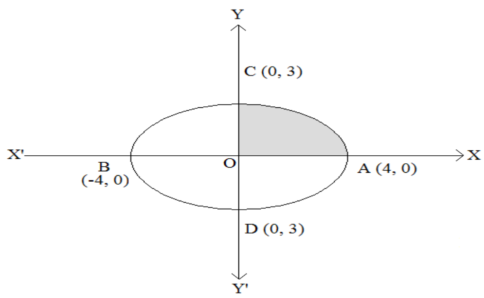
Ans4. $x^2 = 4y$, $y = 2$, $y = 4$ y – axis in the first quadrant

$= 2\int_2^4 \sqrt{y}\,dy$

$= \left(\frac{32 - 8\sqrt{2}}{3}\right) sq\ unit$



Ans5. $\frac{x^2}{16}+\frac{y^2}{9}=1$



$\frac{y^2}{9}=1-\frac{x^2}{16}$

$y^2=\frac{9}{16}\left(16-x^2\right)$

$y=\frac{3}{4}\sqrt{16-x^2}$

Required area $=\cancel{4}\int_o^4 \frac{3}{\cancel{4}}\sqrt{16-x^2}\,dx$

$=3\left[\frac{x}{4}\sqrt{16-x^2}+\frac{16}{2}\sin^{-1}\left(\frac{x}{4}\right)\right]_o^4$

$\left[\because \sqrt{a^2-x^2}=\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]$

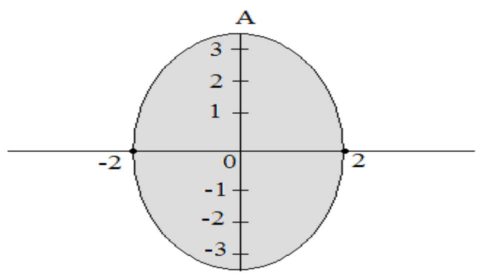
$=3\left[\left(o+8\sin^{-1}(1)\right)-(0)\right]$

$=3\left[8\sin^{-1}\left(\sin\frac{\pi}{2}\right)\right]$

$=3.\cancel{8}.\frac{\pi}{\cancel{2}}$

$=12\pi sq\ unit$

Ans6. $\frac{x^2}{4}+\frac{y^2}{9}=1$

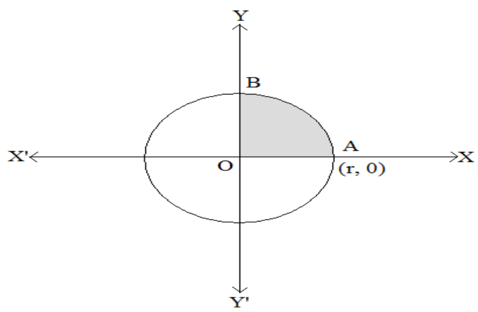


$=4\int_o^2 \frac{3}{2}\sqrt{4-x^2}\,dx$

$=4\int_o^2 \frac{3}{2}\sqrt{2^2-x^2}\,dx$

$=6\pi sq\ unit$

Ans7. $x^2+y^2=r^2$



$=4\int_o^r \sqrt{r^2-x^2}\,dx$

$put\ x=r\sin\theta$

$$dx = r\cos\theta \; d\theta$$

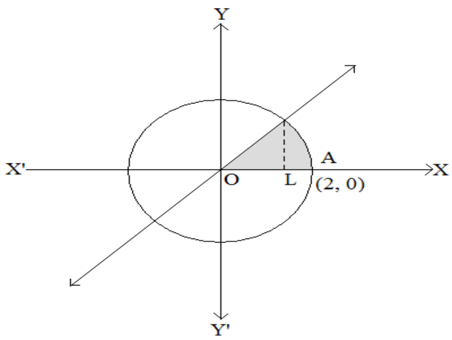
$$= 4\int_o^{\pi/2} r\cos\theta \; d\theta . \; r\cos\theta$$

$$= 4\int_o^{\pi/2} r^2 \cos^2\theta \; d\theta$$

$$= 4r^2 \int_o^{\pi/2} \left(\frac{1-\cos 2\theta}{2}\right) d\theta$$

$$= \pi r^2 sq \; unit$$

Ans8. x – axis



$$x = \sqrt{3}y$$

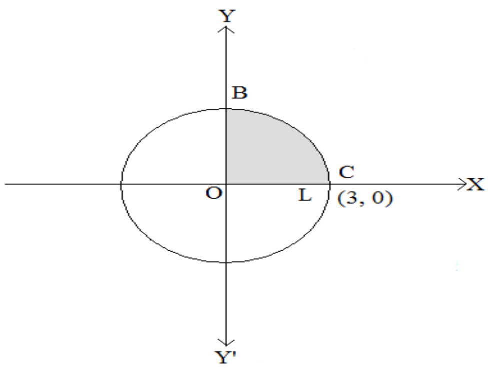
$$x^2 + y^2 = 4$$

in first quadrant.

$$= \frac{1}{\sqrt{3}}\int_o^{\sqrt{3}} x \; dx + \int_{\sqrt{3}}^{2} \sqrt{4-x^2}\,dx$$

$$= \frac{\pi}{3} sq \; unit$$

Ans9.

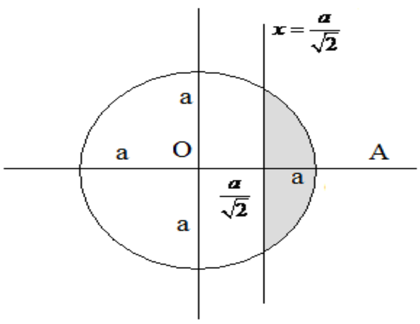


$$= \int_o^{3} \sqrt{9-x^2}\,dx$$

$$= \int_o^{3} \sqrt{3^2-x^2}\,dx$$

$$= \frac{9\pi}{4} sq \; unit$$

Ans10. $x^2 + y^2 = a^2$



$$x = \frac{a}{\sqrt{2}}$$

$$= 2\int_{\frac{a}{\sqrt{2}}}^{a} \sqrt{a^2-x^2}\,dx$$

$$= 2\left[\frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_{\frac{a}{\sqrt{2}}}^{a}$$

$$= \frac{\pi a^2}{4} - \frac{a}{2} sq \; unit.$$

CBSE TEST PAPER-02

CLASS - XII MATHEMATICS (Calculus : Application of Integrals)

Topic: - Application of Integrals

Note : Each Question carries 6 marks.

1. The area between $x = y^2$ and $x = 4$ is divided into equal parts by the line $x = a$, find the value of a.

2. Find the area of the region bounded by the parabola $y = x^2$ and $y = |x|$.

3. Find the area of ellipse $\frac{x^2}{a^2}\frac{y^2}{b^2} = 1$.

4. Find the area bounded by the curve $x^2 = 4y$ and the line $x = 4y – 2$.

5. Find the area of the region bounded by the curve $y^2 = 4x$ and the line $x = 3$.

6. Find the area between the curve $y = |x + 3|$, the x – axis and the lines $x = -6$ and $x = 0$.

7. Find the Area lying in the first quadrant and bounded by the circle $x^2 + y^2 = 4$ and the lines $x = 0$ and $x = 2$.

8. Find the Area of the region bounded by the curve y2 = 4x, y – axis and the line y = 3.

9. Find the area enclosed between the curve $y = x^3$ and the line $y = x$.

10. Find the area of the circle $4x^2 + 4y^2 = 9$ which is interior to the parabola $y^2 = 4x$.

Topic: - Application of Integrals (ANSWERS)

Ans1. x = y2

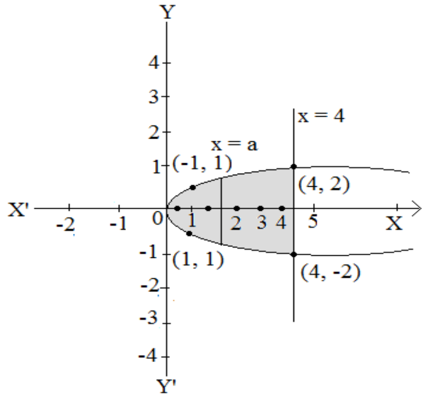
x = 4

x = a

ATQ $2\int_o^a \sqrt{x}dx = 2\int_2^4 \sqrt{x}dx$

$$\left[\frac{x^{\frac{3}{2}}}{3/2}\right]_o^1 = \left[\frac{x^{3/2}}{3/2}\right]_2^4$$

$a = (4)^{2/3}$ *sq unit*



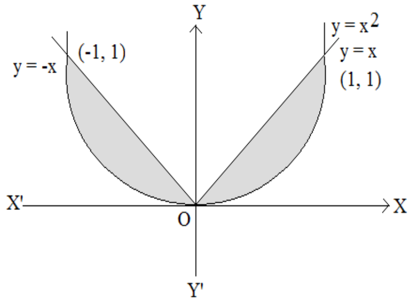
Ans2. $y = x^2$

$y = |x|$

$\Rightarrow y = x$

$y = -x$

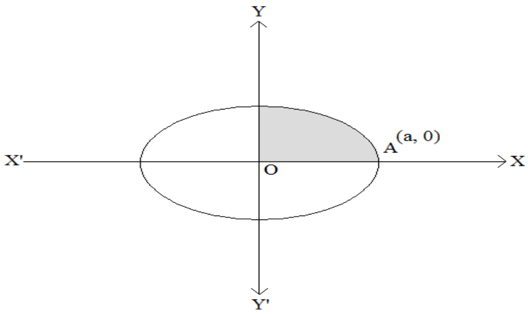
$= 2\int_o^1 (x - x^2)\,dx$



Ans3. $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$= 4\int_o^a \frac{b}{a}\sqrt{a^2 - x^2}\,dx$

$= \pi ab$ *sq unit*

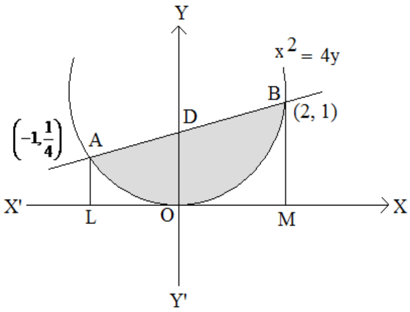


Ans4. $x^2 = 4y$

x = 4y – 2

Req. area = $\int_{-1}^{2} \frac{1}{4}(x+2)\,dx - \frac{1}{4}\int_{-1}^{2} x^2 dx$

$= \frac{9}{8}$ *sq unit*

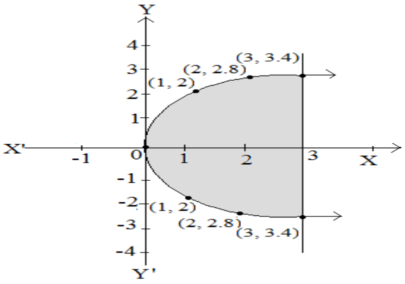


Ans5. $y^2 = 4x$

$x = 3$

$= 2\int_o^3 \sqrt{4x}dx$

$= 8\sqrt{3}sq\ unit$



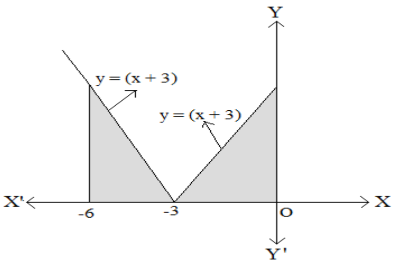
Ans6. $y = |x+3|$

$x - aixs$

$x = -6,\ x = 0$

$\int_{-6}^{o} |x+3| dx = \int_{-6}^{-3} -(x+3)\,dx + \int_{-3}^{o} (x+3)\,dx$

$= 9sq\ unit.$



Ans7. $x^2 + y^2 = 4$

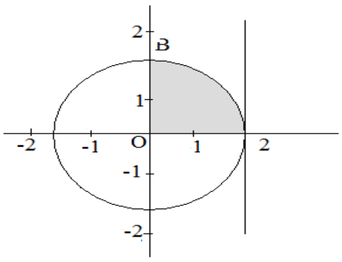
$x = 0$

$x = 2$

Area $= \int_o^{\pi/2} \sqrt{4 - x^2}dx$

$= \int_o^{\pi/2} \sqrt{2^2 - x^2}dx$

$= \pi sq\ unit$



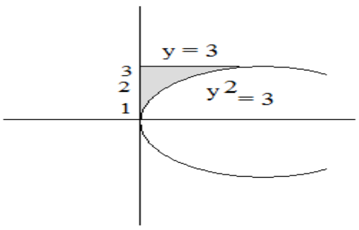
Ans8. $y^2 = 4x$

$y - axis$

$y = 3$

$Area = \int_o^3 \frac{y^2}{4} dy$

$= \frac{9}{4} sq.unit$



Ans9. $y = x^3,\ y = x$

$\Rightarrow x^3 = x$

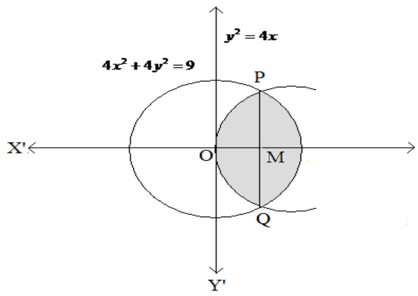
$x = 0,\ x = -1,\ x = 1$

$= 2\int_o^1 \left(x - x^3\right) dx$

$= 2\left[\frac{x^2}{2} - \frac{x^4}{4}\right]_o^1$

$= 2\left[\left(\frac{1}{2} - \frac{1}{4}\right) - (0)\right]$

$= 2\left(\frac{2-1}{4}\right) = \frac{1}{2}\, sq\ unit.$



Ans10. $4x^2 + 4y^2 = 9 ------(1)$

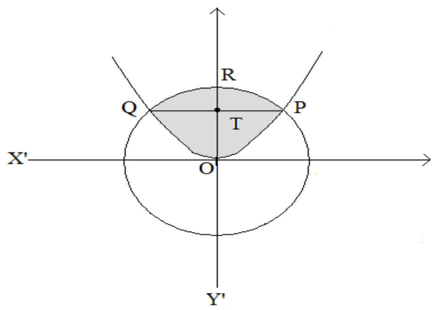
$y^2 = 4x ----------(2)$

On solving (1) and (2)

$y = 1/2$

$= 2\left(\int_o^{1/2} 2\sqrt{y}\, dy + \int_{1/2}^{3/2} \sqrt{\frac{9}{4} - y^2}\, dy\right)$

$= \left[\frac{\sqrt{2}}{6} + \frac{9\pi}{8} - \frac{9}{4}\sin^{-1}\left(\frac{1}{3}\right)\right] sq.\ unit.$



CBSE TEST PAPER-03
CLASS - XII MATHEMATICS (Calculus : Application of Integrals)

Topic: - Application of Integrals

Note : Each Question carries 6 marks.

1. Find the area bounded by the curves $(x - 1)^2 + y^2 = 1$ and $x^2 + y^2 = 1$.

2. Find the area of the region bounded by the parabolas $y^2 = 4ax$ and $x^2 = 4ay$, $a > 0$.

3. Find the area of the region bounded by the curves $y = x^2 + 2$, $y = x$, $x = 0$ and $x = 3$.

4. Find the area of the region $\left\{(x, y) : x^2 \leq y \leq x\right\}$.

5. Find the area bounded by the curves $\left\{(x, y) : x^2 + y^2 \leq 2ax, y^2 > ax, a > 0, x > 0, y > 0\right\}$.

6. Using integration, find the area of region bounded by the triangle whose vertices are (-1, 0), (1, 3) and (3, 2).

7. Find the area of the region: $\left\{(x, y) : x^2 + y^2 \leq 1 \leq x + y\right\}$.

8. Draw a rough sketch of the region $\left\{(x, y) : y^2 \leq 3x, 3x^2 + 3y^2 = 16\right\}$ and find the area enclosed by the region using method of integration.

9. Using integration find the area of the triangular region whose side have the equations $y = 2x + 1$, $y = 3x + 1$, and $x = 4$.

10. Calculate the area of the region enclosed between eh circles: $x^2 + y^2 = 16$ and $(x + 4)^2 + y^2 = 16$.

Topic: - Application of Integrals (ANSWERS)

Ans1. $(x-1)^2 + y^2 = 1 ------(1)$

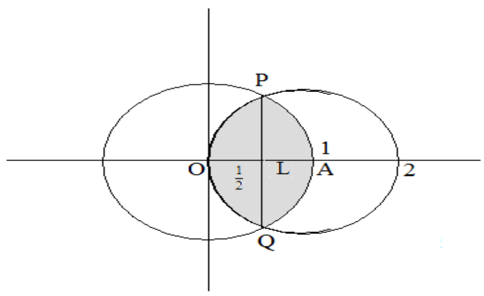
$x^2 + y^2 = 1 --------(2)$

On solving (1) and (2)

$x = \frac{1}{2}, y = \frac{\sqrt{3}}{2}$

$Area = 2\left[\int_o^{1/2} \sqrt{1-(x-1)^2}\,dx + \int_{1/2}^{1} \sqrt{1-x^2}\,dx\right]$

$= \left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right) sq.\ unit$



Ans2. $y^2 = 4ax$

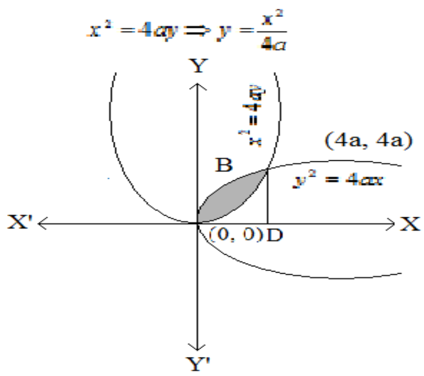
$x^2 = 4ay$

on solving

$x = 4a,\ y = 4a$

$Area = \int_o^{4a} \sqrt{ax}\,dx - \int_o^{4a} \frac{x^2}{4a}\,dx$

$= \frac{16a^2}{3} sq\ unit.$



Ans3. $y = x^2 + 2$

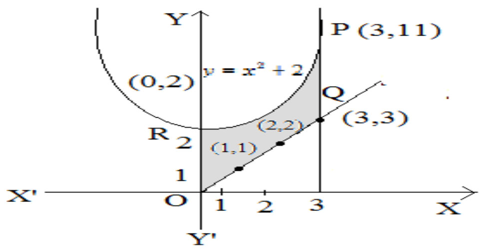
$y = x$

$x = 0$

$x = 3$

$Area = \int_o^3 (x^2+2)dx - \int_o^3 x\,dx$

$= \frac{21}{2} sq\ unit.$



Ans4. $y = x^2$

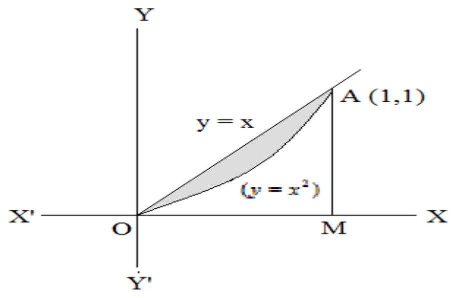
$y = x$

$\Rightarrow x = 0, y = 0$

$x = 1, y = 1$

$Area = \int_o^1 x dx - \int_o^1 x^2 dx$

$= \frac{1}{6} sq.\ unit$



Ans5. $x^2 + y^2 = 2ax$

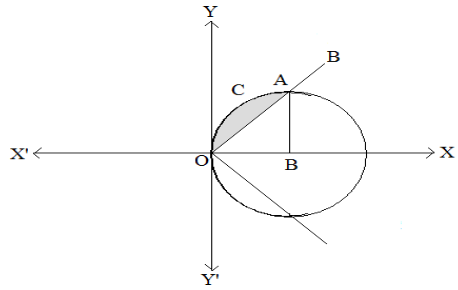
$y^2 = ax$

$\Rightarrow x = a, y = a$

$x = 0, y = 0$

$Area = \int_o^a \sqrt{2ax - x^2} dx - \int_o^a \sqrt{ax} dx$

$= \frac{a^2}{12}(3\pi - 8)\ sq.\ unit$



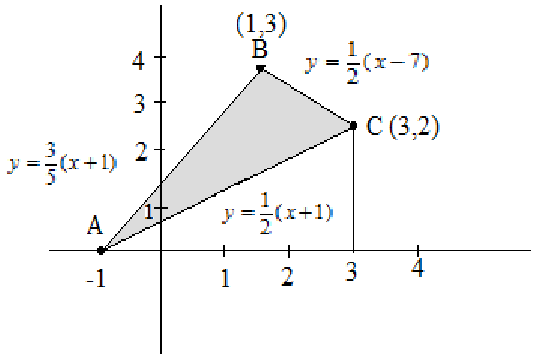
Ans6. A (-1, 0) B (1, 3) C (3, 2)

Equation of AB

$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$y - 0 = \frac{3-0}{1+1}(x+1)$

$y = \frac{3}{2}(x+1)$



Similarly

Equation of BC $y = \frac{-1}{2}(x-7)$

Equation of AC$= \frac{1}{2}(x+1)$

$Area\ \Delta ABC = \int_{-1}^{1} \frac{3}{2}(x+1)dx + \int_1^3 \frac{1}{2}(x-7)dx$

$-\int_{-1}^{3} \frac{1}{2}(x+1)dx$

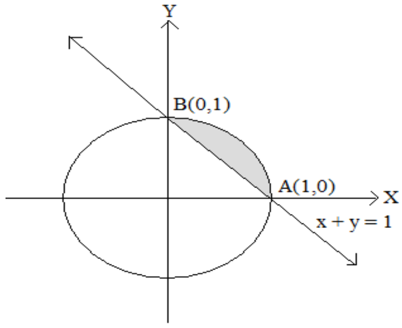
$= 4 sq.\ unit$

Ans7. $x^2+y^2=1$

$x+y=1$

$Area=\int_o^1\sqrt{1-x^2}dx-\int_o^1(1-x)\,dx$

$=\frac{\pi}{4}-\frac{1}{2}sq\ unit$



Ans8. $y^2=3x$

$3x^2+3y^2=16$

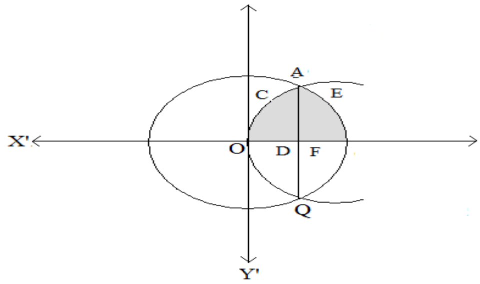
On solving

$x=\frac{-9+\sqrt{273}}{6}=p$

$Area=2\left[\int_0^p\sqrt{3x}dx+\int_p^{4\sqrt{3}}\sqrt{\frac{16-3x^2}{3}}dx\right]$

$=\frac{4}{\sqrt{3}}(p)^{3/2}+\frac{4\pi}{3}-\frac{p}{2}\sqrt{\frac{16}{3}-p^2}$

$-\frac{8}{3}\sin^{-1}\left(\frac{p}{4/\sqrt{3}}\right)$



Ans9. $y=2x+1$

$y=3x+1$

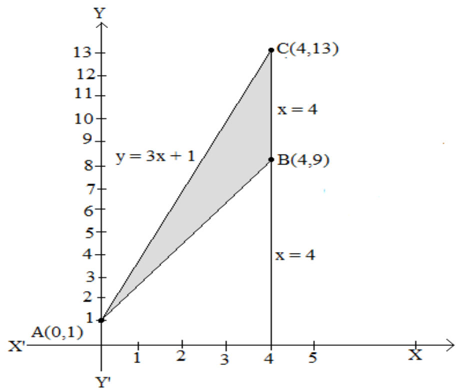
$x=4$

On solving

$A(0,1),\ B(4,9),\ C(4,13)$

$Area=\int_o^4(3x+1)dx-\int_o^4(2x+1)dx$

$=8sq\ unit$



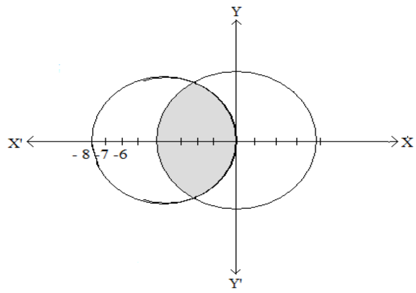
Ans10. $x^2+y^2=16$

$(x+4)^2+y^2=16$

Intersecting at x = -2

$Area=4\int_{-4}^{-2}\sqrt{16-x^2}dx$

$=\left(-8\sqrt{3}+\frac{32\pi}{3}\right)sq\ unit$



CBSE TEST PAPER-04

CLASS - XII MATHEMATICS (Calculus : Application of Integrals)

Topic: - Application of Integrals

Note : Each Question carries 6 marks.

1. Using integration, find the area of the region given below:

   $\left\{(x, y): 0 \le y \le x^2 + 1, 0 \le y \le x + 1, 0 \le x \le 2\right\}$.

2. Compute the area bounded by the lines x + 2y = 2, y – x = 1 and 2x + y = 7.

3. Find Smaller area enclosed by the circle $x^2 + y^2 = 4$ and the lines x + y = 2.

4. Find the area between the curves y = x and $y = x^2$.

5. Sketch the graph of y = |x + 3| and evaluate $\int_{-6}^{0} |x+3|\, dx$.

6. Find the area bounded by the curve y = sinx between x = 0 and x = 2π

7. Find the area enclosed by the parabola $y^2 = 4ax$ and the line y = mx.

8. Find the area of the region $\left\{(x, y): 0 \le y \le (x^2 + 1), 0 \le y \le (x+1), 0 \le x \le 2\right\}$.

9. Find the area enclosed by the parabola $4y = 3x^2$ and the line 2y = 3x + 12.

10. Find the area of the smaller region bounded by the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ and the line

    $\frac{x}{3} + \frac{y}{2} = 1$.

CBSE TEST PAPER-04

CLASS - XII MATHEMATICS (Calculus : Application of Integrals)

Topic: - Application of Integrals (ANSWERS)

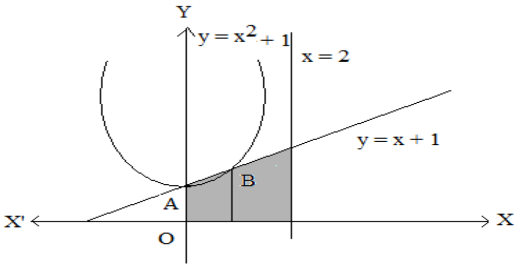
Ans1. $y = x^2 + 1$

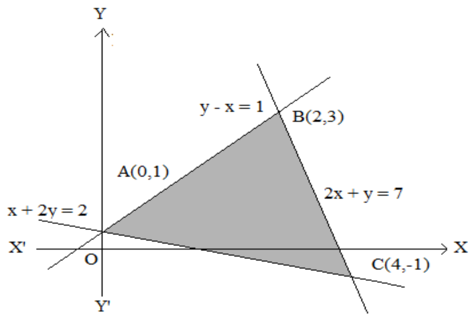
$y = x + 1$

$x = 2$

$Area = \int_o^1 (x^2 + 1)\,dx + \int_1^2 (x+1)\,dx$

$= \dfrac{23}{6}\, sq\ unit$





Ans2. $x + 2y = 2 - - - - - (1)$

$y - x = 1 - - - - - - (2)$

$2x + y = 7 - - - - - (3)$

$$Area = \int_o^2 \left[(1+x) - \left(\frac{2-x}{2} \right) \right] dx + \int_2^4 \left[(7-2x) - \left(\frac{2-x}{2} \right) \right] dx$$

$= 6 sq.\ unit$

Ans3. $x^2 + y^2 = 4 - - - - - (1)$

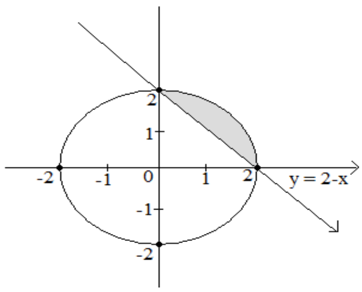
$x + y = 2 - - - - - - (2)$

Finding smaller area. On solving (1) and (2)

$x = 0, 2$

$Area = \int_o^2 \sqrt{4 - x^2}\,dx - \int_o^2 (2-x)\,dx$

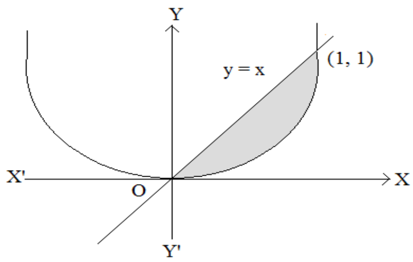
$= (\pi - 2)\, sq\ unit$



Ans4. $y = x$

$y = x^2$

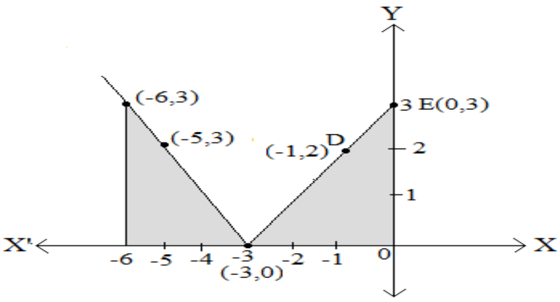
On solving x = 0, 1



$$Area = \int_o^1 \left(x - x^2\right) dx$$

$$= \frac{1}{6} sq\ unit.$$

Ans5. $y = |x+3|$



$$\Rightarrow y = (x+3)$$

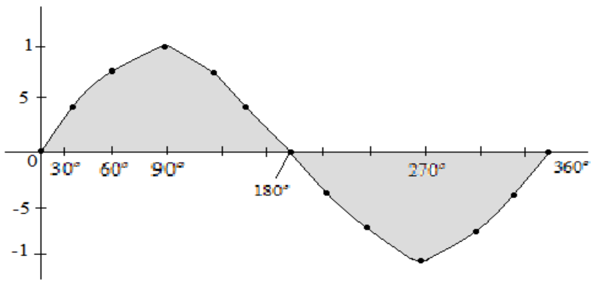
$$y = -(x+3)$$

$$\int_{-6}^{o} |x+3| dx = ?$$

$$Area = \int_{-6}^{-3} (x+3)\, dx + \int_{-3}^{o} (x+3) dx$$

$$= 9 sq.\ unit.$$

Ans6. $y = \sin x$



$$x = 0,\ x = 2\pi$$

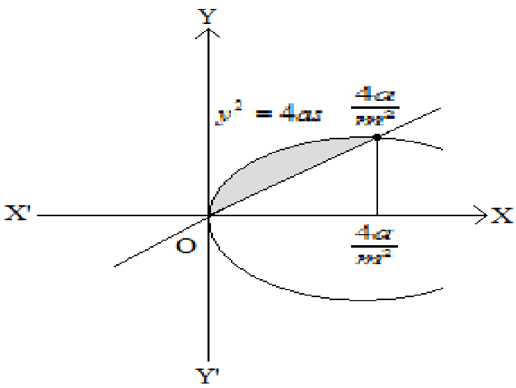
$$Area = 2\int_o^{\pi} \sin x dx$$

$$= -2[\cos x]_o^{\pi}$$

$$= -2[\cos \pi - \cos \theta]$$

$$= -2[-1-1] = 4 sq\ unit$$

Ans7. $y^2 = 4ax$



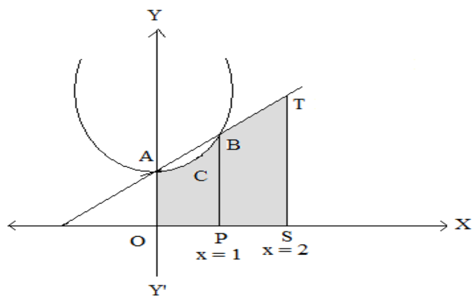
$$y = mx$$

$$x = \frac{4a}{m^2}$$

$$Area = \int_o^{4a/m^2} \sqrt{4ax} dx - \int_o^{\frac{4a}{m^2}} mx\ dx$$

$$= \frac{8a^2}{3m^2} sq\ unit.$$

Ans8. $y = x^2 + 1$



$$y = x + 1$$

$$x \leq 2$$

$$Area = \int_o^1 (x^2+1)\,dx + \int_o^1 (x+1)\,dx$$

$$= \frac{23}{6} sq\ unit$$

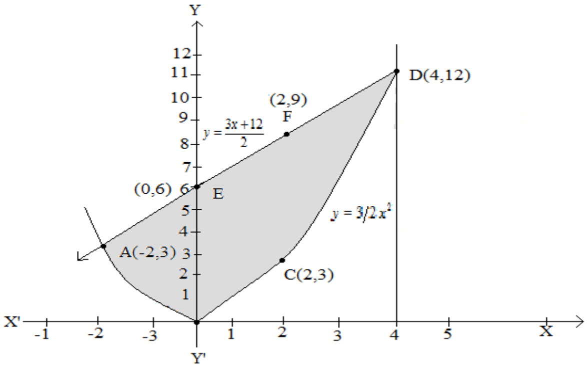
Ans9. $4y = 3x^2$

$2y = 3x + 12$

$x = -2, 4$

$$Area = \int_{-2}^{4} \frac{3x+12}{2} dx - \int_{-2}^{4} \frac{3}{4} x^2 dx$$

$= 27 sq\ unit.$



Ans10. $\frac{x^2}{9} + \frac{y^2}{4} = 1$

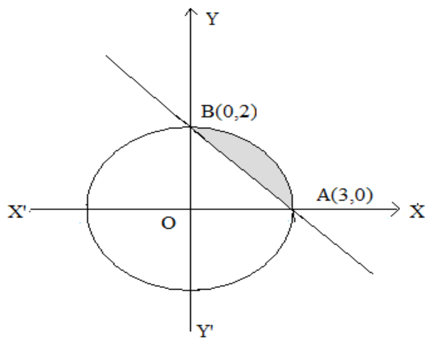
$\frac{x}{3} + \frac{y}{2} = 1$

$\Rightarrow \frac{x^2}{(3)^2} \frac{y^2}{(2)^2} = 1$ is the equation of ellipse and

$\frac{x}{3} + \frac{y}{2} = 1$ is the equation of intercept form

$$Area = \frac{2}{3}\int_o^3 \sqrt{9-x^2}\,dx - \int_o^3 \left(\frac{6-2x}{3}\right) dx$$

$$= \frac{3}{2}(\pi - 2) squnit.$$



CBSE TEST PAPER-05

CLASS - XII MATHEMATICS (Calculus : Application of Integrals)

Topic: - Application of Integrals

Note : Each Question carries 6 marks.

1. Find the area of the smaller region bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and the line $\frac{x}{a}+\frac{y}{b}=1.$

2. Find the area of the region enclosed by the parabola $x^2 = y$, the line $y = x + 2$ and the x – axis.

3. Using method of integration, find the area bounded by the curve $|x| + |y| = 1$.

4. Find area bounded by curves $\{(x, y) : y \geq x^2 \text{ and } y = |x|\}$.

5. Using method of integration find the area of the triangle ABC, coordinates of whose vertices are A (2, 0), B (4, 5) and C (6, 3).

6. Using method of integration, find the area of the region bounded by lines:

   $2x + y = 4$, $3x - 2y = 6$

   and $x - 3y + 5 = 0$.

7. Find the area of two regions $\{(x, y) : y^2 \leq 4x, 4x2 + 4y^2 \leq 9\}$.

8. Find the area of the circle $x^2 = y^2 = 15$ exterior to the parabola $y^2 = 6x$

9. Find the area bounded by the y – axis, y = cosx and y = sinx, $0 \leq x \leq \frac{\pi}{2}$

10. Using integration, find the area of the region in the first quadrant enclosed by the x – axis, the line $y = x$ and the circle $x^2 + y^2 = 32$.

CBSE TEST PAPER-05

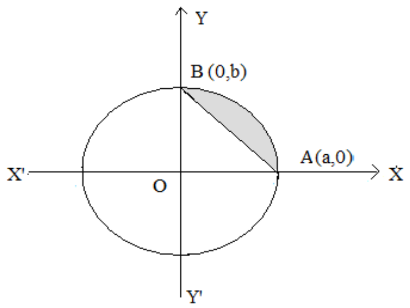
CLASS - XII MATHEMATICS (Calculus : Application of Integrals)

Topic: - Application of Integrals (ANSWERS)

Ans1. $y=\dfrac{b}{a}\sqrt{a^2-x^2}$

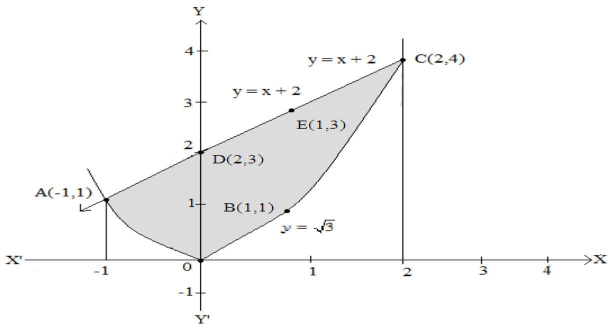
$$Area=\int_o^a \frac{b}{a}\sqrt{a^2-x^2}\,dx-\int_o^a \frac{b}{a}(a-x)$$

$$=\frac{ab}{4}(\pi-2)\ sq\ unit$$



Ans2. $Area=\int_{-1}^{2}(x+2)\,dx-\int_{-1}^{2}x^2dx$

$$=\frac{9}{2}sq.\ unit$$

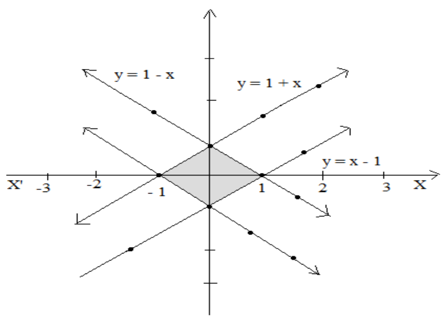


Ans3. $|x|+|y|=1$

$$\Rightarrow x+y=1$$
$$-x+y=1$$
$$x-y=1$$
$$-x-y=1$$

$$Area=4\int_o^1(1-x)\,dx$$

$$=2sq\ unit$$

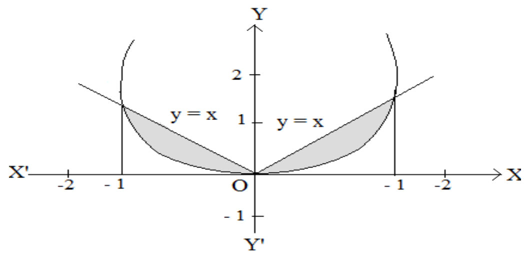


Ans4.

$y=|x|$

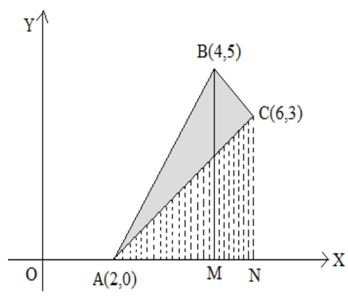
$$Area=2\int_o^1\left(x-x^2\right)dx$$

$$=\frac{1}{3}sq\ unit.$$



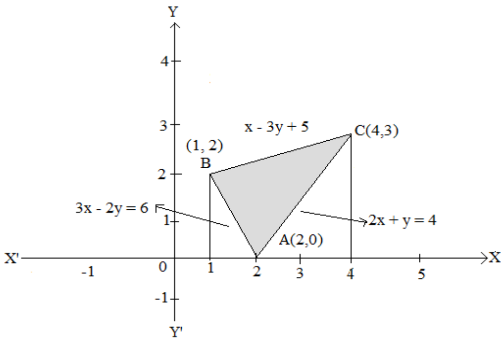
Ans5. $Area=\dfrac{5}{2}\int_2^4(x-2)\,dx+\int_4^6-(x-9)dx-\dfrac{3}{4}\int_2^6(x-2)\,dx$

$$=7sq\ unit$$



Ans6. $Area = \int_1^4 \frac{x+5}{3} dx + \int_1^2 -(2x-4) + \int_2^4 \frac{3x-6}{2}$

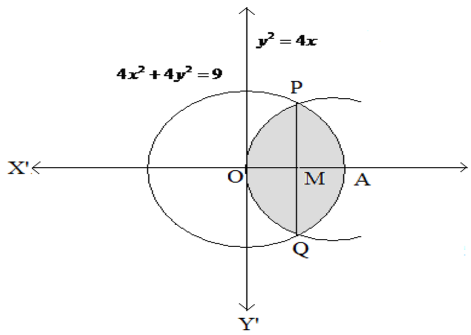
$= \frac{7}{2} sq\ unit.$



Ans7. $y^2 = 4x,\ 4x^2 + 4y^2 = 9$

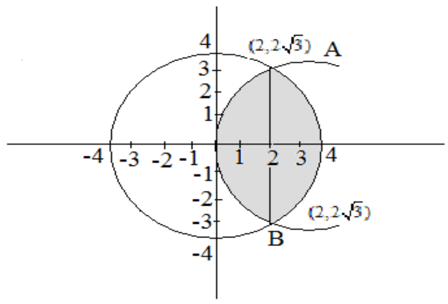
$$Area = 2\left[\int_o^{1/2} 2\sqrt{x}dx + \int_{1/2}^{3/2} \sqrt{\left(\frac{3}{2}\right)^2 - x^2}\, dx\right]$$

$$= \frac{\sqrt{2}}{6} + \frac{9\pi}{8} - \frac{9}{4}\sin^{-1}\left[\frac{1}{3}\right]$$



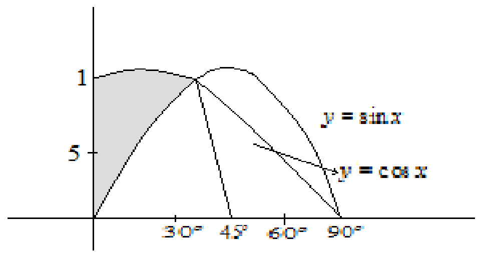
Ans8. $Area = 2\int_0^2 \sqrt{6x} \quad dx + 2\int_2^4 \sqrt{16 - x^2}$

$= \frac{4}{3}\left[8\pi - \sqrt{3}\right]$



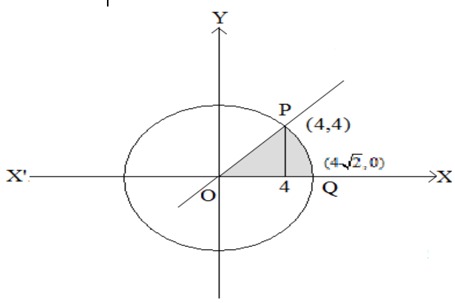
Ans9. $Area = \int_0^{\pi/4} (\cos x - \sin x)\ dx$

$= \sqrt{2} - 1 \ \ sq\ unit$



Ans10. $Area = \int_o^4 x\ dx + \int_4^{4\sqrt{2}} \sqrt{32 - x^2} \quad dx$

$= 4\pi sq\ unit$



CBSE TEST PAPER-01
CLASS - XII MATHEMATICS (Calculus: Differential Equations)

1. Find the order and degree. [1]
$y''' + y^2 + e^{y'} = 0$

2. Verify that the functions is a sol of the corresponding diff. req. [1]
$y = x\sin x \quad ; xy^1 = y + x\sqrt{x^2 - y^2}$

3. Form the differential equation of the family of hyper bolas having foci On x-axis and center at origin. [1]

4. Form the differential equation of the family of circles having centre on y-axis and radius 3 units. [4]

5. Solve the diff. equ. $\sec^2 x.\tan y\, dx + \sec^2 y \tan x\, dy = 0$ [4]

6. Solve the diff eq. $y \log y\, dx - x\, dy = 0$ [4]

7. Solve x cos $\left(\frac{y}{x}\right)\frac{dy}{dx} = y\cos\left(\frac{y}{x}\right) + x$ [4]

8. Solve [4]
$2y\,e^{\frac{x}{y}}dx + (y - 2xe^{x/y})dy = 0$
$and \;\; x = 0 \; when \;\; y = 1$

9. Find the general sol. of the diff eq. [6]
$\frac{dy}{dx} - y = \cos x$

10. Find the particular sol of the diff. eq. [6]
$\frac{dy}{dx} + y\cot x = 2x + x^2 \cot x$
Given that y = 0 when x = $\pi/2$

CBSE TEST PAPER-01

CLASS - XII MATHEMATICS (Calculus: Differential Equations)

[ANSWERS]

1. order =3
Degree = not define.

2. $y = x.\sin x ...(i)$

$y^1 = x.\cos x + \sin x.1$

$\Rightarrow xy^1 = x^2 \cos x + x.\sin x$

$xy^1 = x^2\sqrt{1-\sin^2 x} + x.\sin x$

$xy^1 = x^2\sqrt{1-\left(\frac{y}{x}\right)^2} + x.\sin x$ $\qquad [\because \frac{y}{x} = \sin x$

$xy^1 = x^{\cancel{2}}\frac{\sqrt{x^2-y^2}}{\cancel{x}} + x.sinx$

$xy^1 = x\sqrt{x^2-y^2} + y$

Hence prove.

3. $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 ...(i)$

diff

$\frac{2x}{a^2} - \frac{2y}{b^2}y' = 0$

$\frac{\cancel{2}x}{a^2} = \frac{2y}{b^2}y'$

$\frac{b^2}{a^2} = \frac{y}{x}y'$

again diff

$0 = \left(\frac{y}{x}\right)y'' + y'\left(\frac{x.y' - y'}{x^2}\right)$

$\frac{0}{1} = \frac{xy\,y'' + xy'^2 - yy'}{x^2}$

$xy\,y'' + xy'^2 - yy' = 0$

$yy' = xyy'' + xy'^2$

4. $(0, a)$

$r = 3$

$x^2 + (y-a)^2 = (3)^2 \ldots(i)$

$2x + 2(y-a)y^1 = 0$

$x + (y-a)y^1 = 0$

$y - a = \frac{-x}{y}$

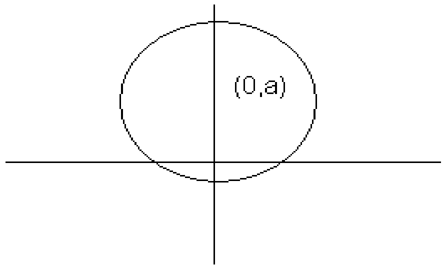
put the value of $y - a$

in eq...(i)

$x^2 + \left(\frac{-x}{y}\right)^2 = 9$

$x^2 + \frac{x^2}{y'^2} = 9$

$x^2(y^{1^2} + 1) = 9y'^2$



5. Sec^{2}x. tan y dx = -sec^{2}y tan x dy

$\int \frac{\sec^2 x}{\tan x} dx = -\int \frac{\sec^2 y}{\tan y} dy$

$\log(\tan x) = -\log(\tan y) + c$

$\log(\tan x . \tan y) = \log c$

$\tan x \quad \tan y = c$

6. $y \ \log y \ dx = x \ dy$

$\int \frac{dx}{x} = \int \frac{dy}{y \log y}$

$\int \frac{dx}{x} = \int \frac{1/y}{\log \ y} dy$

$\log(x) = \log(\log y) + c$

$\log \left(\frac{x}{\log y}\right) = \log c$

$\frac{x}{\log y} = c$

x = c log y

7. $$x\cos\left(\frac{y}{x}\right)\frac{dy}{dx}=y\cos\left(\frac{y}{x}\right)+x$$

$$\frac{dy}{dx}=\frac{y\cos\left(\frac{y}{x}\right)+x}{x\cos\left(\frac{d}{x}\right)}....(i)$$

$$let\ \ y=vx$$

$$\frac{dy}{dx}=v.1+x.\frac{dv}{dx}....(ii)$$

$$Put\ \frac{dy}{dx}in\ eq....(i)$$

$$v+x\frac{dv}{dx}=\frac{vx\cos v+x}{x\cos v}$$

$$v+x\frac{dv}{dx}=\frac{v\cos v+1}{\cos v}$$

$$x\frac{dv}{dx}=\frac{v\cos v+1}{\cos v}-v$$

$$x\frac{dv}{dx}=\frac{v\cancel{\cos}\,v+1-v\cancel{\cos}\,v}{\cos v}$$

$$x\frac{dv}{dx}=\frac{1}{\cos v}$$

$$\int\cos v\,dv=\int\ \frac{dx}{x}$$

$$\sin v=\log x+\log c$$

$$\sin v=\log|cx|[\because\ y=vx$$

$$\sin\left(\frac{y}{x}\right)=\log|cx|$$

8. $$\frac{d\cancel{x}}{d\cancel{y}}=$$

$$2y\,e^{x/y}dx+(y-2xe^{x/y})dy=0$$

$$\frac{dx}{dy}=\frac{2xe^{x/y}-y}{2y\,e^{x/y}}.....(i)$$

$$let\ x=vy$$

$$diff\ \ w.r.to\ y$$

$$\frac{dx}{dy}=v.1+y.\frac{dv}{dy}$$

$$put\ \frac{dx}{dy}\ in\ eq......(i)$$

$$v + y\ \frac{dv}{dy} = \frac{2ve^v - 1}{2e^v}$$

$$y\frac{dv}{dy} = \frac{2ve^v - 1}{2e^v} - v$$

$$= \frac{2v\cancel{e}^v - 1 - 2\cancel{v}e^v}{2e^v}$$

$$y\frac{dv}{dy} = \frac{-1}{2e^v}$$

$$\int 2e^v\ dv = \int \frac{dy}{y}$$

$$2e^v = -\log|y| + c$$

$$replace\ v\ by\ \frac{x}{y}$$

$$2e^{x/y} + \log y = c$$

$$put\ x = 0\ and\ y = 1$$

$$C = 2$$

$$2e^{x/y} + \log y = 2$$

9. $$\frac{dy}{dx} - y = \cos x$$

$$given\ diff\ eq. is\ of\ the\ form$$

$$\frac{dy}{dx} + py = Q$$

$$P = -1, Q = \cos x$$

$$I.F = e^{\int p\,dx}$$

$$= e^{\int -1 dx}$$

$$= e^{-x}$$

$$y \times e^{-x} = \int \cos x \times e^{-x} dx + c$$

$$let\ I = \int \cos x \times e^{-x} dx$$

$$= \cos x.\frac{e^{-x}}{-1} - \int -\sin x.(-e^{-x})dx$$

$$= -\cos x \times e^{-x} - \int \sin x \times e^{x} dx$$

$$= -\cos x \times e^{-x} - [\sin x(-e^{-x}) - \int \cos x(-e^{-x})dx$$

$$= -\cos x \times e^{-x} + \sin x \times e^{-x} - \int \cos x \times e^{-x} dx$$

$$I = -\cos x\ e^{-x} + \sin x\ e^{-x} - I$$

$$2I = e^{-x}(\sin x - \cos x)$$

$$I = \frac{e^{-x}}{2}(\sin x - \cos x)$$

$$y \times e^{-n} = \frac{1}{2}e^{-x}(\sin - \cos x) + c[\ from(i)]$$

$$y = \frac{1}{2}(\sin x - \cos x) + ce^{x} + c$$

10. $$\frac{dy}{dx} + y\cot x = 2x + x^2 \cot x$$

given diff eq is of the form

$$\frac{dy}{dx} + py = Q$$

$$p = \cot x$$

$$Q = 2x + x^2 \cot x$$

$$I.F = e^{\int \cot x\, dx}$$

$$= e^{\log \sin x}$$

$$= \sin x$$

$$y \times \sin x = \int (2x + x^2 \cot x).\sin x\, dx + c$$

$$= \int 2x \sin x\ dx + \int x^2 \cos x\, dx + c$$

$$2[\sin x.\frac{x^2}{2} - \int \cos x.\frac{x^2}{2}dx] + \int x^2 \cos dx + c$$

$$= \cancel{2}.\frac{\sin x.x^2}{\cancel{2}} - \int \frac{x^2 \cos x}{2}dx + \int x^2 \cos x\, dx + c$$

$$= \sin x.x^2 - \cancel{2}\int \frac{x^2\ \cancel{c}os\, x}{\cancel{2}}dx + \int \cancel{x}^2\ \cancel{c}os\, x\, dx + c$$

$$y \times \sin x = x^2.\sin x + c.....(i)$$

$$y = 0, x = \pi/2$$

$$o = \frac{\pi^2}{4} + c$$

$$c = \frac{-\pi^2}{4}$$

$$y \sin x = x^2.\sin x - \frac{\pi^2}{4} + c$$

$$y = x^2 - \frac{\pi^2}{4\sin x}$$

CBSE TEST PAPER-02
CLASS - XII MATHEMATICS (Calculus: Differential Equations)

1. Find the order and degree [1]

$$\left(\frac{ds}{dt}\right)^4 + 3s\frac{d^2s}{dt^2} = 0$$

2. Verity that the function is a solution of the corresponding diff eq. [1]

$x + y = \tan^{-1} y$; $y^2 y^1 + y^2 + 1 = 0$

3. Form the differential equation representing the family of ellipses having foci on x – axis and centre at the origin. [4]

4. Form the diff. eq of the family of circles touching the x – axis at origin. [4]

5. Solve the diff eq. [4]

$e^x \tan y\, dx + \left(1-e^x\right) \operatorname{Sec}^2 y\, dy = 0$

6. Solve $\operatorname{Cos}\left(\frac{dy}{dx}\right) = a,$; $y = 1$ when $x = 0$ [4]

7. Solve. $\left(x^2 - y^2\right)dx + 2xy\, dy = 0$ [4]

8. Solve [4]

$$\left\{x \operatorname{Cos}\left(\frac{y}{x}\right) + y \operatorname{Sin}\left(\frac{y}{x}\right)\right\} y\, dx = \left\{y \operatorname{Sin}\left(\frac{y}{x}\right) - x \operatorname{Cos}\left(\frac{y}{x}\right)\right\} x\, dy$$

9. Find the particulars solution of diff . equation. [6]

$\left(1+x^2\right)dy + 2xy\, dx = Cotx\, dx$

10. Find the particular solution of diff. equation [6]

$x\frac{dy}{dx} + y - x + xy \operatorname{Cot} x = 0$

CBSE TEST PAPER-02

CLASS - XII MATHEMATICS (Calculus: Differential Equations)

[ANSWERS]

Ans 01. order = 2

degree = 1

Ans 02. $x+y=\tan^{-1} y$

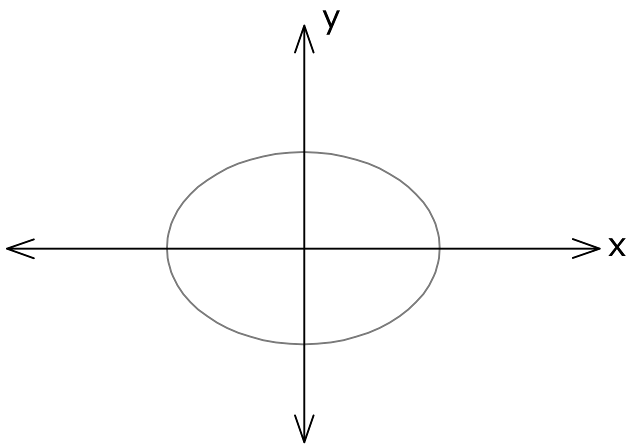
$$1+y^1=\frac{1}{1+y^2}y^1$$

$$1+y^2+\cancel{y^1}+y^1y^2=\cancel{y^1}$$

$$1+y^2+y^1y^2=0$$

proved.

Ans 03. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ (i)



diff eq (i) w. r. t. x

$$\frac{2x}{a^2}+\frac{2y}{b^2}.\frac{dy}{dx}=0$$

$$\frac{\cancel{2}y}{b^2}\ \frac{dy}{dx}=\frac{-\cancel{2}x}{a^2}$$

$$\frac{y}{x}.\ \frac{dy}{dx}=\frac{-b^2}{a^2}$$

diff w. r. t. x

$$\left(\frac{y}{x}\right)\frac{d^2y}{dx^2}+\frac{dy}{dx}\left(\frac{x\frac{dy}{dx}-y}{x^2}\right)=0$$

$$xy\frac{d^2y}{dx^2}+x\left(\frac{dy}{dx}\right)^2-y\frac{dy}{dx}=0$$

Ans 04. Let (o, a) be

The coordinate of circle.

$$x^2+\left(y-a\right)^2=a^2$$

diff

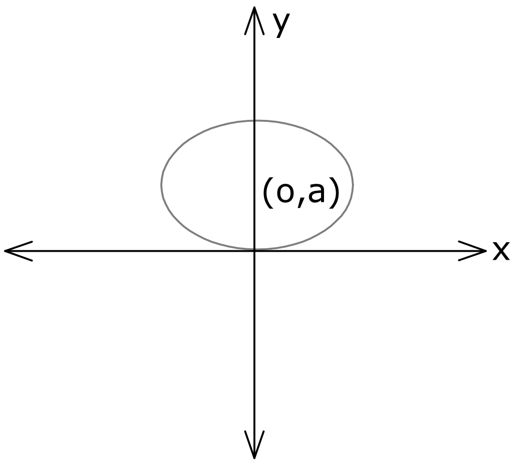
$$2x+2\left(y-a\right)y^1=0$$

$$x+\left(y-a\right)y^1=0$$

$$y-a=\frac{-x}{y^1}$$

$$y+\frac{x}{y^1}=a$$

Put a and y – a in eq (1)



$$x^2+\left(\frac{-x}{y'}\right)^2=\left(y+\frac{x}{y'}\right)^2$$

$$x^2+\cancel{\frac{x^2}{y'^2}}=y^2+\cancel{\frac{x^2}{y'^2}}+2.y\frac{x}{y'}$$

$$x^2-y^2=\frac{2xy}{y'}$$

$$y'=\frac{2xy}{x^2-y^2}$$

Ans. 05. $e^x \tan y\, dx + (1-e^x)\, \text{Sec}^2 y\, dy = 0$

$e^x \tan y\, dx = -(1-e^x)\, \text{Sec}^2 y\, dy$

$$\int\frac{e^x}{1-e^x}dx=-\int\frac{Sec^2 y}{\tan y}dy$$

$-\log(1-e^x) = -\log\tan y + \log C$

$$\log\left(\frac{\tan y}{1-e^x}\right)=\log C$$

$$\frac{\tan y}{1-e^x}=C$$

$\tan y = C\,(1-e^x)$

Ans 06. $$cos\left(\frac{dy}{dx}\right)=a$$

$$\frac{dy}{dx}=\cos^{-1} 9$$

$$\int dy=\int \text{Cos}^{-1}a\, dx$$

$y = \text{Cos}^{-1}\text{a. x+c (1)}$

$1 = \text{o+c} \left[\because \begin{array}{lll} y & = & 1 \\ x & = & 0 \end{array}\right.$

$c = 1$

$y = \text{Cos}^{-1}\text{a. x+1}$

Ans 07. $\left(x^2 - y^2\right)dx + 2xy \text{ dy=0}$

$\left(x^2 - y^2\right)dx = -2xy \text{ dy}$

$$\frac{dy}{dx} = \frac{y^2 - x^2}{2xy}$$

$$\frac{dy}{dx} = \frac{\left(\frac{y}{x}\right)^2 - 1}{2\left(\frac{y}{x}\right)} \quad (1)$$

let y = vx

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$but \ \frac{dy}{dx} in$ eq (i)

$$\text{v+x } \frac{dv}{dx} = \frac{v^2 - 1}{2v}$$

$$x \ \frac{dv}{dx} = \frac{v^2 - 1}{2v} - v$$

$$x \ \frac{dv}{dx} = \frac{v^2 - 1 - 2v^2}{2v}$$

$$x \ \frac{dv}{dx} = \frac{-1 - v^2}{2v}$$

$$\int \frac{2vdv}{v^2 + 1} = \int \frac{-dx}{x}$$

$\log \ \left(v^2 + 1\right) = -\log \ \text{x+c}$

$\log\left(\left(v^2 + 1\right).x\right) = c$

$\left(v^2 + 1\right).x = e^c$

$\left(\frac{y^2}{x^2} + 1\right).x = e^c \quad \left[v = \frac{y}{x}\right.$

$$\frac{x^2+y^2}{x}=A \quad \left[\because e^c = A\right.$$

$$x^2+y^2=Ax$$

Ans 08. $\left\{\text{x Cos}\left(\frac{y}{x}\right)+y\ \text{Sin}\ \left(\frac{y}{x}\right)\right\}y\ \text{dx}=\left\{\text{y Sin}\left(\frac{y}{x}\right)-x\ \text{Cos}\left(\frac{y}{x}\right)\right\}x\ \text{dy}$

$$\frac{dy}{dx}=\frac{\left\{\text{x Cos}\ \left(\frac{y}{x}\right)+y\ \text{Sin}\left(\frac{y}{x}\right)\right\}y}{\left\{\text{y Sin}\left(\frac{y}{x}\right)-x\ \text{Cos}\ \left(\frac{y}{x}\right)\right\}x}$$

$$\frac{dy}{dx}=\frac{\frac{y}{x}\left\{\text{Cos}\ \frac{y}{x}+\frac{y}{x}.\text{Sin}\ \frac{y}{x}\right\}}{\frac{y}{x}.\ \text{Sin}\left(\frac{y}{x}\right)-Cos\,\frac{y}{x}} \quad (1)$$

$$let\ y=v\ x$$

$$\frac{dy}{dx}=v+x\frac{dv}{dx},\ \text{put}\ \frac{dy}{dx}in\ \text{eq (1)}$$

$$\text{v+x}\frac{dv}{dx}=v\frac{(\text{Cos v + v Sin v})}{\text{v Sin v - Cos v}}$$

$$\text{x}\frac{dv}{dx}=v\frac{(\text{Cos v + v Sin v})}{\text{v Sin v - Cos v}}\ \text{- v}$$

$$\text{x}\frac{dv}{dx}=\frac{2v\ \text{Cos v}}{v\ \text{Sin v - Cos v}}$$

$$\int\left(\frac{\text{v Sin v - Cos v}}{\text{v Cos v}}\right)dv=\int\frac{2}{x}dx$$

$$\int\left(\text{tan v -}\ \frac{1}{v}\right)dv\int\frac{2}{x}dx$$

$$-\log(\text{Cos v})-\log v=2\log x+c$$

$$-\log v\ \text{Cos v = 2 log x + c}$$

$$\log\ \left((v\ \text{Cos v}).x^2\right)=-c$$

$$(\text{v Cos v}).x^2=e^{-c}$$

$$\cancel{x}^2.\frac{y}{\cancel{x}}.\text{Cos}\ \frac{y}{x}=A$$

$$xy\ \text{Cos}\ \frac{y}{x}=A$$

Ans 09. $\left(1+x^2\right)$dy+2xy dx = Cot x dx

$$\frac{dy}{dx}+\frac{2x}{1+x^2}y=\frac{\cot x}{1+x^2}$$

Given diff. eq is of the form

$$\frac{dy}{dx}+py=Q$$

$$p=\frac{2x}{1+x^2}, \text{ Q}=\frac{\cot x}{1+x^2}$$

$$I.F=e^{\int \text{p dx}}$$

$$= e^{\int\frac{2x}{1+x^2}}$$

$$= e^{\log(1+x^2)}$$

$$=1+x^2$$

$$y\times(1+x^2)=\int\frac{Cot\ \text{x}}{\cancel{1+x^2}}\times\left(\cancel{1+x^2}\right)dx+c$$

$$y\left(1+x^2\right)=\log\left(\text{Sin x}\right)+c$$

$$y=\frac{\log\text{ (Sin x)}}{1+x^2}+\frac{c}{1+x^2}$$

Ans 10. $x\ \dfrac{dy}{dx}+y-x+xy$ Cot x = o

$$\text{x }\frac{dy}{dx}+y\left(1+x\text{ Cot x}\right)=x$$

$$\frac{dy}{dx}+\frac{y}{x}\ \left(1+x\text{ Cot x}\right)=1$$

$$\frac{dy}{dx}+\left(\frac{1+x\ \text{ cot x}}{x}\right)y=1$$

given diff eq is of the

form $\dfrac{dy}{dx}+py=Q$

$$p=\frac{1+x\text{ cot x}}{x}=\frac{1}{x}+\cot x$$

Q = 1

$$IF=e^{\int\left(\frac{1}{x}+\cot x\right)dx}$$

$$=e^{\text{log x+log Sin x}}$$

$$=e^{\log(x.Sinx)}\quad\left[\because e^{\log\theta}=\theta\right.$$

$$=x.Sin\text{ x}$$

$$\text{y x sin x}=\int 1\times\text{ x sin x dx + c}$$

$$\text{y x sin x = -x cos x - }\int 1(-cos\text{ x) dx + c}$$

$$=\text{- x cos x + Sin x + c}$$

$$\text{y = - cot x + }\frac{1}{x}+\frac{c}{x\text{ sin x}}$$

CBSE TEST PAPER-03
CLASS - XII MATHEMATICS (Calculus: Differential Equations)

1. Find order and degree. [1]

$$\left(\frac{d^2y}{dx^2}\right)^3+\left(\frac{dy}{dx}\right)^2+Sin\left(\frac{dy}{dx}\right)+1=0$$

2. Verify that the function is a solution of the corresponding diff eq. [1]

$$y=\sqrt{1+x^2}\ ;\ y^1=\frac{xy}{1+x^2}$$

3. Form the differential equation representing the family of curves given by $(x-a)^2 + 2y^2 = a^2$, where a is an arbitrary constant. [4]

4. Form the diff. equation of the family of circles in the second quadrant and touching the coordinate axes. [4]

5. Solve the diff eq. [4]

$$\left(x^3+x^2+x+1\right)\frac{dy}{dx}=2x^2+x\ ;\ y=1 \text{ when } x=0$$

6. Solve $x\left(x^2-1\right)\frac{dy}{dx}=1$; y=0 when x=2 [4]

7. Solve $\left(1+e^{\frac{x}{y}}\right)dx+e^{\frac{x}{y}}\left(1-\frac{x}{y}\right)dy=0$ [4]

8. Solve [4]

$$\left[x\ \mathrm{Sin}^2\left(\frac{y}{x}\right)-y\right]dx+x\ dy=0\ ;\ y=\pi/4, when\ x=1$$

9. Solve the eq. [6]

$$\left(1+y^2\right)dx=\left(\tan^{-1}y-x\right)dy$$

10. Solve the diff eq. [6]

$$\left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}}-\frac{y}{\sqrt{x}}\right)\frac{dx}{dy}=1$$

CBSE TEST PAPER-03

CLASS - XII MATHEMATICS (Calculus: Differential Equations)

[ANSWERS]

Ans 01. order = 3

degree = not define

Ans 02. $y=\sqrt{1+x^2}$ (i)

$y^1=\frac{1}{\cancel{2}\sqrt{1+x^2}}.\cancel{2}x$ (ii)

(ii) $\div$ (i)

$\frac{y^1}{y}=\frac{\frac{x}{\sqrt{1+x^2}}}{\sqrt{1+x^2}}$

$\frac{y^1}{y}=\frac{x}{1+x^2}$

$y^1=\frac{xy}{1+x^2}$

Ans 03. $(x-a)^2+2y^2=a^2$ (i)

diff both side w.r.t.x

$2(x-a)+4y\ \mathrm{y}^1=0$

$(x-a)+2y\ \mathrm{y}^1=0$

$(x-a)=-2y\ \mathrm{y}^1$ (ii)

x+2y y^1=a (iii)

put the vlue of (x-a) and a in eq (i)

$\left(x-2y\ \mathrm{y}^1\right)^2+2y^2=\left(x+2y\ \mathrm{y}^1\right)^2$

$2y^2=x^2+4xy\frac{dy}{dx}$

Ans 04. Eq. of circle is

$$(x+a)^2+(y-a)^2=a^2 \quad (1)$$

$x^2+y^2+2ax-2ay+a^2=0$

diff both side w.r.to x

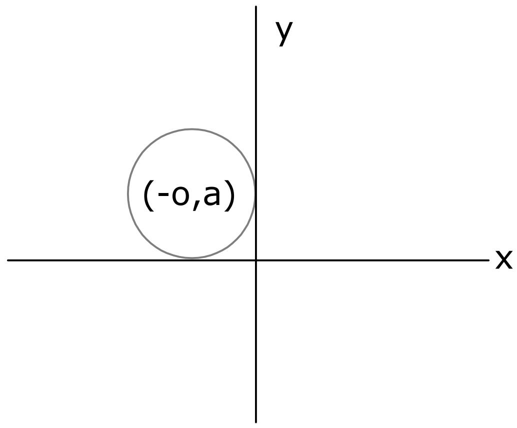
$2x+2y\,y^1+2a-2ay^1=0$

$x+y\,y^1=a\left(y^1-1\right)$

$$\frac{x+y\,y^1}{y^1-1}=a$$

put the velue of a in eq (1)

$$(x+y)^2\left[\left(y^1\right)^2+1\right]=\left(x+y\,y^1\right)^2$$



Ans 05. $$\frac{dy}{dx}=\frac{2x^2+x}{x^3+x^2+x+1}$$

$$\int dy=\int\frac{2x^2+x}{x^3+x^2+x+1}dx$$

$$\int dy=\int\frac{2x^2+x}{(x+1)\left(x^2+1\right)} \quad \text{(i)}$$

$$let\frac{2x^2+x}{(x+1)\left(x^2+1\right)}=\frac{A}{x+1}+\frac{Bx+c}{x^2+1}$$

A=$1/2$, B=$3/2$, C=-$1/2$

$$y=\int\left(\frac{\frac{1}{2}}{x+1}dx+\frac{\frac{3}{2}x-\frac{1}{2}}{x^2+1}\right)\text{dx} \ \left[from(i)\right.$$

$$y=\frac{1}{2}\ \text{log (x+1)} + \frac{3}{4}\int\frac{2x}{x^2+1}dx-\frac{1}{2}\int\frac{dx}{x^2+1}$$

$$y=\frac{1}{2}\ \text{log (x+1)} + \frac{3}{4}\ \text{log}\left(x^2+1\right)-\frac{1}{2}\tan^{-1}x+c$$

when x = 0, y = 1

$$1=\frac{1}{2}\ \text{log1} + \frac{3}{4}\ \text{log1}-\frac{1}{2}\ \tan^{-1}(0) + c$$

1=c

c=1

$$y=\frac{1}{2}\log(x+1)+\frac{3}{4}\log(x^2+1)-\frac{1}{2}\tan^{-1}x+1$$

Ans 06. $$x\left(x^2-1\right)\frac{dy}{dx}=1$$

$$dy=\frac{1}{x\left(x^2-1\right)}dx$$

$$\int dy=\int\frac{dx}{x(x+1)(x-1)} \quad \text{(i)}$$

$$\frac{let\ 1}{x(x+1)(x-1)}=\frac{A}{x}+\frac{B}{x+1}+\frac{C}{x-1}$$

A=-1, B=$1/2$, C=$1/2$

$$y=\int\left[\frac{-1}{x}+\frac{\frac{1}{2}}{x+1}+\frac{\frac{1}{2}}{x-1}\right]dx \ \left[\text{form (i)}\right.$$

$$y=-\log x+\frac{1}{2}\log(x+1)+\frac{1}{2}\log(x-1)+c$$

$$y=\frac{-1}{2}(2)\log x+\frac{1}{2}\log\left(x^2-1\right)+c$$

$$y=\frac{1}{2}\log\left(\frac{x^2-1}{x^2}\right)+c$$

$$C=0\ \left[\begin{array}{lll} x & = & 0 \\ y & = & 2\end{array}\right.$$

$$y=\frac{1}{2}\log\left(\frac{x^2-1}{x^2}\right)$$

Ans 07. $$\left(1+e^{\frac{x}{y}}\right)dx+e^{\frac{x}{y}}\left(1-\frac{x}{y}\right)dy=0$$

$$\left(1+e^{\frac{x}{y}}\right)dx=-e^{x/y}\left(1-\frac{x}{y}\right)dy$$

$$\frac{dx}{dy}=-\frac{e^{x/y}\left(1-\frac{x}{y}\right)}{1+e^{x/y}} \quad \text{(i)}$$

$$\frac{dy}{dx}=\frac{e^{x/y}\left(\frac{x}{y}-1\right)}{1+e^{x/y}}$$

let x = vy

$$\frac{dx}{dy}=v+y\frac{dv}{dy}$$

$put\ \frac{dx}{dy}$ in eq (1)

$$v+y\frac{dv}{dx}=\frac{e^{v}\left(v-1\right)}{e^{v}+1}$$

$$y\ \frac{dv}{dx}=\frac{ve^{v}-e^{v}}{e^{v}+1}\ \text{-v}$$

$$y\ \frac{dv}{dy}=\frac{\cancel{ve^{v}}-e^{v}-\cancel{ve^{v}}-v}{e^{v}+1}$$

$$-\int\frac{dy}{y}=\int\frac{e^{v}+1}{v+e^{v}}dv$$

log $\left(e^{v}+v\right)=-$ log (y) + c

log $\left(\left(e^{v}+v\right).y\right)=c$

$\left(e^{v}+v\right)y=e^{c}$

$\left(e^{v}+v\right)y=A$

$$\left(e^{x/y}+\frac{x}{y}\right)y=A$$

$ye^{x/y}+x=A$

Ans 08. $\left\{x\ \sin^{2}\ \left(\frac{y}{x}\right)-y\right\}dx+x$ dy=0

$$\text{Sin}^{2}\ \left(\frac{y}{x}\right)-\frac{y}{x}+\frac{dy}{dx}=0$$

$$\frac{dy}{dx}=\frac{y}{x}-\text{Sin}^{2}\ \left(\frac{y}{x}\right)\ \text{(i)}$$

let y = vx

$$\frac{dy}{dx}=v+x\frac{dv}{dx}$$

$put\ \frac{dy}{dx}$ in eq (i)

v + x $\frac{dv}{dx} = v - Sin^2 v$

$\int \text{cosec}^2 \text{ v dv} = \int -\frac{dx}{x}$

$-$ cot v = - log x + c

log x - cot v = c

log x - cot $\left(\frac{y}{x}\right) = c$

when x = 1, y = $\pi/4$

$c = -1$

log x - cot $\left(\frac{y}{x}\right) = -1$

$[-1 = -\log \text{e}$

log ex = cot $\left(\frac{y}{x}\right)$

Ans 09. $\left(1+y^2\right)dx = \left(\tan^{-1} \text{ y - x}\right)dy$

$\frac{dx}{dy} = \frac{\tan^{-1} y - x}{1+y^2}$

$\frac{dx}{dy} = \frac{\tan^{-1} y}{1+y^2} - \frac{x}{1+y^2}, \frac{dx}{dy} + \frac{x}{1+y^2} = \frac{\tan^{-1} y}{1+y^2}$

given diff eq. is of the form

$\frac{dx}{dy} + Px = Q$

$P = \frac{\tan^{-1} y}{1+y^2}$, Q= $\frac{\tan^{-1} y}{1+y^2}$

$I.F = e^{\int P \text{ dy}}$

$= e^{\int \frac{1}{1+y^2} dy}$ = $\text{e}^{\tan^{-1} y}$

$x \times$ I. F = $\int Q.\left(I.F\right)dy + c$

$x.\ \text{e}^{\tan^{-1} y} = \int e^{\tan^{-1} y}\left(\frac{\tan^{-1} y}{1+y^2}\right)dy + c$ (1)

$let\ \tan^{-1}y=t$

$\frac{1}{1+y^2}\ dy=dt$

$I=\int t\ e^t dt$

$= t\ e^t - \int e^t\ dt + c$

$= t\ e^t - e^t + c$

$= \tan^{-1}y\ e^{\tan^{-1}y} - e^{\tan^{-1}y} + c$

$x\ e^{\tan^{-1}y} = \tan^{-1}y.\ e^{\tan^{-1}y} - e^{\tan^{-1}y} + c\ [\text{from (i)}$

$x = \tan^{-1}y - 1 + ce^{-\tan^{-1}y}$

Ans 10. $\left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}\right)\frac{dx}{dy} = 1$

$\frac{dy}{dx} = \frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}$

$\frac{dy}{dx} + \frac{y}{\sqrt{x}} = e^{\frac{-2\sqrt{x}}{\sqrt{x}}}$ (i)

given diff eq is the form of

$\frac{dy}{dx} + Py = Q$

$P = \frac{1}{\sqrt{x}},\ Q=e^{\frac{-2\sqrt{x}}{\sqrt{x}}}$

$I.F = e^{\int \frac{1}{\sqrt{x}}dx}$

$= e^{2\sqrt{x}}$

$y \times e^{2\sqrt{x}} = \int e^{2\sqrt{x}} \times \frac{e^{-2\sqrt{x}}}{\sqrt{x}} dx + c$

$y \times e^{2\sqrt{x}} = 2\sqrt{x} + c$

CBSE TEST PAPER-04
CLASS - XII MATHEMATICS (Calculus: Differential Equations)

1. Find order and degree. $\frac{d^4y}{dx^4}+\sin\left(y'''\right)=0$ [1]

2. Verify that the function is a solution of the corresponding diff eq. [1]

   $y=x^2+2x+c$; y^1-2x-2=0

3. Form a differential equation representing the given family of curve by eliminating arbitrary constants a and b. [4]

   $y=e^{2x}\ \left(a+bx\right)$

4. Form a differential equation representing the given family of curve by eliminating arbitrary constants a and b. [4]

   Y = e^x (a Cos x + b Sin x)

5. Solve the diff eq. [4]

   $\frac{dy}{dx}+\sqrt{\frac{1-y^2}{1-x^2}}=0$

6. Find the equation of the curve passing through the point $\left(o,\frac{\pi}{4}\right)$ whose diff eq. [4]

   is Sin x Cos y dx + Cos x . Sin y dy = 0

7. Solve $\left(\text{x dy - y dx}\right)$ y Sin $\left(\frac{y}{x}\right)=\left(\text{y dx + x dy}\right)x$ Cos $\frac{y}{x}$ [4]

8. Solve the diff eq. [4]

   y $e^{x/y}$ dx = $\left(x\ e^{x/y}+y^2\right)dy$

9. Find a particular solution of the diff eq. $\frac{dy}{dx}+y$ cot x=4x. cosec x [6]

   Given that y = 0 when x = $\pi/2$.

10. Solve the diff eq. [6]

    $Cos^2x\ \frac{dy}{dx}$ + y = tan x

CBSE TEST PAPER-04

CLASS - XII MATHEMATICS (Calculus: Differential Equations)

[ANSWERS]

Ans 01. order = 4

degree = not define

Ans 02. $y^1 = 2x + 2$

$y^1 - 2x - 2 = 0$

Proved

Ans03. $y = e^{2x}\left(a + bx\right)$

$ye^{-2x} = \left(a + bx\right)$

diff

$ye^{-2x}(-2) + e^{-2x}.y^1 = b$

agein diff

$(-2)\left[ye^{-2x}\left(-2\right) + e^{-2x}.y^1\right] + e^{-2x}.y^2 + y^1\left(-2e^{-2x}\right) = 0$

$4ye^{-2x} - 2y^1e^{-2x} + y^2e^{-2x} - 2y^1e^{-2x} = 0$

$e^{-2x}\left(4y - 2y^1 + y^2 - 2y^1\right) = 0$

$y^2 - 4y^1 + 4y = 0$

Ans 04. $y = e^x\left(\text{a Cos x + b Sin x}\right)$ (i)

$ye^{-x} = a$ cos x + b sin x

$ye^{-x}(-1) + e^{-x}.y^1$ = - a sin x + b cos x

again diff

$-1\left(-ye^{-x} + e^{-x}.y^1\right) + \left(e^{-x}y'' + y'\left(-e^{-x}\right)\right) = -a$ cos x - b sin x

$ye^{-x} - e^{-x}y' + e^{-x}y'' - y'e^{-x} = -\left(\frac{y}{e^x}\right)$ $[\because from(i)$

$e^{-x}\left(y - \cancel{y^1} + y'' - \cancel{y^1}\right) = -y\ e^{-x}$

$-y + y' - y'' + y' = y$

$$y'' - 2y^1 + 2y = 0$$

Ans 05. $$\frac{dy}{dx} = -\frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$$

$$\int \frac{dy}{\sqrt{1-y^2}} = \int \frac{-dx}{\sqrt{1-x^2}}$$

$\sin^{-1}(y') + \sin^{-1}(x) = c$

Ans 06. Sin x. Cos y dx = - Cos x. Sin y dy

$$\left|\frac{Sin\ x}{Cos\ x} dx = -\int \frac{\text{Sin y}}{\text{Cos y}} dy\right.$$

$\int$ tan x dx = - $\int$ tan y dy

log (Sec x) = - log (Sec y) + log c

log(*Sec* x. Sec y) = log c

Sec x. Sec y = c (1)

When x = 0, y = $\pi/4$

$C = \sqrt{2}$

put the velue of c in eq (i)

Sec x. Sec y = $\sqrt{2}$

Ans 07. $$(x \text{ dy - y dx}) \text{ y Sin } \left(\frac{y}{x}\right) = (\text{y dx + x dy}) \text{ x Cos } \frac{y}{x}$$

$$\left[\text{xy sin } \left(\frac{y}{x}\right) - x^2 \text{ cos } \left(\frac{y}{x}\right)\right] \text{dy} = \left[\text{xy cos } \frac{y}{x} + y^2 \text{ sin } \frac{y}{x}\right] \text{dx}$$

$$\frac{dy}{dx} = \frac{\text{xy cos } \left(\frac{y}{x}\right) + y^2 \text{ sin } \left(\frac{y}{x}\right)}{\text{xy sin } \left(\frac{y}{x}\right) \text{ - x}^2 \text{ cos } \left(\frac{y}{x}\right)}$$

÷ N ans D by x^2

$$\frac{dy}{dx} = \frac{\frac{y}{x} \text{ cos } \left(\frac{y}{x}\right) + \left(\frac{y^2}{x^2}\right) \text{ sin } \left(\frac{y}{x}\right)}{\frac{y}{x} \text{ sin } \left(\frac{y}{x}\right) \text{ - cos} \left(\frac{y}{x}\right)} \quad (1)$$

let y = vx

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

put $\frac{dy}{dx}$ in eq (1)

$$\text{v+x } \frac{dv}{dx} = \frac{\text{v cos v + v}^2 \text{ sin v}}{\text{v sin v - cos v}}$$

$$x\frac{dv}{dx} = \frac{\text{2v cos v}}{\text{v sin v - cos v}}$$

$$\int\left(\frac{\text{v sin v - cos v}}{\text{v cos v}}\right)\text{dv} = \int\frac{2}{x}\text{dx}$$

$$\int \text{tanv dv} - \int\frac{1}{v}dv = 2\int\frac{1}{x}dx$$

$$\log\ |\sec v| - \log v = 2\log x + \log c$$

$$\log\ \frac{\sec v}{vx^2} = \log c$$

$$\frac{\sec v}{vx^2} = c$$

replace v by $\frac{y}{x}$

$$\frac{\sec\left(\frac{y}{x}\right)}{\frac{y}{\cancel{x}}\cdot x^{\cancel{2}}} = c$$

$$\sec\left(\frac{y}{x}\right) = c\ xy$$

Ans 08. $ye^{x/y}\ \text{dx} = \left(xe^{x/y} + y^2\right)dy$

$$\frac{dx}{dy} = \frac{x\ e^{x/y} + y^2}{y\ e^{x/y}}$$

$$\frac{dx}{dy} = \frac{x}{y} + y\ e^{-x/y}\quad (1)$$

let x = vy

$$\frac{dx}{dy} = v + y\frac{dv}{dy}$$

put $\frac{dx}{dy}$ in eq (1)

$$\cancel{v} + \cancel{y}\frac{dv}{dy} = \cancel{v} + \cancel{y}\, e^{-v}$$

$$\int e^{v}\, dv = \int dy$$

$$e^{v} = y + c$$

$$e^{x/y} = y + c$$

Ans 09. $\frac{dy}{dx} + y \cot x = 4x. \operatorname{cosec} x$

given diff eq is the form of

$$\frac{dy}{dx} + Py = Q$$

$P = cot\ x,$ Q = 4x. cosec x

I. F = $e^{\int Pdx}$

$= e^{\int Cot\ x\ dx}$

$= e^{\log |\text{Sin x}|} = \sin x$

y x sin x = $\int$ 4x. cos ec x. sin x dx + c

$= \int 4x. \frac{1}{\cancel{Sin\ x}}. \cancel{sin\ x}\ dx + c$

y. Sin x = ${}^{2}\cancel{4}\ \frac{x^2}{\cancel{2}} + c$

y. Sin x = 2 x ^{2} + c (1)

When x = $\pi/2$, then y = 0

c = - $\frac{\pi^2}{2}$

y. Sin = 2x^2 - $\frac{\pi^2}{2}$

$$y = \frac{2x^2}{\text{Sin x}} - \frac{\pi^2}{2\ \text{Sin x}}$$

Ans 10. $\text{Cos}^2\ x\ \frac{dy}{dx} + y = \tan x$

$\frac{dy}{dx} +\ \sec^2\ x$ y= Sec^2 x. tan x (1)

given diff eq. is of the form

$$\frac{dy}{dx} + Py = Q$$

$P =$ Sec^2 x, Q = Sec^2 x tan x

I.F= $e^{\int Pdx}$

$= e^{\int \text{Sec}^2 \text{ x dx}}$

$= e^{\tan x}$

$y \times e^{\tan x} = \int \text{Sec}^2 \text{ x. tan x. } e^{\tan x} \text{ dx} + c$ (ii)

$y \times e^{\tan x} = \int e^t \text{ t dt} + c$ when tan x = t , Sec^2 x dx = dt

$y\, e^{\tan x} = t\, e^t - \int e^t.1 \text{ dt} + c$

$= t\, e^t - e^t + c$

$= e^t (t - 1) + c$

$= e^{\tan x} (\tan x - 1) + c$

$$y = \frac{\cancel{e^{\tan x}} (\tan x - 1)}{\cancel{e^{\tan x}}} + \frac{c}{e^{\tan x}}$$

$y = (\tan x - 1) + c\, e^{-\tan x}$

CBSE TEST PAPER-05
CLASS - XII MATHEMATICS (Calculus: Differential Equations)

1. Write the order and degree of the diff equation $y = x\frac{dy}{dx} + a\sqrt{1 + \left(\frac{dy}{dx}\right)^2}$ [1]

2. Verify that the given functions is a solution of the corresponding diff eq. [1]

   $y = \text{Cos x + c ; y}^1 \text{ + Sin x = 0}$

3. Form a differential equation representing the given family of curve by elimination arbitrary Constants a and b. [4]

   $y = a e^{3x} + b e^{-2x}$

4. Form a differential equation representing the given family of curve by elimination arbitrary Constants a and b. [4]

   $y^2 = a\left(b^2 - x^2\right)$

5. Find the particular Solution of the diff. equation [4]

   $\left(1+e^{2x}\right)dy + \left(1+y^2\right)e^x \text{ dx} = 0$ given that y = 1, when x = 0

6. Solve the diff. eq [4]

   $\frac{dy}{dx} + \frac{y^2+y+1}{x^2+x+1} = 0$

7. Solve the diff. eq [4]

   $\frac{dy}{dx} = \frac{x(2y-x)}{x(2y+x)}$, if y = 1 when x = 1

8. Solve the following diff eq. [4]

   $\left(3xy + y^2\right) \text{dx} + (x^2 + xy) \text{ dy} = 0$

9. Solve the following diff. eq. [6]

   $\left(x^2+1\right)\frac{dy}{dx} + 2xy = \sqrt{x^2+4}$

10. Solve the diff. eq. $\frac{dy}{dx} + 2y \tan x = \sin x$ [6]

CBSE TEST PAPER-05

CLASS - XII MATHEMATICS (Calculus: Differential Equations)

[ANSWERS]

Ans 01. $y = x\frac{dy}{dx} + a\sqrt{1+\left(\frac{dy}{dx}\right)^2}$

$$\left(y - x\frac{dy}{dx}\right)^2 = \left(a\sqrt{1+\left(\frac{dy}{dx}\right)^2}\right)^2$$

$$\left(y - x\frac{dy}{dx}\right)^2 = a^2\left[1+\left(\frac{dy}{dx}\right)^2\right]$$

$$y^2 + x^2\left(\frac{dy}{dx}\right)^2 - 2.xy\frac{dy}{dx} = a^2 + a^2\left(\frac{dy}{dx}\right)^2$$

$$\left(x^2 - a^2\right)\left(\frac{dy}{dx}\right)^2 - 2xy\frac{dy}{dx} + y^2 - a^2 = 0$$

$order = 1$

$\deg ree = 2$

Ans 02. y = Cos x + c

$y' = -\text{Sin x}$

$y' + \text{Sin x} = 0$

Ans 03. $y = ae^{3x} + be^{-2x}$

$$\frac{dy}{dx} = a\ 3\ e^{3x}\ -2\ b\ e^{-2x}\quad \text{(i)}$$

$$\frac{d^2y}{dx^2} = 9\ a\ e^{3x}\ +4\ b\ e^{-2x}\quad \text{(ii)}$$

(ii) - (i)

$$\frac{d^2y}{dx2} - \frac{dy}{dx} = 6ae^{3x} + 6be^{-2x}$$

1, $\quad = 6\ \left(ae^{3x} + be^{-2x}\right)$

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} = 6\ y$$

Ans 04. $y^2 = a\left(b^2 - x^2\right)$ (i)

$$2y\frac{dy}{dx} = a\left[0 - 2x\right] \Rightarrow y\frac{dy}{dx} = -ax(ii)$$

again diff.

$$\text{y}\frac{d^2y}{dx^2} + \frac{dy}{dx}.y^1 = -a \text{ (iii)}$$

$$\text{y}\frac{d^2y}{dx^2} + \frac{dy}{dx}y^1 = \frac{y\frac{dy}{dx}}{x} \quad \left[from(ii)\right.$$

$$xy\frac{d^2y}{dx^2} + xy^1\frac{dy}{dx} = y\frac{dy}{dx}$$

Ans 05. $\left(1+e^{2x}\right)dy = -\left(1+y^2\right)e^x$ dx

$$\int\frac{dy}{\left(1+y^2\right)} = -\int\frac{e^x}{1+e^{2x}}dx$$

$\tan^{-1}$ (y) + $\tan^{-1}$ e^x = c

when x = 0, y = 1

c = $\pi/2$

$\tan^{-1}$ y + $\tan^{-1}$ e^x = $\pi/2$

Ans 06. $\frac{dy}{dx} + \frac{y^2+y+1}{x^2+x+1} = 0$

$$\frac{dy}{dx} = -\frac{y^2+y+1}{x^2+x+1}$$

$$\int\frac{dy}{y^2+y+1} = -\int\frac{dx}{x^2+x+1}$$

$$\int\frac{dy}{y^2+y+\left(\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2+1} = -\int\frac{dx}{x^2+x+\left(\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2+1}$$

$$\int\frac{dy}{\left(y+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2} = -\int\frac{dx}{\left(x+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}$$

$$\frac{1}{\frac{\sqrt{3}}{2}}\ \tan^{-1}\left(\frac{y+\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right)=-\frac{1}{\frac{\sqrt{3}}{2}}\ \tan^{-1}\left(\frac{x+\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right)+c$$

$$\tan^{-1}\left(\frac{2y+1}{\sqrt{3}}\right)+\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)=\frac{\sqrt{3}}{2}c$$

$$\tan^{-1}\left[\frac{\frac{2y+1}{\sqrt{3}}+\frac{2x+1}{\sqrt{3}}}{1+\left(\frac{2y+1}{\sqrt{3}}\right)\left(\frac{2x+1}{\sqrt{3}}\right)}\right]=\frac{\sqrt{3}}{2}c$$

$$let\ \frac{\sqrt{3}}{2}c=A_1$$

$$\frac{\frac{2y+1}{\sqrt{3}}+\frac{2x+1}{\sqrt{3}}}{1-\left(\frac{2y+1}{\sqrt{3}}\right)\left(\frac{2x+1}{\sqrt{3}}\right)}=\tan A_1$$

$$\frac{2\sqrt{3}(x+y+1)}{3-(4xy+2x+2y+1)}=\tan\ A_1$$

$$x+y+1=\frac{1}{\sqrt{3}}\tan\ A_1\ (1-x-y-2\ xy)$$

$$x+y+1=A(1-x-y-2\ xy)$$

$$\left[\because\ \frac{1}{\sqrt{3}}\ \tan A_1=A\right.$$

Ans 07. $$\frac{dy}{dx}=\frac{\cancel{x}(2y-x)}{\cancel{x}(2y+x)}\ \text{(i)}$$

if y=1, when x = 1

Let y = v x

$$\frac{dy}{dx}=v+x\frac{dv}{dx}$$

put $\frac{dy}{dx}$ in eq (i)

$$v+x\frac{dv}{dx}=\frac{(2vx-x)}{(2vx+x)}$$

$$v+x\frac{dv}{dx}=\frac{2v-1}{2v+1}$$

$$x\frac{dv}{dx}=-\frac{2v-1}{2v+1}-v$$

$$x\frac{dv}{dx}=\frac{-2v^2+v-1}{2v+1}$$

$$\int\frac{2v+1}{2v^2-v+1}dv=\int-\frac{dx}{x}$$

$$\frac{1}{2}\int\frac{4v+2}{2v^2-v+1}dv=-\int\frac{dx}{x}$$

$$\frac{1}{2}\int\frac{4v+1}{2v^2-v+1}\,dv+\frac{1}{2}\int\frac{3}{2v^2-v+1}dv=\int-\frac{dx}{x}$$

$$\frac{1}{2}\int\frac{4v-1}{2v^2-v+1}dv+\frac{3}{4}\int\frac{dv}{\left(v-\frac{1}{4}\right)^2+\left(\frac{\sqrt{7}}{4}\right)^2}=\int-\frac{dx}{x}$$

$$\frac{1}{2}\ \log\ \left(2v^2-v+1\right)+\frac{3}{4}.\frac{4}{\sqrt{7}}\ \ \text{tan-1}\left(\frac{v-\frac{1}{4}}{\frac{\sqrt{7}}{4}}\right)=-\log x+\log c$$

$$\frac{1}{2}\log\left(2\frac{y^2}{x^2}-\frac{y}{x}+1\right)+\frac{3}{\sqrt{7}}\tan^{-1}\left(\frac{4y-x}{\sqrt{7x}}\right)=-\log x+\log c$$

put x = 1, y = 1

$$\log\ c\frac{1}{2}\log 2+\frac{3}{\sqrt{7}}\tan^{-1}\left(\frac{3}{\sqrt{7}}\right)$$

$$\Rightarrow\frac{1}{2}\log\left(\frac{2y^2-xy+x^2}{x^2}\right)+\frac{3}{\sqrt{7}}\tan^{-1}\left(\frac{4y-x}{\sqrt{7}x}\right)$$

$$=-\log x+\frac{1}{2}\log 2+\frac{3}{\sqrt{7}}\tan^{-1}\frac{3}{\sqrt{7}}$$

Ans 08. $\left(3xy+y^2\right)dx+\left(x^2+xy\right)dy=0$

$$\left(3xy+y^2\right)dx=-\left(x^2+xy\right)dy$$

$$\frac{dy}{dx}=-\frac{\left(3xy+y^2\right)}{x^2+xy}(i)$$

let y=v x

$$\frac{dy}{dx}=v.1+x.\frac{dv}{dx}$$

put $\frac{dy}{dx}$ in rq (i)

$$v+x\frac{dv}{dx}=-v\frac{(3+v)}{1+v}$$

$$\int\frac{v+1}{v^2+2v}.dv=-2\int\frac{dx}{x}$$

$$\frac{1}{2}\int\frac{2v+2}{v^2+2v}dv=-\int\frac{dx}{x}$$

$$\frac{1}{2}\log\ \left(v^2+2v\right)=-\log x+\frac{1}{2}\log c$$

$$\log\left|\left(v^2+2v\right)x^2\right|=\log c$$

$$y^2+2xy=c$$

Ans 09. $$\left(x^2+1\right)\frac{dy}{dx}+2xy=\sqrt{x^2+4}$$

$$\frac{dy}{dx}+\frac{2xy}{x^2+1}=\frac{\sqrt{x^2+4}}{x^2+1}$$

given diff. eq is the form of

$$\frac{dy}{dx}+Py=Q$$

$$P=\frac{2x}{x^2+1},\ Q=\frac{\sqrt{x^2+4}}{x^2+1}$$

$$I.F=e^{\int\frac{2x}{x^2+1}dx}$$

$$=e^{\log\left(x^2+1\right)}=x^2+1$$

$$y\ \text{x}\ \left(x^2+1\right)=\int\frac{\sqrt{x^2+4}}{\cancel{x^2+1}}\times\left(\cancel{x^2+1}\right)dx+c$$

$$y\left(x^2+1\right)=\int\sqrt{x^2+4}\ \ dx+c$$

$$y\left(x^2+1\right)=\frac{1}{2}\left[x\sqrt{x^2+4}+4\log\ x+\sqrt{x^2+4}\right]+c$$

Ans 10. $$\frac{dy}{dx}+2y\ \tan x=\sin x$$

Given diff eq is the form of

$$\frac{dy}{dx}+Py=Q$$

P = 2 tan x, Q = Sin x

$$\text{I. F.} = e^{\int P\, dx}$$

$$= e^{\int 2\tan x\, dx}$$

$$= e^{2\log \text{Sec x}}$$

$$= e^{\log \text{Sec}^2 x}$$

$$= sec^2 x$$

$$y \times sec^2 x = \int sin\ x\ \sec^2 x\ dx + c$$

$$= \int \sec x. \tan x\ dx + c$$

$$y \times \sec^2 x = \sec x + c$$

$$y = \frac{sec\ x + c}{\sec^2 x}$$

$$y = \frac{1}{sec\ x} + \frac{c}{\sec^2 x}$$

$$y = \cos x + c. \cos^2 x$$

CBSE TEST PAPER-01

CLASS - XII MATHEMATICS (Vectors & Three Dimensional Geometry)

<u>Topic: - Vectors</u>

1. Is the measure of 5 seconds is scalar or vector? [1]

2. Find the sum of the vectors. [1]

   $\vec{a} = \vec{i} - 2\vec{j} + \vec{k}, \quad \vec{b} = -2\vec{i} + 4\vec{j} + 5\vec{k} \quad \vec{c} = \hat{i} - 6\hat{j} - 7\hat{k}$

3. Find the direction ratios and the direction cosines of the vector $\vec{r} = 2\hat{i} - 7\hat{j} - 3\hat{k}$ [1]

4. Find the angle between vectors $\vec{a}$ *and* $\vec{b}$ *if* $|\vec{a}| = \sqrt{3}, \ |\vec{b}| = 2 \quad \vec{a}.\vec{b} = \sqrt{6}$ [1]

5. Vectors $\vec{a}$ *and* $\vec{b}$ be such that $|\vec{a}| = 3,$ *and* $|\vec{b}| = \frac{\sqrt{2}}{3}$, then $\vec{a} \times \vec{b}$ is a unit vector. Find angle between $\vec{a}$ *and* $\vec{b}$. [1]

6. Find the unit vector in the direction of the sum of the vectors $\vec{a} = 2\hat{i} + 2\hat{j} - 5\hat{k}, \ \vec{b} = 2\hat{i} + \hat{j} + 3\hat{k}$ [4]

7. Show that the points $A\left(2\hat{i} - \hat{j} + \hat{k}\right), B\left(\hat{i} - 3\hat{j} - 5\hat{k}\right), C\left(3\hat{i} - 4\hat{j} - 4\hat{k}\right)$ are the vertices of right angled triangle. [4]

8. Show that the points $A\left(-2\hat{i} + 3\hat{j} + 5\hat{k}\right), B\left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$ *and* $C\left(7\hat{i} - \hat{k}\right)$ are collinear. [4]

9. If $\vec{a}, \vec{b}, \vec{c}$ are unit vector such that $\vec{a} + \vec{b} + \vec{c} = 0$ find the value of $\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}$ [4]

10. If $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$, $\vec{c} = 3\hat{i} + \hat{j}$ are such that $\vec{a} + \lambda\vec{b}$ *is* $\perp$ *to* $\vec{c}$ is then find the value of λ. [4]

CBSE TEST PAPER-01

CLASS - XII MATHEMATICS (Vectors & Three Dimensional Geometry)

<u>Topic: - Vectors</u> [ANSWERS]

Ans1. Scalar

Ans2. $\vec{d} = \vec{a} + \vec{b} + \vec{c}$

$= 0\hat{i} - 4\hat{j} - \hat{k}$

Ans3. D.R of $\vec{r}$ *are* $2, -7, -3$

$\left|\vec{r}\right| = \sqrt{4+49+9} = \sqrt{62}$

D.C of $\vec{r}$ *are* $\frac{2}{\sqrt{62}}, \frac{-7}{\sqrt{62}}, \frac{-3}{\sqrt{62}}$

Ans4. $\cos\theta = \frac{\vec{a}.\vec{b}}{\left|\vec{a}\right|\left|\vec{b}\right|}$

$= \frac{\sqrt{6}}{\left(\sqrt{3}\right).(2)} = \frac{\sqrt{2}\times\cancel{\sqrt{3}}}{\cancel{\sqrt{3}}.2} = \frac{\cancel{\sqrt{2}}}{\cancel{\sqrt{2}}.\sqrt{2}}$

$\cos\theta = \frac{1}{\sqrt{2}} \quad \theta = \pi/4$

Ans5. $\left|\vec{a}\times\vec{b}\right| = \left|\vec{a}\right|\left|\vec{b}\right|\sin\theta$

$1 = \cancel{3}\times\frac{\sqrt{2}}{\cancel{3}}\times\sin\theta$

$\frac{1}{\sqrt{2}} = \sin\theta$

$\theta = \pi/4$

Ans6. Let $\vec{c}=\vec{a}+\vec{b}$

$$=\left(2\hat{i}+2\hat{j}-5\hat{k}\right)+\left(2\hat{i}+\hat{j}+3\hat{k}\right)$$

$$=4\hat{i}+3\hat{j}-2\hat{k}$$

$$\left|\vec{c}\right|=\sqrt{16+9+4}$$

$$=\sqrt{29}$$

The required unit vector is

$$\hat{c}=\frac{\vec{c}}{\left|\vec{c}\right|}$$

$$=\frac{4\hat{i}+3\hat{j}-2\hat{k}}{\sqrt{29}}$$

$$=\frac{4}{\sqrt{29}}\hat{i}+\frac{3}{\sqrt{29}}\hat{j}-\frac{2}{\sqrt{29}}\hat{k}$$

Ans7. $\overrightarrow{AB}=-\hat{i}-2\hat{j}-6\hat{k}$

$$\overrightarrow{BC}=2\hat{i}-\hat{j}+\hat{k}$$

$$\overrightarrow{CA}=-\hat{i}+3\hat{j}+5\hat{k}$$

$$\left|\overrightarrow{AB}\right|^2=41$$

$$\left|\overrightarrow{BC}\right|=6$$

$$\left|\overrightarrow{CA}\right|=35$$

$$\left|\overrightarrow{AB}\right|^2=\left|\overrightarrow{BC}\right|^2+\left|\overrightarrow{CA}\right|^2$$

Hence, the Δ is a right angled triangle.

Ans8. $\overrightarrow{AB}=3\hat{i}-\hat{j}-2\hat{k}$

$$\overrightarrow{BC}=6\hat{i}-2\hat{j}-4\hat{k}$$

$$\overrightarrow{CA}=9\hat{i}-3\hat{j}-6\hat{k}$$

$$\left|\overrightarrow{AB}\right|=\sqrt{14},\overrightarrow{BC}=2\sqrt{14}$$

$and \left|\overrightarrow{AC}\right| = 3\sqrt{14}$

$\left|\overrightarrow{AC}\right| = \left|\overrightarrow{AB}\right| + \left|\overrightarrow{BC}\right|$

Hence points A, B, C are collinear.

Ans9. $\left|\vec{a}\right| = 1, \left|\vec{b}\right| = 1, \left|\vec{c}\right| = 1,$

$\vec{a} + \vec{b} + \vec{c} = 0 \quad (Given)$

$\vec{a}.\left(\vec{a} + \vec{b} + \vec{c}\right)$

$\vec{a}.\vec{a} + \vec{a}.\vec{b} + \vec{a}.\vec{c} = 0$

$\left(\vec{a}\right)^2 + \vec{a}.\vec{b} + \vec{a}.\vec{c} = 0$

$1 + \vec{a}.\vec{b} + \vec{a}.\vec{c} = 0$

$\vec{a}.\vec{b} + \vec{a}.\vec{c} = -1 - - - - - - - - (i)$

similiorly

$\vec{b}.\vec{a} + \vec{b}.\vec{c} = -1 - - - - - - - - (ii)$

again

$\vec{c}.\vec{a} + \vec{c}.\vec{b} = -1 - - - - - - - - (iii)$

$adding (i), (ii) and (iii)$

$2\left(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}\right) = -3 \qquad \left[\vec{a}.\vec{b} = \vec{b}.\vec{a}\right.$

$\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a} = -3/2$

Ans10. $\vec{a} + \lambda\vec{b} = \left(2\hat{i} + 2\hat{j} + 3\hat{k}\right) + \lambda\left(-\hat{i} + 2\hat{j} + \hat{k}\right)$

$= (2 - \lambda)\hat{i} + (2 + 2\lambda)\hat{j} + (3 + \lambda)\hat{k}$

$\left(\vec{a} + \lambda\vec{b}\right).\vec{c} = 0 \left[\because \vec{a} + \lambda\vec{b} \perp \vec{c}\right.$

$\left[(2 - \lambda)\hat{i} + (2 + 2\lambda)\hat{j} + (3 + \lambda)\hat{k}\right].\left(3\hat{i} + \hat{j}\right) = 0$

$3(2 - \lambda) + (2 + 2\lambda) = 0$

$-\lambda = -8$

$\lambda = 8$

CBSE TEST PAPER-02

CLASS - XII MATHEMATICS (Vectors & Three Dimensional Geometry)

Topic: - Vectors

1. Is the measure of 10 Newton is scalar or vector. [1]

2. Write two different vectors having same magnitude. [1]

3. Find the direction ratios and the direction cosines of the vector $\vec{r} = \hat{i} + \hat{j} + \hat{k}$ [1]

4. Find $\left|\vec{a} - \vec{b}\right|$ *if* $\left|\vec{a}\right| = 2,\ \left|\vec{b}\right| = 3$ *and* $\vec{a}.\vec{b} = 4$ [1]

5. If $\vec{a} = 4\hat{i} + 3\hat{j} + 2\hat{k}\ \vec{b} = 3\hat{i} + 2\hat{k}$ *find* $\left|\vec{b} \times 2\vec{a}\right|$ [1]

6. Consider two point P and Q with position vectors $\overrightarrow{OP} = 3\vec{a} - 2\vec{b}$ *and* $\overrightarrow{OQ} = \vec{a} + \vec{b}$. Find the positions vector of a point R which divides the line joining P and Q in the ratio 2:1 (i) internally (ii) externally. [4]

7. Show that the points A, B, C with position vectors $-2\vec{a} + 3\vec{b} + 5\vec{c},\ \vec{a} + 2\vec{b} + 3\vec{c}$ *and* $7\vec{a} - \vec{c}$ respectively are collinear. [4]

8. Find a unit vector $\perp$ to each of the vectors $\left(\vec{a} + \vec{b}\right)$ *and* $\left(\vec{a} - \vec{b}\right)$ *where* $\vec{a} = \hat{i} + \hat{j} + \hat{k},\ \vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$ [4]

9. The scalar product of the vector $\hat{i} + \hat{j} + \hat{k}$ with a unit vector along the sum of vectors $2\hat{i} + 4\hat{j} - 5\hat{k}$, and $\lambda\hat{i} + 2\hat{j} + 3\hat{k}$ is equal the one. Find the value of λ [4]

10. Find the area of the Δ with vertices A (1, 1, 2) B (2, 3, 5) and C (1, 5, 5). [4]

CBSE TEST PAPER-02

CLASS - XII MATHEMATICS (Vectors & Three Dimensional Geometry)

Topic: - Vectors [ANSWERS]

Ans1. Vector

Ans2. $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$

$$\left|\vec{a}\right| = \sqrt{1+4+9} = \sqrt{14}$$
$$\vec{b} = 3\hat{i} + 2\hat{j} + 1\hat{k}$$
$$\left|\vec{b}\right| = \sqrt{9+4+1} = \sqrt{14}$$

Ans3. D.R of $\vec{r}$ *are* 1,1,1

$$\vec{r} = \sqrt{1+1+1} = \sqrt{3}$$
$$D.C \ of \ are \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$$

Ans4. $\left|\vec{a} - \vec{b}\right|^2 = \left(\vec{a} - \vec{b}\right).\left(\vec{a} - \vec{b}\right)$

$$= \vec{a}.\vec{a} - \vec{a}.\vec{b} - \vec{b}.\vec{a} + \vec{b}.\vec{b}$$
$$= \left|\vec{a}\right| - 2\vec{a}.\vec{b} + \left|\vec{b}\right|^2$$
$$= 4 - 2\times 4 + 9$$
$$= 5$$
$$\left|\vec{a} - \vec{b}\right| = \sqrt{5}$$

Ans5. $\vec{b} = 3\hat{i} + 2\hat{k}$

$$2\vec{a} = 8\hat{i} + 6\hat{j} + 4\hat{k}$$

$$\vec{b}\times 2\vec{a} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 0 & 2 \\ 8 & 6 & 4 \end{vmatrix}$$

$$= \hat{i}(0-12) - \hat{j}(12-16) + \hat{k}(18-0)$$

$$-12\hat{i} + 4\hat{j} + 18\hat{k}$$

$$\left|\vec{b}\times 2\vec{a}\right| = \sqrt{(-12)^2 + (4)^2 + (18)^2}$$

$$= \sqrt{484}$$

$$= 22$$

Ans6. (i) $O\vec{R} = \dfrac{2(\vec{a}+\vec{b}) + 1(3\vec{a}-2\vec{b})}{2+1}$

$$= \frac{5\vec{a}}{3}$$

(ii) $O\vec{R} = \dfrac{2(\vec{a}+\vec{b}) - (3\vec{a}-2\vec{b})}{2-1}$

$$= \frac{2\vec{a} + 2\vec{b} - 3\vec{a} + 2\vec{b}}{1}$$

$$= 4\vec{b} - \vec{a}$$

Ans7. $O\vec{A} = -2\vec{a} + 3\vec{b} + 5\vec{c}$

$$O\vec{B} = \vec{a} + 2\vec{b} + 3\vec{c}$$

$$O\vec{C} = 7\vec{a} - \vec{c}$$

$$O\vec{A} = O\vec{B} - O\vec{A} = 3\vec{a} - \vec{b} - 2\vec{c}$$

$$\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB} = 6\vec{a} - 2\vec{b} - 4\vec{c}$$

$$= 2(3\vec{a} - \vec{b} - 2\vec{c})$$

$$\overrightarrow{BC} = 2\overrightarrow{AB}$$

Thus $\overrightarrow{AB} \parallel \overrightarrow{BC}$ but one point B is common to both vectors hence A, B, C are collinear.

Ans8. A vector which is $\perp$ to both $(\vec{a}+\vec{b})$ are $(\vec{a}-\vec{b})$ is giving by

$$(\vec{a}+\vec{b})\times(\vec{a}-\vec{b})=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 3 & 4\\ 0 & -1 & -2\end{vmatrix}$$

$$=-2\hat{i}+4\hat{j}-2\hat{k}$$

$$Let\ \vec{c}=-2\hat{i}+4\hat{j}-2\hat{k}$$

$$|\vec{c}|=\sqrt{4+16+4}$$

$$=\sqrt{24}$$

$$=2\sqrt{6}$$

Req. unit vector is

$$\frac{\vec{c}}{|\vec{c}|}=-\frac{1}{\sqrt{6}}\hat{i}+\frac{2}{\sqrt{6}}\hat{j}-\frac{1}{\sqrt{6}}\hat{k}$$

Ans9. $\vec{a}=2\hat{i}+4\hat{j}-5\hat{k}$

$$\vec{b}=\lambda\hat{i}+\hat{j}+3\hat{k}$$

$$\vec{a}+\vec{b}=(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}$$

Unit vector along

$$\vec{a}+\vec{b}=\frac{\vec{a}+\vec{b}}{|\vec{a}+\vec{b}|}$$

$$=\frac{(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{(2+\lambda)^2+\ (6)^2\ +(-2)^2}}$$

$$=\frac{(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{(2+\lambda)^2+\ 40}}$$

$$ATQ\ \vec{c}.(\vec{a}+\vec{b})=1$$

$$(\hat{i}+\hat{j}+\hat{k}).\left(\frac{(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}}{(2+\lambda)^2+40}\right)=1$$

$$\frac{(2+\lambda)+6-2}{\sqrt{(2+\lambda)^2+40}}=1$$

$$2+\lambda+4=\sqrt{(2+\lambda)^2+40}$$

sq.both site

$$\lambda^2+36+12\lambda=(2+\lambda)^2+40$$

$$\lambda=1$$

Ans10. A (1, 1, 2) B(2, 3, 5) C (1, 5, 5)

$$\overrightarrow{OA}=\hat{i}+\hat{j}+2\hat{k}$$

$$\overrightarrow{OB}=2\hat{i}+3\hat{j}+5\hat{k}$$

$$\overrightarrow{OC}=\hat{i}+5\hat{j}+5\hat{k}$$

$$\overrightarrow{OA}=\overrightarrow{OB}-\overrightarrow{OA}=\hat{i}+2\hat{j}+2\hat{k}$$

$$\overrightarrow{AC}=\overrightarrow{OC}-\overrightarrow{OA}=4\hat{j}+3\hat{k}$$

$$\overrightarrow{AB}\times\overrightarrow{AC}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 2 & 2\\ 0 & 4 & 3\end{vmatrix}$$

$$=-2\hat{i}-3\hat{j}+4\hat{k}$$

$$Arae\ of\ \Delta ABC=\frac{1}{2}\left|\overrightarrow{AB}\times\overrightarrow{AC}\right|$$

$$=\frac{1}{2}\left|\sqrt{(-2)^2+(-3)^2+(4)^2}\right|$$

$$=\frac{1}{2}\sqrt{29}\ sq.unit$$

CBSE TEST PAPER-03
CLASS - XII MATHEMATICS (Vectors & Three Dimensional Geometry)

Topic: - Vectors

1. Is the measure of 20 m/s towards north is scalar or vector. [1]

2. $\vec{a}=\hat{i}+2\hat{j}\ \vec{b}=2\hat{i}+\hat{j}\ Is\left|\vec{a}\right|=\left|\vec{b}\right|$ [1]

3. Find the direction ratios and the direction cosines of the vector $\vec{r}=\hat{i}+2\hat{j}+3\hat{k}$ [1]

4. Evaluate the product $\left(3\vec{a}-5\vec{b}\right).\left(2\vec{a}+7\vec{b}\right)$ [1]

5. Find $\vec{a}\times\vec{b}$ if $\vec{a}=2\hat{i}+\hat{j}+3\hat{k},\ \vec{b}=3\hat{i}+5\hat{j}-2\hat{k}$ [1]

6. Show that the points A (1, -2, -8) B (5, 0, -2) and C (11, 3, 7) are collinear, and find the ratio in which B divides AC. [4]

7. Find a vector $\vec{d}$ which is $\perp$ to both $\vec{a}$ and $\vec{b}$ and $\vec{c}\ .\vec{d}$ =15 [4]

   Let $\vec{a}=\hat{i}+4\hat{j}+2\hat{k}$

   $\vec{b}=3\hat{i}-2\hat{j}+7\hat{k}$

   $\vec{c}=2\hat{i}-\hat{j}+4\hat{k}$

8. Let $\vec{a},\vec{b}\ and\ \vec{c}$ be three vectors such that $\left|\vec{a}\right|=3,\ \left|\vec{b}\right|=4,\ \left|\vec{c}\right|=5$ and each one of them being $\perp$ to the sum of the other two, find $\left|\vec{a}+\vec{b}+\vec{c}\right|$ [4]

9. If $\vec{a}=4\hat{i}+2\hat{j}-\hat{k}$ [4]

   $\vec{b}=5\hat{i}+2\hat{j}-3\hat{k}$

   Find the angel between the vectors $\vec{a}+\vec{b}\ and\ \vec{a}-\vec{b}$

10. Find the sine of the angel between the vectors. [4]

   $\vec{a}=2\hat{i}-\hat{j}+3\hat{k}$

   $\vec{b}=\hat{i}+3\hat{j}+2\hat{k}$

CBSE TEST PAPER-03

CLASS - XII MATHEMATICS (Vectors & Three Dimensional Geometry)

<u>Topic: - Vectors</u> [ANSWERS]

Ans1. Vector

Ans2. $\left|\vec{a}\right| = \sqrt{(1)^2 + (2)^2} = \sqrt{5}$

$\left|\vec{b}\right| = \sqrt{(2)^2 + (1)^2} = \sqrt{5}$

Ans3. D.R of $\vec{r}$ *are* $1, 2, 3$

$\left|\vec{r}\right| = \sqrt{1+4+9} = \sqrt{4}$

D.C of $\vec{r}$ *are* $\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}$

Ans4. $\left(3\vec{a} - 5\vec{b}\right).\left(2\vec{a} + 7\vec{b}\right)$

$= 6\left|\vec{a}\right|^2 - 11\vec{a}.\vec{b} - 35\left|\vec{b}\right|^2$

$\left[\because \vec{a}.\vec{a} = \left|\vec{a}\right|^2 \quad \vec{b}.\vec{b} = \left|\vec{b}\right|^2 \quad \vec{a}.\vec{b} = \vec{b}.\vec{a}\right.$

Ans5. $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 3 \\ 3 & 5 & -2 \end{vmatrix}$

$= \hat{i}(-2-15) - \hat{j}(-4-9) + \hat{k}(10-3)$

$= -17\hat{i} + 13\hat{j} + 7\hat{k}$

Ans6. A (1, -2, -8), B (5, 0, -2), C (11, 3, 7)

$\overrightarrow{OA} = 1\hat{i} - 2\hat{j} - 8\hat{k}$

$\overrightarrow{OB} = 5\hat{i} - 0\hat{j} - 2\hat{k}$

$\overrightarrow{OC} = 11\hat{i} + 3\hat{j} + 7\hat{k}$

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$= 4\hat{i} + 2\hat{j} + 6\hat{k}$

$$\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB}$$

$$= 3\left(2\hat{i} + \hat{j} + 3\hat{k}\right)$$

$$= \frac{3}{2}\left(4\hat{i} + 2\hat{j} + 6\hat{k}\right)$$

$$\overrightarrow{BC} = \frac{3}{2}\overrightarrow{AB}$$

Thus $\overrightarrow{BC} \parallel \overrightarrow{AB}$ and one point B is common there fore A, B, C are collinear.

Ans7. $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$

$\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$

$\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$

$Let\ \vec{d} = x\hat{i} + y\hat{j} + z\hat{k}$

$ATQ\ \vec{d}.\vec{a} = 0,\ \vec{d}.\vec{b} = 0\ and\ \vec{c}.\vec{d} = 15$

$x + 4y + 2z = 0 ---------(1)$

$3x - 2y + 7z = 0 --------(2)$

$2x - y + 4z = 15 --------(3)$

On solving equation (i) and (ii)

$$\frac{x}{\begin{array}{cc}4 & 2\\ -2 & 7\end{array}} = \frac{y}{\begin{array}{cc}2 & 1\\ 7 & 3\end{array}} = \frac{z}{\begin{array}{cc}1 & 4\\ 3 & -2\end{array}} = K$$

$$\frac{x}{28+4} = \frac{y}{6-7} = \frac{z}{-2-12} = K$$

$x = 32k, y = -k, z = -14k$

Put x, y, z in equation (iii)

$2(32k) - (-k) + 4(-14k) = 15$

$64k + k - 56k = 15$

$9k = 15$

$$k = \frac{15}{9}$$

$$k = \frac{5}{3}$$

$$x = 32 \times \frac{5}{3} = \frac{160}{3}$$

$$y = -\frac{5}{3} = -\frac{5}{3}$$

$$z = -14 \times \frac{5}{3} = -\frac{70}{3}$$

$$\vec{d} = \frac{160}{3}\hat{i} - \frac{5}{3}\hat{j} - \frac{70}{3}\hat{k}$$

Ans8. $\vec{a}.\left(\vec{b}+\vec{c}\right) = 0,\ \vec{b}.\left(\vec{c}+\vec{a}\right) = 0\ \vec{c}.\left(\vec{a}+\vec{b}\right) = 0, (Given)$

$$\left|\vec{a}+\vec{b}+\vec{c}\right|^2 = \left(\vec{a}+\vec{b}+\vec{c}\right).\left(\vec{a}+\vec{b}+\vec{c}\right)$$

$$= \vec{a}.\vec{a} + \vec{a}.\left(\vec{b}+\vec{c}\right) + \vec{b}.\vec{b} + \vec{b}.\left(\vec{a}+\vec{c}\right) + \vec{c}.\vec{c} + \left(\vec{a}+\vec{b}\right)$$

$$= \left|\vec{a}\right|^2 + \left|\vec{b}\right|^2 + \left|\vec{c}\right|^2$$

$$= 9 + 16 + 25$$
$$= 50$$

$$\left|\vec{a}+\vec{b}+\vec{c}\right| = \sqrt{50}$$

$$= 5\sqrt{2}$$

Ans9. $\vec{a}+\vec{b} = 9\hat{i} + 4\hat{j} - 4\hat{k}$

$$\vec{a}-\vec{b} = -\hat{i} + 2\hat{k}$$

$$\left(\vec{a}+\vec{b}\right).\left(\vec{a}-\vec{b}\right) = -17$$

$$\left|\vec{a}+\vec{b}\right| = \sqrt{113}$$

$$\left|\vec{a}-\vec{b}\right| = \sqrt{5}$$

$$\cos\theta = \frac{\left(\vec{a}+\vec{b}\right).\left(\vec{a}-\vec{b}\right)}{\left|\vec{a}+\vec{b}\right|\left|\vec{a}-\vec{b}\right|}$$

$$= \frac{-17}{\sqrt{113}.\sqrt{5}}$$

$$\cos\theta = \frac{-17}{\sqrt{565}}$$

$$\theta = \cos^{-1}\left(\frac{-17}{\sqrt{565}}\right)$$

Ans10. $\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 3 \\ 1 & 3 & 2 \end{vmatrix}$

$$= -11\hat{i} - \hat{j} + 7\hat{k}$$

$$\left|\vec{a}\times\vec{b}\right| = \sqrt{(-11)^2 + (-1)^2 + (7)^2}$$

$$= \sqrt{171} = 3\sqrt{19}$$

$$\sin\theta = \frac{\left|\vec{a}\times\vec{b}\right|}{\left|\vec{a}\right|\left|\vec{b}\right|} = \frac{3\sqrt{19}}{\sqrt{14}.\sqrt{14}} = \frac{3}{14}\sqrt{19}$$

CBSE TEST PAPER-04

CLASS - XII MATHEMATICS (Vectors & Three Dimensional Geometry)

Topic: - Vectors

1. Is the measure of 30 m/s towards north is scalar or vector. [1]

2. Complete the magnitude. $\vec{b}=2\hat{i}-7\hat{j}-3\hat{k}$ [1]

3. Find the direction ratios and the direction cosines of the vector $\vec{r}=\hat{i}+2\hat{j}-\hat{k}$ [1]

4. $\vec{a}$ Is unit vector and $\left(\vec{x}-\vec{a}\right)\left(\vec{x}+\vec{a}\right)=8$, Then find $\left|\vec{x}\right|$ [1]

5. Show that $\left(\vec{a}-\vec{b}\right)\times\left(\vec{a}+\vec{b}\right)=2\left(\vec{a}\times\vec{b}\right)$ $\vec{a}$ *and* $\vec{b}$. [1]

6. Three vectors $\vec{a},\vec{b}$ *and* $\vec{c}$ satisfy the condition $\vec{a}+\vec{b}+\vec{c}=0$ Evaluate the quantity $\mu=\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}$ if $\left|\vec{a}\right|=1,\ \left|\vec{b}\right|=4,\ \left|\vec{c}\right|=2$ [4]

7. If with reference to the right handed system of mutually $\perp$ unit vectors $\hat{i}$, $\hat{j}$, $\hat{k}$ *and* $\hat{k}$, $\vec{\alpha}=3\hat{i}-\hat{j}$, $\vec{\beta}=2\hat{i}+\hat{j}-3\hat{k}$ then express $\vec{\beta}$ in the form $\vec{\beta}=\vec{\beta_1}+\vec{\beta_2}$, where $\vec{\beta_1}$ is || to $\vec{\alpha}$ and $\vec{\beta_2}$ is $\perp$ to $\vec{\alpha}$ [4]

8. If $\vec{a},\vec{b}$ *and* $\vec{c}$ be three vectors such that $\vec{a}+\vec{b}+\vec{c}=0$ and $\left|\vec{a}\right|=3,\ \left|\vec{b}\right|=5,\ \left|\vec{c}\right|=7$ find the angle between $\vec{a}$ *and* $\vec{b}$. [4]

9. Find the area of the ||gm whose adjacent sides are represented by the vectors, $\vec{a}=3\hat{i}+\hat{j}-2\hat{k},\ \vec{b}=\hat{i}-3\hat{j}+4\hat{k}$ [4]

10. Find the vector joining the points P (2, 3, 0) and Q (-1, -2, -4) directed from P to Q. Also find direction ratio and direction cosine. [4]

CBSE TEST PAPER-04

CLASS - XII MATHEMATICS (Vectors & Three Dimensional Geometry)

Topic: - Vectors [ANSWERS]

Ans1. Scalar

Ans2. $\left|\vec{b}\right| = \sqrt{(2)^2 + (-7)^2 + (-3)^2}$

$= \sqrt{4+49+9}$

$= \sqrt{62}$

Ans3. D.R of $\vec{r}$ *are* $1, 2, -1$

$\vec{r} = \sqrt{1+4+1} = \sqrt{6}$

D.C of $\vec{r}$ *are* $\frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}, \frac{-1}{\sqrt{6}}$

Ans4. $\left|\vec{a}\right| = 1$

$\left(\vec{x} - \vec{a}\right).\left(\vec{x} + \vec{a}\right) = 8$

$\left|\vec{x}\right|^2 - 1 = 8$

$\left|\vec{x}\right|^2 = 9$

$\left|\vec{x}\right|^2 = 3$

Ans5. $L.H.S = \left(\vec{a} - \vec{b}\right) \times \left(\vec{a} + \vec{b}\right)$

$= \vec{a} \times \vec{a} + \vec{a} \times \vec{b} - \vec{b} \times \vec{a} - \vec{b} \times \vec{b}$

$= 0 + \vec{a} \times \vec{b} + \vec{a} \times \vec{b} - 0$

$= 2\left(\vec{a} \times \vec{b}\right) \left[\begin{array}{l} \because \vec{a} \times \vec{b} = 0 \\ \vec{a} \times \vec{b} = -\vec{b} \times \vec{a} \end{array}\right.$

Ans6. $\vec{a} + \vec{b} + \vec{c} = 0$

$\vec{a}.\left(\vec{a} + \vec{b} + \vec{c}\right) = 0$

$\vec{a}.\vec{a} + \vec{a}.\vec{b} + \vec{a}.\vec{c} = 0$

$\vec{a}.\vec{b}+\vec{a}.\vec{c}=-\left|\vec{a}\right|^2$

$\vec{a}.\vec{b}+\vec{a}.\vec{c}=-1--------(i)$

$\vec{b}.\vec{a}+\vec{b}.\vec{c}=-16------(ii)$

$\vec{a}.\vec{c}+\vec{b}.\vec{c}=-4------(iii)$

Adding (i) (ii) and (iii)

$2\left(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{a}.\vec{c}\right)=-21$

$\mu=\frac{-21}{2}$

Ans7. $Let\ \vec{\beta}_1=\lambda\vec{\alpha}$ $\left[\because \vec{\beta}_1 \parallel to \vec{\alpha}\right.$

$\vec{\beta}_1=\lambda\left(3\hat{i}-\hat{j}\right)$

$=3\lambda\hat{i}-\lambda\hat{j}$

$\vec{\beta}_2=\vec{\beta}-\vec{\beta}_1$

$=\left(2\hat{i}+\hat{j}-3\hat{k}\right)-\left(3\lambda\hat{i}-\lambda\hat{j}\right)$

$=(2-3\lambda)\hat{i}+(1+\lambda)\hat{j}-3\hat{k}$

$\vec{\alpha}.\vec{\beta}_2=0$ $\left[\because \vec{\beta}_2 \perp \vec{\alpha}\right.$

$3(2-3\lambda)-(1+\lambda)=0$

$\lambda=\frac{1}{2}$

$\vec{\beta}_1=\frac{3}{2}\hat{i}-\frac{1}{2}\hat{j}$

$\vec{\beta}_2=\frac{1}{2}\hat{i}+\frac{3}{2}\hat{j}-3\hat{k}$

Ans8. $\vec{a}+\vec{b}+\vec{c}=0$

$\vec{a}+\vec{b}=-\vec{c}$

$\left(\vec{a}+\vec{b}\right).\left(-\vec{c}\right)=-\vec{c}.\left(-\vec{c}\right)$

$\left(\vec{a}+\vec{b}\right).\left(\vec{a}+\vec{b}\right)=\vec{c}.\vec{c}$

$\left|\vec{a}\right|^2+2\vec{a}\vec{b}+\left|\vec{b}\right|^2=\left|\vec{c}\right|^2$

$\vec{a}.\vec{b}=\frac{49-9-25}{2}=\frac{15}{2}$

$$\cos\theta = \frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|}$$

$$= \frac{1}{2}$$

$$\theta = 60$$

Ans9. $\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -2 \\ 1 & -3 & 4 \end{vmatrix}$

$$= -2\hat{i} - 14\hat{j} - 10\hat{k}$$

$$req.area = |\vec{a}\times\vec{b}|$$

$$= \sqrt{(-2)^2 + (-14)^2 + (-10)^2} = 10\sqrt{3}$$

Ans10. $\overrightarrow{PQ} = (-1-2)\hat{i} + (-2-3)\hat{j} + (-4-0)\hat{k}$

$$= -3\hat{i} - 5\hat{j} - 4\hat{k}$$

$DR\ are -3, -5, -4$

$$|\overrightarrow{PQ}| = \sqrt{9+25+16}$$

$D.C\ are \frac{-3}{\sqrt{50}}, \frac{-5}{\sqrt{50}}, \frac{-4}{\sqrt{50}}$

CBSE TEST PAPER-05
CLASS - XII MATHEMATICS (Vectors & Three Dimensional Geometry)

Topic: - Vectors

1. Is the measure of 1000 cm^3 is scalar or vector. [1]

2. Write two different vectors having same direction. [1]

3. Find the direction ratios and the direction cosines of the vector $\vec{r} = \hat{i} + \hat{j} - 2\hat{k}$ [1]

4. Find angle between two vectors $\vec{a}$ *and* $\vec{b}$ *if* $|\vec{a}| = 1$, $|\vec{b}| = 2$ $\vec{a}.\vec{b} = 1$ [1]

5. Find a vector in the direction of vector $\vec{a} = \hat{i} - 2\hat{j}$ that has magnitude 7 units. [1]

6. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = \hat{j} - \hat{k}$ find a vector $\vec{c}$ such that $\vec{a} \times \vec{c} = \vec{b}$. *and* $\vec{a}.\vec{c} = 3$ [4]

7. Find the value of λ so that the vectors $2\hat{i} - 4\hat{j} + \hat{k}$ *and* $4\hat{i} - 8\hat{j} + \lambda\hat{k}$ are (i) parallel (ii) perpendicular [4]

8. Show that $\vec{a} = \frac{1}{7}\left(2\hat{i} + 3\hat{j} + 6\hat{k}\right)$ $\vec{b} = \frac{1}{7}\left(6\hat{i} + 2\hat{j} - 3\hat{k}\right)$ $\vec{c} = \frac{1}{7}\left(3\hat{i} - 6\hat{j} + 2\hat{k}\right)$ [4]

9. If $\vec{a} = 3\hat{i} + \hat{j} + 2\hat{k}$ $\vec{b} = 2\hat{i} - 2\hat{j} + 4\hat{k}$ find [4]

   (i) Magnitude of $\vec{a} \times \vec{b}$

   (ii) A unit vector $\perp$ $\vec{a}$ *and* $\vec{b}$ to both

   (iii) The cosine and cosine of the angle b/w the vectors $\vec{a}$ *and* $\vec{b}$

10. The vectors $\vec{a} = 3\hat{i} + x\hat{j} - \hat{k}$ and $\vec{b} = 2\hat{i} - \hat{j} + y\hat{k}$ are mutually $\perp$. Given $|\vec{a}| = |\vec{b}|$, find x and y [4]

CBSE TEST PAPER-05

CLASS - XII MATHEMATICS (Vectors & Three Dimensional Geometry)

Topic: - Vectors [ANSWERS]

Ans1. Scalar

Ans2. $\vec{a}=1\hat{i}+3\hat{j}+5\hat{k}$

$\vec{b}=2\hat{i}+4\hat{j}+6\hat{k}$

$\vec{b}=2\left(\hat{i}+2\hat{j}+3\hat{k}\right)$

$\vec{b}=2\vec{a}$

Ans3. $D.R \ of \ \vec{r} \ are \ 1,1,-2$

$\left|\vec{r}\right|=\sqrt{1+1+4}=\sqrt{6}$

$D.C \ of \ \vec{r} \ are \dfrac{1}{\sqrt{6}},\dfrac{1}{\sqrt{6}},\dfrac{-2}{\sqrt{6}}$

Ans4. $\cos\theta=\dfrac{\vec{a}.\vec{b}}{\left|\vec{a}\right|\left|\vec{b}\right|}$

$\cos\theta=\dfrac{1}{(1)(2)}=\dfrac{1}{2}$

$\cos\theta=\cos\dfrac{\pi}{3}\Rightarrow\theta=\dfrac{\pi}{3}$

Ans5. $\hat{a}=\dfrac{\vec{a}}{\left|\vec{a}\right|}$

$=\dfrac{\left(\hat{i}-2\hat{j}\right)}{\sqrt{5}}$

Vector having magnitude equal to 7 and in the direction of $\vec{a}$ is

$7\hat{a}=7\left(\dfrac{\hat{i}-2\hat{j}}{\sqrt{5}}\right)$

$=\dfrac{7}{\sqrt{5}}\left(\hat{i}-2\hat{j}\right)$

Ans6. $\vec{a}=\hat{i}+\hat{j}+\hat{k}$

$\vec{b}=\hat{j}-\hat{k}$

$Let\ \vec{c}=c_1\hat{i}+c_2\hat{j}+c_3\hat{k}$

$\vec{a}\times\vec{c}=\vec{b}$

$$\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 1 & 1\\ c_1 & c_2 & c_3\end{vmatrix}=\vec{b}$$

$\hat{i}.(c_3-c_2)-\hat{j}(c_3-c_1)+\hat{k}(c_2-c_1)=\hat{j}-\hat{k}$

$c_3-c_2=0$

$c_3=c_2---------(i)$

$-c_3+c_1=1-----(ii)$

$c_2-c_1=-1-----(iii)$

$also\ \vec{a}.\vec{c}=3$

$c_1+c_2+c_3=3----(iv)$

on solving equaiton (i) (ii) (iii) and (iv)

$c_1=5/3,\ c_2=2/3,\ c_3=\frac{2}{3}$

$\vec{c}=5/3\,\hat{i}+2/3\,\hat{j}+\frac{2}{3}\hat{k}.$

Ans7. (i) Given vectors are parallel

$$if\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -4 & 1\\ 4 & -8 & \lambda\end{vmatrix}=0$$

$\lambda=2$

(ii) For $\perp$

$(4\hat{i}-8\hat{j}+\lambda\hat{k}).(2\hat{i}-4\hat{j}+\hat{k})=0$

$8+32+\lambda=0$

$\lambda=-40$

Ans8. $|\vec{a}|=1$

$|\vec{b}|=1$

$|\vec{c}|=1$

Hence they are unit vectors

$\vec{a}.\vec{b} = 0$

$\vec{b}.\vec{c} = 0$

$\vec{c}.\vec{a} = 0$

$\vec{a} \perp \vec{b},\ \vec{b} \perp \vec{c}\ and\ \vec{c} \perp \vec{a}$

So they are $\perp$ to each other.

Ans9. (i) $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 2 \\ 2 & -2 & 4 \end{vmatrix}$

$= 8\hat{i} - 8\hat{j} - 8\hat{k}$

$\left|\vec{a} \times \vec{b}\right| = 8\sqrt{3}$

(ii) $= \dfrac{\vec{a} \times \vec{b}}{\left|\vec{a} + \vec{b}\right|}$

$= \dfrac{8\hat{i} - 8\hat{j} - 8\hat{k}}{8\sqrt{3}}$

(iii) $\vec{a}.\vec{b} = 12$

$\left|\vec{a}\right| = \sqrt{14}$

$\left|\vec{b}\right| = \sqrt{24}$

$\cos\theta = \dfrac{\vec{a}.\vec{b}}{\left|\vec{a}\right|\left|\vec{b}\right|}$

$= \dfrac{12}{\sqrt{14}.\sqrt{24}} = \dfrac{\sqrt{3}}{\sqrt{7}}$

$Also\ \sin\theta = \dfrac{\left|\vec{a} + \vec{b}\right|}{\left|\vec{a}\right|\left|\vec{b}\right|}$

$= \dfrac{8\sqrt{3}}{\sqrt{14}\sqrt{24}}$

$= \dfrac{2}{\sqrt{7}}$

Ans10. $\vec{a}.\vec{b} = 0 \qquad \left[\because \vec{a} \perp \vec{b}\right.$

$y - x = 6 --------- (1)$

$\left|\vec{a}\right| = \left|\vec{b}\right| \qquad (Given)$

$3^2 + x^2 + 1 = 2^2 + 1^2 + y^2$

$y^2 - x^2 = 5$

$(y - x)(y + x) = 5$

$6(y - x) = 5$

$y - x = 5/6$

$x = \dfrac{-31}{12},\ y = \dfrac{41}{12}$

CBSE TEST PAPER-06
CLASS - XII MATHEMATICS (Vectors & Three Dimensional Geometry)

Topic: - Three Dimensional Geometry

1. Find the directions cosines of x, y and z axis. [1]

2. Find the vector equation for the line passing through the points (-1,0,2) and (3,4,6) [1]

3. Find the angle between the vector having direction ratios 3,4,5 and 4, -3, 5. [1]

4. Find the vector and Cartesian equation of the line through the point (5, 2,-4) and which is parallel to the vector $3\hat{i}+2\hat{j}-8\hat{k}$ [4]

5. Find the angle between the lines [4]

$$\vec{r}=(3\hat{i}+\hat{j}-2\hat{k})+\lambda(\hat{i}-\hat{j}-2\widehat{k})$$
$$\vec{r}=(2\hat{i}-\hat{j}-56\hat{k})+\mu(3\hat{i}-5\hat{j}-4\hat{k})$$

6. Find the shortest distance between the lines [4]

$$\vec{r}=(\hat{i}+2\hat{j}+\hat{k})+\lambda(\hat{i}-\hat{j}+\hat{k})$$
$$\vec{r}=(2\hat{i}-\hat{j}-\vec{k})+\mu(2\hat{i}+\hat{j}+2\hat{k})$$

7. Find the direction cosines of the unit vector $\perp$ to the plane $\vec{r}.(6\hat{i}-3\hat{j}-2\hat{k})+1=0$ passing through the origin. [4]

8. Find the angle between the two planes 3x – 6y + 2z = 7 and 2x + 2y - 2z = 5 [4]

9. Find the vector equation of the plane passing through the intersection of planes $\vec{r}.(\hat{i}+\hat{j}+\hat{k})=6$ *and* $\vec{r}.(2\hat{i}+3\hat{j}+4\hat{k})=-5$ And the point (1,1,1) [6]

10. Find the coordinate where the line thorough (3,-4,-5) and ((2,-3,1) crosses the plane 2x + y + z = 7 [6]

CBSE TEST PAPER-06

CLASS - XII MATHEMATICS (Vectors & Three Dimensional Geometry)

Topic: - Three Dimensional Geometry [ANSWERS]

Ans1. 1,0,0, 0,1,0 0,0,1

Ans2. Let $\vec{a}$ *and* $\vec{b}$ be the p.v of the points A (-1,0,2) and B (3, 4 6)

$$\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$$
$$= (-\hat{i} + 2\hat{j}) + \lambda(4\hat{i} + 4\hat{j} + 4\hat{k})$$

Ans3. Let $a_1 = 3$, $b_1 = 4$, $c_1 = 5$ and $a_2 = 4$, $b_2 = -3$, $c_2 = 5$

$$\cos\theta = \left(\frac{a_1 \quad a_2 + b_1 \quad b_2 + c_1 \quad c_2}{\sqrt{a_1{}^2 + b_1{}^2 + c_1{}^2} \quad \sqrt{a_2{}^2 + b_2{}^2 + c^2{}_2}}\right) = \frac{1}{2}$$
$$\theta = 60^0$$

Ans4. $\vec{a} = 5\hat{i} + 2\hat{j} - 4\hat{k},\ \vec{b} = 3\hat{i} + 2\hat{j} - 8\hat{k}$

Vector equation of line is

$$\vec{r} = \vec{a} + \lambda\vec{b}$$
$$= 5\hat{i} + 2\hat{j} - 4\hat{k} + \lambda(3\hat{i} + 2\hat{j} - 8\hat{k})$$

Cartesian equation is

$$x\hat{i} + y\hat{j} + z\hat{k} = 5\hat{i} + 2\hat{j} - 4\hat{k} + \lambda(3\hat{i} + 2\hat{j} - 8\hat{k})$$
$$x\hat{i} + y\hat{j} + z\hat{k} = (5 + 3\lambda)\hat{i} + (2 + 2\lambda)\hat{j} + (-4 - 8\lambda)\hat{k}$$
$$x = 5 + 3\lambda,\ y = 2 + 2\lambda,\ z = -4 - 8\lambda$$
$$\frac{x-5}{3} = \frac{y-2}{2} = \frac{z+4}{-8}$$

Ans5. Let θ is the angle between the given lines

$$\vec{b}_1 = \hat{i} - \hat{j} - 2\hat{k}$$

and

$$\vec{b}_2 = 3\hat{i} - 5\hat{j} - 4\hat{k}$$
$$\cos\theta = \left|\frac{\vec{b}_1 \cdot \vec{b}_2}{|\vec{b}_1||\vec{b}_2|}\right|$$

$$\left|\frac{(\hat{i}-\hat{j}-2\hat{k})\cdot(3\hat{i}-5\hat{j}-4\hat{k})}{\left|\hat{i}-\hat{j}-2\hat{k}\right|\left|3\hat{i}-5\hat{j}-4\hat{k}\right|}\right|$$

$$=\left|\frac{3+5+8}{\sqrt{6}\sqrt{50}}\right|=\frac{16}{\sqrt{50}}$$

$$\frac{16}{\sqrt{6} \qquad 5\sqrt{2}}$$

$$\frac{16}{\sqrt{2}\times\sqrt{3}\times\sqrt{5}\times\sqrt{2}}\times\frac{\sqrt{3}}{\sqrt{3}}$$

$$=\frac{8\cancel{16}\sqrt{3}}{\cancel{2}\times3\times5}$$

$$\cos\theta=\frac{8\sqrt{3}}{15}$$

$$\theta=\cos^{-1}\left(\frac{8\sqrt{3}}{15}\right)$$

Ans6. $\vec{a}_1=\hat{i}+2\hat{j}+\hat{k},\vec{b}_1=\hat{i}-\hat{j}+\hat{k}$

$$\vec{a}_2=2\hat{i}-\hat{j}-\hat{k},\vec{b}_1=2\hat{i}+\hat{j}+2\hat{k}$$

$$d=\left|\frac{\left(\vec{a}_2-\vec{a}_1\right).(\vec{b}_1\times\vec{b}_2)}{\left|\vec{b}_1\times\vec{b}_2\right|}\right|$$

$$\vec{a}_2-\vec{a}_1=\hat{i}-3\hat{j}-2\hat{k}$$

$$\vec{b}_1\times\vec{b}_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -1 & 1\\ 2 & 1 & 2\end{vmatrix}$$

$$=-3\hat{i}+3\hat{k}$$

$$d=\left|\frac{(\hat{i}-3\hat{j}-2\hat{k}).(-3\hat{i}+3\hat{k}}{\left|-3\hat{i}+3\hat{k}\right|}\right|$$

$$=\left|\frac{-3-6}{\sqrt{9+9}}\right|=\left|\frac{9}{3\sqrt{2}}\right|=\frac{3}{\sqrt{2}}$$

Ans7. $\vec{r}.(6\hat{i}-3\hat{j}-2\hat{k})=-1$

$\hat{r}.(-6\hat{i}+3\hat{j}+2\hat{k})=1....(1)$

$\left|-6\hat{i}+3\hat{j}+2\hat{k}\right|=\sqrt{36+9+4}=7$

Dividing equation 1 by 7

$\vec{r}.\left(\frac{-6}{7}\hat{i}+\frac{3}{7}\hat{j}+\frac{2}{7}\hat{k}\right)=\frac{1}{7}$

$\hat{n}=\frac{-6}{7}\hat{i}+\frac{3}{7}\hat{j}+\frac{2}{7}\hat{k}[\because \vec{r}.\vec{n}=d$

Hence direction cosines of $\hat{n}$ is $\frac{-6}{7},\frac{3}{5},\frac{2}{7}$

Ans8. Comparing the giving eq of the planes with the equations

A_1 x +B_1y +C_1Z + D = 0 , A_2 x + B_2y + C_2 Z + D_2 = 0
A_1 = 3, B_1 = -6, C_1 = 2
A_2 = 2, B_2 = 2, C_2 = -2

$\cos\theta=\left|\frac{A_1A_2+B_1B_2+C_1C_2}{\sqrt{A_1^2+B_{12}+C_1^2}\sqrt{A_2^2+B_2^2+C_2^2}}\right|$

$=\left|\frac{-10}{7\times 2\sqrt{3}}\right|$

$=\frac{5}{7\sqrt{3}}=\frac{5\sqrt{3}}{21}$

$\theta=COS^{-1}\left(\frac{5\sqrt{3}}{21}\right)$

Ans9.

$\vec{n}_1=\hat{i}+\hat{j}+\hat{k},\vec{n}_2=2\hat{i}+3\hat{j}+4\hat{k}$

$\vec{d}_1=-5,d_2=6$

Using the relation

$\vec{r}.(n_1+\lambda n_2)=d_1+\lambda d_2$

$r.[(1+2\lambda)\hat{i}+(1+3\lambda)\hat{j}+(1+4\lambda)\hat{k}]=6-5\lambda......(1)$

$taking \quad \vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$

$(x\hat{i}+y\hat{j}+z\hat{k})[(1+2\lambda)\hat{i}+(1+3\lambda)\hat{j}+(1+4\lambda)\hat{k}]=6-5\lambda$

$(1+2\lambda)x+(1+3\lambda)y+(1+4\lambda)z=6-5\lambda$

$(x+y+z-6)+\lambda(2x+3y+4y+4z+5)=0....(2)$

$plane \quad passes \quad through \quad the\ po\text{int} \quad (1,1,1)$

$\lambda=\frac{3}{14}$

$put \quad \lambda\, in \quad eq \quad (1)$

$\vec{r}.\left[\left(1+\frac{3}{7}\right)\hat{i}+\left(1+\frac{9}{14}\right)\hat{j}+\left(1+\frac{6}{7}\right)\hat{k}\right]=6-\frac{15}{14}$

$\vec{r}.\left(\frac{10}{7}\hat{i}+\frac{23}{14}\hat{j}+\frac{13}{7}\hat{k}\right)=\frac{69}{14}$

$\vec{r}.(20\hat{i}+23\hat{j}+26\hat{k})=69$

Ans10. Given points are A(3,-4,-5)

B(2,-3,1)

Direction ration of AB are 3-2, -4+3, -5-1

1,-1,-6

Eq. of line AB

$\frac{x-3}{1}=\frac{y+4}{-1}=\frac{Z+5}{-6}=\lambda(say)$

$x=\lambda+3, y=-\lambda-4, Z=-6\lambda-5$

let

$(\lambda+3, \ -\lambda-4, \ -6\lambda-5)\ lies\, in$

$the \quad plane \quad 2x+y+Z=7$

$2(\lambda+3)+(-\lambda-4)+(-6\lambda-5)=7$

$\lambda=-2$

$(1,-2,7)$

are the required point

CBSE TEST PAPER-07
CLASS - XII MATHEMATICS (Vectors & Three Dimensional Geometry)

Topic: - Three Dimensional Geometry

1. What is the direction ratios of the line segment joining P(x_1 y_1 z_1) and Q (x_2 y_2 z_2) [1]

2. The Cartesian equation of a line is $\frac{x+3}{2}=\frac{y-5}{4}=\frac{z+6}{2}$ Find the vector equation for the line. [1]

3. Show that the lines $\frac{x+3}{-3}=\frac{y-1}{1}=\frac{z-5}{5} and \frac{x+1}{-1}=\frac{y-2}{2}=\frac{z-5}{5}$ are coplanar. [1]

4. Find the shortest between the l_1 and l_2 whose vectors equations are [4]
$\vec{r}=\hat{i}+\hat{j}+\lambda(2\hat{i}-\hat{j}+\hat{k})$
$\vec{r}=2\hat{i}+\hat{j}-\hat{k}+\mu(3\hat{i}-5\hat{j}+2\hat{k})$

5. Find the angel between lines [4]
$\vec{r}=(2\hat{i}-5\hat{j}+\hat{k})+\lambda(3\hat{i}+2\hat{j}+6\hat{k})$
$\vec{r}=(7\hat{i}-6\hat{k})\mu(\hat{i}+2\hat{j}+2\hat{k})$

6. Show that the lines $\frac{x-5}{7}=\frac{y+2}{-5}=\frac{z}{1} and \frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ Are perpendicular to each others [4]

7. Find the vector equations of the plane passing through the points R(2,5,-3), Q(-2,-3,5) and T (5,3,-3) [4]

8. Find the Cartesian equation of the plane $\vec{r}.(\hat{i}+\hat{j}-\hat{k})=2$ [4]

9. Find the equation of the plane through the intersection of the planes [6]

   3x – y + 2z -4 = 0 and x + y + z – 2 = 0 and the point (2,2,1)

10. If the points (1,1p) and (-3,0,1)be equidistant from the plane $\vec{r}.(3\hat{i}+4\hat{j}-12\hat{k})+13=0$, then find the value of p. [6]

CBSE TEST PAPER-07

CLASS - XII MATHEMATICS (Vectors & Three Dimensional Geometry)

Topic: - Three Dimensional Geometry [ANSWERS]

Ans1. $x_2 - x_1$, $y_2 - y$, and z_2-z_1 are the direction ratio of the line segment PQ.

Ans2. Comparing the given equation with the standard equation form

$$\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}$$

$$\vec{r}=(-3\hat{i}+5\hat{j}+6\hat{k})+\lambda(2\hat{i}+4\hat{j}+2\hat{k})$$

Ans3. x_1=-3, y_1 = 1, z_1 = 5
a_1 = -3, b_1=1, c_1= 5
x_2 = -1, y_2=2, z_2 = 5
a_2 = -1, b_2 = 2, c_2 = 5

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ z_2 & b_2 & c_2 \end{vmatrix}=\begin{vmatrix} 2 & 1 & 0 \\ -3 & 1 & 5 \\ -1 & 2 & 5 \end{vmatrix}=0$$

Therefore lines are coplanar.

Ans4. $\vec{a}_1=\hat{i}+\hat{j},\vec{b}_1=2\hat{i}-\hat{j}+\hat{k}$

$\vec{a}_2=2\hat{i}+\hat{j}-\hat{k},\vec{b}_2=3\hat{i}-5\hat{j}+2\hat{k}$

$\vec{a}_2-\vec{a}_1=\hat{i}-\hat{k}$

$$\vec{b}_1\times\vec{b}_2=\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 3 & -5 & 2 \end{vmatrix}$$

$=3\hat{i}-\hat{j}-7\hat{k}$

$\left|\vec{b}_1\times\vec{b}_2\right|=\sqrt{59}$

$$d=\left|\frac{(\vec{b}_1\times\vec{b}_2).(\vec{a}_2-\vec{a}_1)}{\left|\vec{b}_1\times\vec{b}_2\right|}\right|=\frac{|3-0+7|}{\sqrt{59}}=\frac{10}{\sqrt{59}}$$

Ans5. $\vec{b}_1 = 3\hat{i} + 2\hat{j} + 6\hat{k}$

$\vec{b}_2 = \hat{i} + 2\hat{j} + 2\hat{k}$

The angle θ between them is given by

$$\cos\theta = \left| \frac{\vec{b}_1 . \vec{b}_2}{|\vec{b}_1| \quad |\vec{b}_2|} \right|$$

$$\left| \frac{(3\hat{i} + 2\hat{j} + 6\hat{k}).(\hat{i} + 2\hat{j} + 2\hat{k})}{|3\hat{i} + 2\hat{j} + 6\hat{k}||\hat{i} + 2\hat{j} + 2\hat{k}|} \right|$$

$$\left| \frac{3+4+12}{\sqrt{49}\sqrt{9}} \right|$$

$$= \frac{19}{7\times 3} = \frac{19}{21}$$

Ans6. $$\frac{x-5}{7} = \frac{y-(-2)}{-5} = \frac{z-0}{1}$$

$$\frac{x-0}{1} = \frac{y-0}{2} = \frac{z-0}{3}$$

$a_1 = 7, b_1 = -5, c_1 = 1$

$a_2 = 1, b_2 = 2, c_2 = 3$

For $\perp$

$a_1a_2+b_1b_2+c_1c_2=0$

L.H. S

$= 7\times 1 + (-5\times 2) + 1\times 3$

$= 7 - 10 + 3$

$= 0$

hence $\quad l_1 \perp l_2$

Ans7. Let

$\vec{a} = 2\hat{i} + 5\hat{j} - 3\hat{k}$

$\vec{b} = -2\hat{i} - 3\hat{j} + 5\hat{k}$

$\vec{c} = 5\hat{i} + 3\hat{j} - 3\hat{k}$

Vector equation is

$$(\vec{r} - \vec{a}).[(\vec{b} - \vec{a}) \times (\vec{c} - \vec{a})] = 0$$

$$[\vec{r} - (2\hat{i} + 5\hat{j} - 3\hat{k})].[(-4\hat{i} - 8\hat{j} + 8\hat{k}) \times (3\hat{i} - 2\hat{j})] = 0$$

Ans8. Let

$$\vec{r} = x\hat{i} + y\hat{i} + z\hat{k}$$
$$\vec{r}.(\hat{i} + \hat{j} - \hat{k}) = 2$$
$$(x\hat{i} + y\hat{i} + z\hat{k})(\hat{i} + \hat{j} - \hat{k}) = 2$$
$$x + y - z = 2$$

Which is the required equation of plane.

Ans9. Equation of any plane through the intersection of given planes can be taken as $(3x - y + 2z - 4) + \lambda(x + y + z - 2) = 0.....(i)$

The point (2,2,1) lies in this plane

$\lambda = -2/3$ put in eq ....(i)

$$(3x - y + 2z - 4) - \frac{2}{3}(x + y + z - 2) = 0$$
$$7x - 5y + 4z - 8 = 0$$

Ans10. The given plane is

$$\vec{r}.(3\hat{i} + 4\hat{j} - 12\hat{k}) + 13 = 0$$
$$(x\hat{i} + y\hat{j} + z\hat{k}).(3\hat{i} + 4\hat{j} - 12\hat{k}) + 13 = 0$$
$$3x + 4y - 12z + 13 = 0....(i)$$

This plane is equidistant from the points (1,1,P) and (-3,0,1)

$$\frac{|3(1) + 4(1) - 12p + 13|}{\sqrt{3^2 + 4^2 + (-12)^2}} = \frac{|3(-3) + 4(0) - 12(1) + 13|}{\sqrt{3^2 + 4^2 + (-12)^2}}$$
$$|20 - 12p| = |-8|$$
$$20 - 12p = \pm 8$$
$$p = -1 or \frac{7}{3}$$

CBSE TEST PAPER-08

CLASS - XII MATHEMATICS (Vectors & Three Dimensional Geometry)

Topic: - Three Dimensional Geometry

1. If a line has the direction ratios -18, 12, -4 then what are its direction cosines [1]

2. Find the angle between the pair of line given by [1]

$\vec{r} = 3\hat{i} + 2\hat{j} - 4\hat{k} + \lambda\left(\hat{i} + 2\hat{j} + 2\hat{k}\right)$

$\vec{r} = 5\hat{i} - 2\hat{j} + \mu(3\hat{i} + 2\hat{j} + 6\hat{k})$

3. Prove that the points A(2,1,3) B(5, 0,5)and C(-4, 3,-1) are collinear [1]

4. find the distance between the lines l_1 and l_2 given by [4]

$\vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$

$\vec{r} = 3\hat{i} + 3\hat{j} - 5\hat{k} + \mu(2\hat{i} + 3\hat{j} + 6\hat{k})$

5. Find the angle between lines [4]

$\frac{x}{2} = \frac{y}{2} = \frac{z}{1}, \frac{x-5}{4} = \frac{y-2}{1} = \frac{z-3}{8}$

6. Find the shortest distance between the lines [4]

$\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1} \, and \, \frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$

7. Find the vector and Cartesian equations of the plane which passes through the point (5,2,-4) and $\perp$ to the line with direction ratios (2,3,-1) [4]

8. Find the Cartesian equation of the plane [4]

$\vec{r}[(5-2t)\hat{i} + (3-t)\hat{j} + (25+t)\hat{k}] = 15$

9. Find the equation of the plane through the line of intersection of the planes x +y +z = 1 and 2x + 3y + 4z = 5 which is $\perp$ of the plane x-y + z = 0 [6]

10. Find the distance of the point (-1,-5,-10) from the point of intersection of the line $\vec{r} = (2\hat{i} - \hat{j} + 2\hat{k}) + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$ and the plane $\vec{r}.(\hat{i} - \hat{j} + \hat{k}) = 5$ [6]

CBSE TEST PAPER-08

CLASS - XII MATHEMATICS (Vectors & Three Dimensional Geometry)

Topic: - Three Dimensional Geometry [ANSWERS]

Ans1. a = -18, b=12, c= -4

$a^2+b^2+c^2 = (-18)^2 + (12)^2 + (-4)^2$

= 484

$$l = \frac{-18}{\sqrt{484}} = \frac{-18}{22} = \frac{-9}{11}$$

$$m = \frac{12}{22} = \frac{6}{11}$$

$$n = \frac{-4}{22} = \frac{-2}{11}$$

Ans2.

$$\vec{b}_1 = \hat{i} + 2\hat{j} + 2\hat{k}$$

$$\vec{b}_2 = 3\hat{i} + 2\hat{j} + 6\hat{k}$$

$$\cos\theta = \left|\frac{\vec{b}_1.\vec{b}_2}{|\vec{b}_1||\vec{b}_2|}\right| = \frac{19}{21}$$

Ans3. The equations of the line AB are

$$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1}$$

$$\frac{x-2}{5-2} = \frac{y-1}{0-1} = \frac{z-3}{5-3}$$

$$\frac{x-2}{3} = \frac{y-1}{-1} = \frac{z-3}{2} \qquad (1)$$

If A, B, C are collinear, C lies in equation (1)

$$\frac{-4-2}{3} = \frac{3-1}{-1} = \frac{-1-3}{2}$$

$$-2 = -2 = -2$$

Hence A,B,C are collinear

Ans4.

$$\vec{a}_1 = \hat{i} + 2\hat{j} - 4\hat{k}$$

$$\vec{a}_2 = 3\hat{i} + 3\hat{j} - 5\hat{k}$$

since $\vec{b}_1 = \vec{b}_2$

Hence line are parallel

$$\vec{a}_2-\vec{a}=2\hat{i}-\hat{j}-\hat{k}$$

$$\vec{b}\times(\vec{a}_2-\vec{a})=\begin{vmatrix} i & j & k \\ 2 & 3 & 6 \\ 2 & 1 & -1 \end{vmatrix}$$

$$d=\left|\frac{\vec{b}\times(\vec{a}_2-\vec{a}_1)}{|\vec{b}|}\right|$$

$$\frac{\left|-9\hat{i}+14\hat{j}-4\hat{k}\right|}{\sqrt{49}}$$

$$\frac{\sqrt{293}}{\sqrt{49}}=\frac{\sqrt{293}}{7}$$

Ans5. $$\frac{x-0}{2}=\frac{y-0}{2}=\frac{z-0}{1}$$

$$\frac{x-5}{4}=\frac{y-2}{1}=\frac{z-3}{8}$$

$$a_1=2, b_1=2, c_1=1$$

$$a_2=4, b_2=1, c_2=8$$

$$\cos\theta=\frac{|\vec{b}_1.\vec{b}_2|}{|\vec{b}_1||\vec{b}_2|}$$

$$=\left|\frac{2(4)+2(1)+1(8)}{\sqrt{2^2+2^2+1}\sqrt{4^2+1^2+8^2}}\right|$$

$$=\left|\frac{8+2+8}{\sqrt{9}\sqrt{81}}\right|$$

$$=\frac{18}{27}$$

$$=\frac{2}{3}$$

$$\theta=\cos^{-1}\left(\frac{2}{3}\right)$$

Ans6. $$\vec{a}_1=-\hat{i}-\hat{j}-\hat{k}$$

$$\vec{a}_2=3\hat{i}+5\hat{j}+7\hat{k}$$

$$\vec{b}_1=7\hat{i}-6\hat{j}+1\hat{k}$$

$$\vec{b}_2=\hat{i}-2\hat{j}+\hat{k}$$

$$\vec{a}_2 - a_1 = 4\hat{i} + 6\hat{j} + 8\hat{k}$$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 7 & -6 & 1 \\ 1 & -2 & 1 \end{vmatrix}$$

$$= -4i - 6j - 8k$$

$$\left|\vec{b}_1 \times \vec{b}_2\right| = \sqrt{(-4)^2 + (-6)^2 + (-8)^2}$$

$$= \sqrt{116}$$

$$= 2\sqrt{29}$$

$$d = \left| \frac{(\vec{a}_2 - \vec{a}_1)(b_1 \times b_2)}{\left|\vec{b}_1 \times \vec{b}_2\right|} \right|$$

$$= \left| \frac{-116}{2\sqrt{29}} \right| = \frac{\cancel{116}58}{\cancel{2}\sqrt{29}}$$

$$= 2\sqrt{29}$$

Ans7. $\vec{a} = 5\hat{i} + 2\hat{j} - 4\hat{k}$

$$\vec{N} = 2\hat{i} + 3\hat{j} - \hat{k}$$

Vector equation is

$$\left(\vec{r} - \vec{a}\right).\vec{N} = 0$$

$$[\vec{r} - (5\hat{i} + 2\hat{j} - 4\hat{k})].(2\hat{i} + 3\hat{j} - \hat{k}) = 0$$

Cartesian equation is

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$[x\hat{i} + y\hat{j} + z\hat{k} - 5\hat{j} - 2\hat{j} + 4\hat{k}].[2\hat{i} + 3\hat{j} - \hat{k}] = 0$$

$$((x-5)\hat{i} + (y-2)\hat{j} + (z+4)\hat{k}).(2\hat{i} + 3\hat{j} - \hat{k}) = 0$$

$$2(x-5) + 3(y-2) - (z+4) = 0$$

$$2x - 10 + 3y - 6 - z - 4 = 0$$

$$2x + 3y - z = 20$$

Ans8. $\vec{r}[(5-2t)\hat{i} + (3-t)\hat{j} + (2s+t)\hat{k}] = 15$

$$(x\hat{i} + y\hat{j} + z\hat{k}).[(s-2t)\hat{i} + (3-t)\hat{j} + (2s+t)\hat{k}] = 15$$

$$(s-2t)n + (3-t)y + (2s+t) - z = 15$$

Ans9. Equations of any plane through the intersection of given planes are be written is

$$(x+y+z-1)+\lambda(2x+3y+4z-5)=0$$
$$(1+2\lambda)x+(1+3\lambda)y+(1+4\lambda)z-1-5\lambda=0...(1)$$

This plane is it right angle to the plane x-y+z=0

$$(1+2\lambda)(1)+(1+3\lambda)(-1)+(1+4\lambda)(1)=0$$
$$\lambda=-1/3$$
$put\lambda in$ equation (1)
$$\left(1-\frac{2}{3}\right)x+\left(1-\frac{3}{3}\right)y+\left(1-\frac{4}{3}\right)Z-1+\frac{5}{3}=0$$
$$x-z+2=0$$

Ans10. $r=(2i-j+2k)+\lambda(3i+4j+2k)$

$$\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{2}=\lambda.......(i)$$
$coordinets\, are$
$$3\lambda+2,4\lambda-1,2\lambda+2$$
$$and\, \vec{r}.(\hat{i}-\hat{j}+\hat{k})=5$$
$$(x\hat{i}+y\hat{j}+z\hat{k}).(\hat{i}-\hat{j}+\hat{k})=5$$
$$x-y+z=5.....(ii)$$
$coordinate\, lies\ in\ \ eq.\ (ii)$
$$\lambda=0$$
$we\ \ get\,(2,-1,2)$

Are the coordinate of the point of intersection of the given line and the plane

$(-1,-5,-10) and\ (2,-1,2)$

req. distance $=\sqrt{(2+1)^2+(-1+5)^2+(2+10)^2}$

$=13$

CBSE TEST PAPER-09
CLASS - XII MATHEMATICS (Vectors & Three Dimensional Geometry)

Topic: - Three Dimensional Geometry

1. Find the direction cosines of the line passing through the two points (2,4,-5) and (1,2,3). [1]

2. Find the equation of the plane with intercepts 2,3 and 4 on the x, y and z axis respectively. [1]

3. If the equations of a line AB is $\frac{x-3}{1}=\frac{y+2}{-2}=\frac{z-5}{4}$ find the directions ratio of line parallel to AB. [1]

4. Find the distance of a point (2,5,-3) from the plane $\vec{r}.(6\hat{i}-3\hat{j}+2\hat{k})=4$ [4]

5. Find the angle b/w the line [4]
$\frac{x-2}{2}=\frac{y-1}{5}=\frac{z+3}{-3} and \frac{x+2}{-1}=\frac{y-4}{8}=\frac{z-5}{4}$

6. Find the shortest distance [4]
$\vec{r}=(\hat{i}+2\hat{j}+3\hat{k})+\lambda(\hat{i}-3\hat{j}+2\hat{k}) and \vec{r}=(4\hat{i}+5\hat{j}+6\hat{k})+\mu(2\hat{i}+3\hat{j}+\hat{k})$

7. Find the vector equation of a plane which is at a distance of 7 units from the origin and normal to the vector $3\hat{i}+5\hat{j}-6\hat{k}$ [4]

8. Find the Cartesian equation of plane $\vec{r}.(\hat{i}+\hat{j}-\hat{k})=2$ [4]

9. Find the equation of the plane that contains the point (1,-1,2) and is $\perp$ to each of the plane 2x+3y-2z=5 and x+2y-3z = 8 [6]

10. Find the vector equation of the line passing through (1,2,3) and $||$ to the planes $\vec{r}.(\hat{i}-\hat{j}+2\hat{k})=5 and \vec{r}.(3\hat{i}+\hat{i}+\hat{k})=6$ [6]

CBSE TEST PAPER-09

CLASS - XII MATHEMATICS (Vectors & Three Dimensional Geometry)

Topic: - Three Dimensional Geometry [ANSWERS]

Ans1. Let P(-2,4,-5) Q (1,2,3)

$$PQ=\sqrt{(1+2)^2+(2-4)^2+(3+5)^2}$$
$$=\sqrt{9+4+64}$$
$$=\sqrt{77}$$
$$the\, direction \cos ines\, of\, the\, line$$
$$Joining\, two\, po\text{int}\, is$$
$$\frac{1+2}{\sqrt{77}},\frac{2-4}{\sqrt{77}},\frac{3+4}{\sqrt{77}}$$
$$\frac{3}{\sqrt{33}},\frac{-2}{\sqrt{77}},\frac{8}{\sqrt{77}}$$

Ans2. Let the equation of the plane be

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$$
$$\frac{x}{2}+\frac{y}{3}+\frac{z}{4}=1$$
$$6x+4y+3z=12$$

Ans3. $\frac{x-3}{1}=\frac{y+2}{-2}=\frac{z-5}{4}$ the direction ratios of a line parallel to AB are 1, -2, 4

Ans4.

$$\vec{a}=2\hat{i}+5\hat{j}-3\hat{k}$$
$$\overrightarrow{N}=6\hat{i}-3\hat{j}+2\hat{k}, d=4$$
$$d=\frac{\left|\vec{a}.\overrightarrow{N}.d\right|}{\left|\overrightarrow{N}\right|} \qquad [\because \vec{r}.\overrightarrow{N}=d$$
$$=\frac{\left|(2\hat{i}+5\hat{j}-3\hat{k}).(6\hat{i}-3\hat{j}+2\hat{k})-4\right|}{\left|6\hat{i}-3\hat{j}+2\hat{k}\right|}$$
$$=\frac{\left|12-15-6-4\right|}{\sqrt{36+9+4}}=\frac{13}{7}$$

Ans5. $\vec{b}_1 = 2\hat{i} + 5\hat{j} - 3\vec{k}$

$\vec{b}_2 = -\hat{i} + 8\hat{j} + 4\hat{k}$

$$\cos\theta = \left|\frac{\vec{b}_1.\vec{b}_2}{|\vec{b}_1||\vec{b}_2|}\right|$$

$$= \left|\frac{2(-1) + 5(8) + (-3)(4)}{\sqrt{38}\sqrt{81}}\right|$$

$$= \frac{26}{9\sqrt{38}}$$

$$\theta = \cos^{-1}\left(\frac{26}{9\sqrt{38}}\right)$$

Ans6. $\vec{a}_1 = \hat{i} + 2\hat{j} + 3\hat{k}$

$\vec{b}_1 = \hat{i} - 3\hat{j} + 2\hat{k}$

$\vec{a}_2 = 4\hat{i} + 5\hat{j} + 6\hat{k}$

$\vec{b}_2 = 2\hat{i} + 3\hat{j} + \hat{k}$

$\vec{a}_2 - \vec{a}_1 = 3\hat{i} + 3\hat{j} + 3\hat{k}$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 2 \\ 2 & 3 & 1 \end{vmatrix}$$

$$= -9\hat{i} + 3\hat{j} + 9\hat{k}$$

$$d = \frac{(\vec{a}_2 - \vec{a}_2).(\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|}$$

$$= \left|\frac{9}{3\sqrt{19}}\right| = \frac{3}{\sqrt{19}}$$

Ans7. $3\hat{i} + 5\hat{j} - 6\hat{k}$

$|(\vec{n})| = \sqrt{70}$

$$\hat{n} = \frac{\vec{n}}{|\vec{n}|}$$

$$= \frac{3}{\sqrt{70}}\hat{i} + \frac{5}{\sqrt{70}}\hat{j} - \frac{6}{\sqrt{70}}\hat{k}$$

$$\vec{r}.\hat{n} = 7$$

$$\vec{r}.\left(\frac{3}{\sqrt{70}}\hat{i} + \frac{5}{\sqrt{70}}\hat{j} - \frac{6}{\sqrt{70}}\hat{k}\right) = 7$$

Ans8. $\vec{r}.(\hat{i}+\hat{j}-\hat{k}) = 2$

$$let \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$(x\hat{i}+y\hat{i}+z\hat{k}).(\hat{i}+\hat{j}-\hat{k}) = 2$$

$$x + y - z = 2$$

Ans9. The equation of the plane containing the given point is

A(x-1)+B(y+1)+C(Z-2)=0….[i]

Condition of $\perp$ to the plane given in (i) with the plane

2x+3y-2z=5, x+2y-3z=8

2A+3B-2C=0

A+2B-3C=8

On solving

A=-5c, B=4C

5x-4y-Z=7

Ans10. $line\ passing\ through\ (1,2,3)$

$$i.e\ \vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$$

$$and \parallel to\ the\ planes$$

$$\vec{b}_1 = \hat{i} - \hat{j} + 2\hat{k}$$

$$\vec{b}_2 = 3\hat{i} + \hat{j} + \hat{k}$$

$$\therefore The\ line\ is\ normal\ to\ the\ vector$$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 2 \\ 3 & 1 & 1 \end{vmatrix}$$

$$= -3\hat{i} + 5\hat{j} + 4\hat{k}$$

$$\therefore The\ req.\ eq. of\ the\ line\ is$$

$$\vec{r} = \hat{i} + 2\hat{i} + 3\hat{k} + \lambda(-3\hat{i} + 5\hat{j} + 4\hat{k})$$

CBSE TEST PAPER-10
CLASS - XII MATHEMATICS (Vectors & Three Dimensional Geometry)

Topic: - Three Dimensional Geometry

1. If the line has direction ratios 2,-1,-2 determine its direction Cosines. [1]

2. The Cartesian equation of a line is $\frac{x-5}{3}=\frac{y+4}{7}=\frac{z-6}{2}$. Write its vector form [1]

3. Cartesian equation of a line AB is $\frac{2x-1}{2}=\frac{4-y}{7}=\frac{z+1}{2}$ write the direction ratios of a line parallel to AB. [1]

4. Find the angle between the line $\frac{x+1}{2}=\frac{y}{3}=\frac{z-3}{6}$ and the plane 10x +2y-11z=3 [4]

5. Find the value of P so that the lines $\frac{1-x}{3}=\frac{7y-14}{2p}=\frac{z-3}{2}$ *and* $\frac{7-7x}{3p}=\frac{y-5}{1}=\frac{6-z}{5}$ are at right angles. [4]

6. Find the shortest distance between the lines whose vector equation are [4]
$\vec{r}=(1-t)\hat{i}+(t-2)\hat{j}+(3-2t)\hat{k}$
$\vec{r}=(s+1)\hat{i}+(2s-1)\hat{j}-(2s+1)\hat{k}$

7. Find x such that four points A(3,2,1) B(4,x,5)(4,2,-2) and D (6,5,-1)are coplanar. [4]

8. Find the angle between the two planes 2x +y-2z=5 and 3x -6y -2z = 7using vector method. [4]

9. Find the equation of the point where the line through the points A(3,4,1) and B(5,1,6) crosses the XY plane. [6]

10. Prove that if a plane has the intercepts a,b,c is at a distance of p units from the origin then [6]
$\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}=\frac{1}{p^2}$

CBSE TEST PAPER-10

CLASS - XII MATHEMATICS (Vectors & Three Dimensional Geometry)

Topic: - Three Dimensional Geometry [ANSWERS]

Ans1. $$\frac{2}{\sqrt{(2)^2+(-1)^2+(-2)^2}}, \frac{-1}{\sqrt{(2)^2+(-1)^2+(-2)^2}}, \frac{-2}{\sqrt{(2)^2+(-1)^2+(-2)^2}} = \frac{2}{3}, \frac{-1}{3}, \frac{-2}{3}$$

Ans2.
$$\vec{r} = \vec{a} + \lambda\vec{b}$$
$$\vec{a} = 5\hat{i} - 4\hat{j} + 6\hat{k}$$
$$\vec{b} = 3\hat{i} + 7\hat{j} + 2\hat{k}$$
$$\vec{r} = (5\hat{i} - 4\hat{j} + 6\hat{k}) + \lambda(3\hat{i} + 7\hat{j} + 2\hat{k})$$

Ans3. Given equation of a line can be written is
$$\frac{x-\frac{1}{2}}{1} = \frac{y-4}{-7} = \frac{z+1}{2}$$
The direction ratios of a line parallel to AB are 1, -7, 2.

Ans4.
$$\vec{r} = (-\hat{i} + o.\hat{j} + 3\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$$
$$and\ \vec{r}.(10\hat{i} + 2\hat{j} - 11\hat{k}) = 3$$
$$here\ \vec{b} = 2\hat{i} + 3\hat{j} + 6\hat{k}$$
$$and\ \ \vec{n} = 10\hat{i} + 2\hat{j} - 11\hat{k}$$
$$\sin\phi = \left|\frac{\vec{b}.\vec{n}}{|\vec{b}||\vec{n}|}\right|$$
$$= \left|\frac{20+6-66}{7\times 15}\right| = \left|\frac{-40}{7\times 15}\right| = \frac{8}{21}$$

Ans5.
$$\frac{x-1}{-3} = \frac{y-2}{\frac{2p}{7}} = \frac{z-3}{3} .....(i)$$
$$\frac{-(x-1)}{\frac{-3p}{7}} = \frac{y-5}{1} = \frac{z-6}{-5} .....(ii)$$
$$a_1 = -3, b_1 = \frac{2p}{7}, c_1 = 2$$

$$a_2 = \frac{-3p}{7}, b_2 = 1, c_2 = -5$$

$$for \ \perp$$

$$a_1a_2 + b_1b_2 + c_1c_2 = 0$$

$$-3\left(\frac{-3p}{7}\right) + \frac{2p}{7}(1) + 2(-5) = 0$$

$$\frac{9p}{7} + \frac{2p}{7} - \frac{10}{1} = 0$$

$$\frac{9p + 2p - 70}{7} = 0$$

$$11p = 70$$

$$p = \frac{70}{11}$$

Ans6.

$$\vec{r} = \hat{i} - 2\hat{j} + 3\hat{k} + t(\hat{i} + \hat{j} - 2\hat{k})$$

$$\vec{r} = \hat{i} - \hat{j} - \hat{k} + s(\hat{i} + 2\hat{j} - 2\hat{k})$$

$$\vec{a}_1 = \hat{i} - 2\hat{j} + 3\hat{k}$$

$$\vec{b}_1 = \hat{i} + \hat{j} - 2\hat{k}$$

$$\vec{a}_2 = \hat{i} - \hat{j} - \hat{k}$$

$$\hat{b}_2 = \hat{i} + 2\hat{j} - 2\hat{k}$$

$$\vec{a}_2 - \vec{a}_1 = \hat{j} - 4\hat{k}$$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & -2 \\ 1 & 2 & -2 \end{vmatrix}$$

$$= 2\hat{i} - 4\hat{j} - 3\hat{k}$$

$$\left|\vec{b}_1 \times \vec{b}_2\right| = \sqrt{(2)^2 + (-4)^2 + (-3)^2}$$

$$= \sqrt{29}$$

$$d = \left|\frac{(\vec{a}_2 - \vec{a}_1)\ (\vec{b}_1 \times \vec{b}_2)}{\left|\vec{b}\hat{1} \times \vec{b}_2\right|}\right| = \frac{8}{\sqrt{29}}$$

Ans7. The equation of plane through A(3,2,1), C(4,2,-2) and D (6,5,-1) is

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix}=0$$

$$\begin{vmatrix} x-3 & y-2 & z-1 \\ 4-3 & 2-2 & -2-1 \\ 6-3 & 5-2 & -1-1 \end{vmatrix}=0$$

$9x-7y+3z-16=0...(i)$

The point A,B,C,D are coplanar

$9\times 4-7x+3\times 5-16=0$

$x=5$

Ans8. $\vec{N}_1=2\hat{i}+\hat{j}-2\hat{k}$

$\vec{N}_2=3\hat{i}-6\hat{j}-2\hat{k}$

$$\cos\theta=\left|\frac{\vec{N}_1.\vec{N}_2}{|\vec{N}_1||\vec{N}_2|}\right|$$

$$=\left|\frac{(2\hat{i}+\hat{j}-2\hat{k}).(3\hat{i}-6\hat{j}-2\hat{k}}{\sqrt{4+1+4}\sqrt{9+36+4}}\right|$$

$$\frac{4}{21}$$

$$\theta=\cos^{-1}\left(\frac{4}{21}\right)$$

Ans9. The vector equation of the line through the point A and B is

$\vec{r}=3\hat{i}+4\hat{j}+k+\lambda[(5-3)\hat{i}+(1-4)\hat{j}+(6-1)\hat{k}]$

$\vec{r}=3\hat{i}+4\hat{j}+\hat{k}+\lambda(2\hat{i}-3\hat{j}+5\hat{k})....(i)$

Let P be the point where the line AB crosses the XY plane. Then the position vector $\vec{r}$ of the point P is the form

$x\hat{i}+y\hat{j}$

$x\hat{i}+y\hat{i}=(3+2\lambda)\hat{i}+(4-3\lambda)\hat{j}+(1+5\lambda)\hat{k}$

$x=3+2\lambda \,|\, y=4-3\lambda$

$x={}^{13}\!/_{5},\ y={}^{23}\!/_{5}$

$req.\ po\text{int}\ is\left(\frac{13}{5},\frac{23}{5},0\right)$

Ans10. The equation of the plane in the intercepts from is $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ distance of this plane from the origin is given to be p.

$$p=\frac{\left|\frac{1}{a}.0+\frac{1}{b}.0+\frac{1}{c}.0-1\right|}{\sqrt{\left(\frac{1}{a}\right)^2+\left(\frac{1}{b}\right)^2+\left(\frac{1}{c}\right)^2}}$$

$$p=\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}$$

$$\Rightarrow \frac{1}{p^2}=\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}$$

CBSE TEST PAPER-01
CLASS - XII MATHEMATICS (Linear Programming)

Topic: - Linear Programming

1. Reshma wishes to mix two types of food P and Q in such a way that the vitamin contents of the mixture contain at least 8 units of vitamin A and 11 units of vitamin B. food P costs Rs 60/kg and Food Q costs Rs 80/kg. Food P contains 3 units/kg of Vitamin A and 5 units/kg of Vitamin B while food Q contains 4 units/kg of Vitamin A and 2 units/kg of vitamin B. Determine the minimum cost of the mixture. [6]
2. One kind of cake requires 200g of flour and 25g of fat, and another kind of cake requires 100g of flour and 50g of fat. Find the maximum number of cakes which can be made from 5kg of flour and 1kg of fat assuming that there is no shortage of the other ingredients used in making the cakes. [6]
3. A factory makes tennis rackets and cricket bats. A tennis racket takes 1.5 hours of machine time and 3 hours of craftman's time in its making while a cricket bat takes 3 hour of machine time an 1 hour of craftman's time. In a day, the factory has the availability of not more than 42 hours of machine time and 24 hours of craftsman's time. [6]
   (i) What number of rackets and bats must be made if the factory is t work at full capacity?
   (ii) If the profit on a racket and on a bat is Rs 20 and Rs 10 respectively, find the maximum profit of the factory when it works at full capacity.
4. A manufacturer produces nuts and bolts. It takes 1 hours of work on machine A and 3 hours on machine B to produce a package of bolts. He earns a profit of Rs 17.50 per package on nuts and Rs 7.00 per package on bolts. How many package of each should be produced each day so as to maximise his profit, if he operates his machines for at the most 12 hours a day? [6]
5. A factory manufactures two types of screws, A and B. Each type of screw requires the use of two machines, an automatic and a hand operated. It takes 4 minutes on the automatic and 6 minutes on hand operated machines to manufacture a package screws A, while it takes 6 minutes on automatic and 3 minutes and on the hand operated machines to manufacture a package of screws B. Each machine is available for at the most 4 hours on any day. The manufacturer can sell a package of screws A at a profit of Rs 7 and screws B at a profit of Rs 10. Assuming that he can sell all the screws he manufactures, how many package of each type should the factory owner produce in a day in order to maximise his profit? Determine the maximum profit. [6]

6. A cottage industry manufactures pedestal lamps and wooden shades, each requiring the use of a grinding/cutting machine and a sprayer. It takes 2 hours on grinding/cutting machine and 3 hours on the sprayer to manufacture a pedestal lamp. It takes 1 hour on the grinding/cutting machine and 2 hour on the sprayer to manufacture a shade. On any day, the sprayer is available for at the most 20 hours and the grinding/cutting machine for at the most 12 hours. The profit form the sale of a lamp is Rs 5 and that from a shade is Rs 3. Assuming that the manufacturer can sell all the lamps and shades that he produces, how should he schedule his daily production in order to maximise his profit? [6]

7. A company manufactures two types of novelty souvenirs made of plywood. Souvenirs of type A require 5 minutes each for cutting and 10 minutes each for assembling. Souvenirs of type B require 8 minutes each for cutting and 8 minutes each for assembling. There are 3 hours 20 minutes available for cutting and 4 hours for assembling. The profit is Rs 5 each for type A and Rs 6 each for type B souvenirs. How may souvenirs of each type should the company manufacture in order to maximise the profit? [6]

8. A merchant plans to sell two types of personal computers – a desktop model and a portable model that will cost Rs 25000 and Rs 40000 respectively. He estimates that the total monthly demand of computers will not exceed 250 units. Determine the number of units of each type of computers which the merchant should stock to get maximum profit if he does not want to invest more than Rs 70 lakhs and if his profit on the desktop model is Rs 4500 and on portable model is Rs 5000. [6]

9. A diet is to contain at least 80 units of vitamin A and 100 units of minerals. Two foods F_1 and F_2 are available. Food F1 costs Rs 4 per unit food and F_2 costs Rs 6 per unit. On unit of food F_1 contains 3 units of vitamin A and 4 units of minerals. One unit of food F_2 contains 6 units of vitamin A and 3 units of minerals. Formulate this as linear programming problem. Find the minimum cost for diet that consists of mixture of these two foods and also meets the minimal nutritional requirements. [6]

10. There are two types of fertilizers F_1 and F_2. F_1 consists of 10% nitrogen and 6% phosphoric acid and F_2 consists of 5% nitrogen and 10% phosphoric acid. After testing the soil conditions, a farmer finds that she needs atleast 14kg of nitrogen and 14kg of phosphoric acid for her crop. If F_1 costs Rs 6/kg and F_2 costs Rs 5/kg, determine requirements are met at a minimum cost. What is the minimum cost? [6]

CBSE TEST PAPER-01

CLASS - XII MATHEMATICS (Linear Programming)
Topic: - Linear Programming [ANSWERS]

Ans 01. Let food P consist of x Kg and food Q consists of Y Kg.

$Z = 60x + 80y$

$3x + 4y \geq 8$

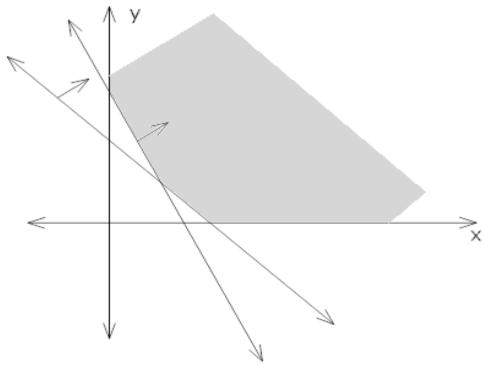
$5x + 2y \geq 11$

$x \geq 0$

$y \geq 0$

Hence, Cost is minimum = Rs 160

When $x = 2$, $y = \frac{1}{2}$



Ans 02. Let x be number of cakes of first kind and y the number of cakes of other kind.

$Z = x + y$

$200x + 100y \leq 5000$

$\Rightarrow 2x + y \leq 50$

$25x + 50y \leq 1000$

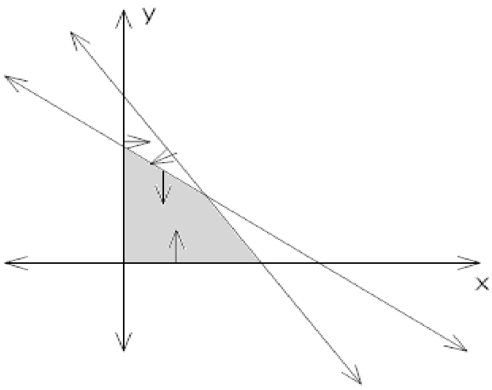
$\Rightarrow x + 2y \leq 40$

$x \geq 0, y \geq 0$

Maximum number of cakes

$Z = 30$

When $x = 20$, $y = 10$.



Ans 03. Let the number of cricket and the number of cricket bats to be made in a day be x and y respectively.

$Z = x + y$

and also $P = 20x + 10y$

$\frac{3}{2}x + 3y \leq 42$

$\Rightarrow x + 2y \leq 28$

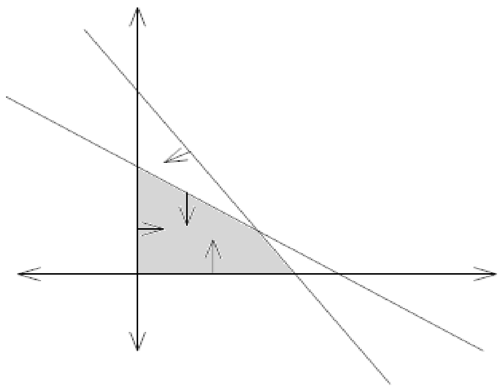
$3x + y \leq 24$

$x \geq 0, y \geq 0$

(i) Maximum $Z = 16$ at $x = 4$ $y = 12$

(ii) $P = 20\times4 + 10\times12$

$= 200$



Ans 04. Let the manufacture produce x nuts and y bolts

$Z = 17.50x + 7y$

$x + 3y \leq 12$

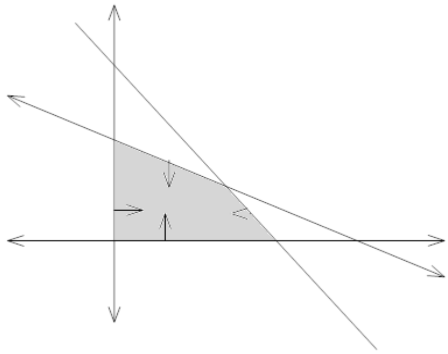
$3x + y \leq 12$

$X_1\ y \geq 0$

Maximum profit

Z = Rs 73. 50 at

X = 3, y = 3



Ans 05. Let the manufacturer produce x packages of screw A and y packages Screw B.

$Z = 7x + 10y$

$4x + 6y \leq 240$

$\Rightarrow 2x + 3y \leq 120$

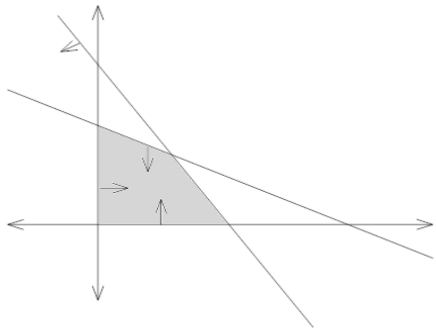
$6x + 3y \leq 240$

$\Rightarrow 2x + y \leq 80$

$x \geq 0, y \geq 0$

profit is maximum = 410

When 30 packages of screw A and 20 package of screw B.



Ans 06. Let x be pedestal lamps and y wooden shades

$Z = 5x + 3y$

$2x + y \leq 12$

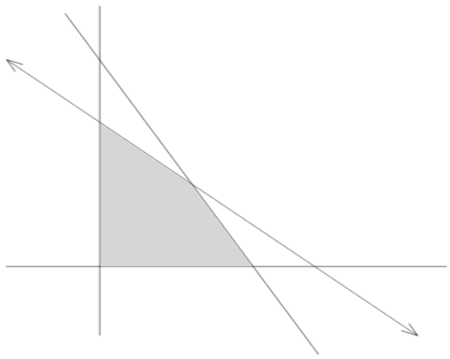
$3x + 2y \leq 20$

$x \geq 0, y \geq 0$

profit maximum

when 4 pedestal lamps

4 wooden shades



Ans 07. Let x souvenirs of type A and y souvenirs of type B

$Z = 5x + 6y$

$5x + 8y \leq 200$

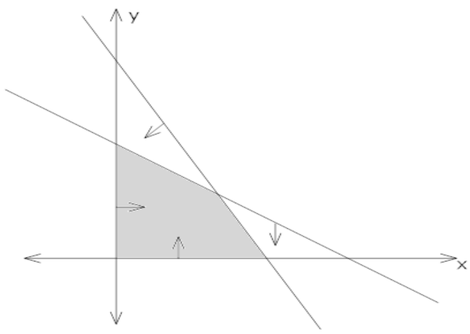
$10x + 8y \leq 240$

$\Rightarrow 5x + 4y \leq 120$

$x \geq 0, y \geq 0.$

Maximum profit is Rs 160

When 8 souvenirs of Type A and 20 souvenirs of type B.



Ans 08. Let the merchant stock x desktop computers and y portable computer.

$Z = 4500x + 5000y$

$x + y \leq 250$

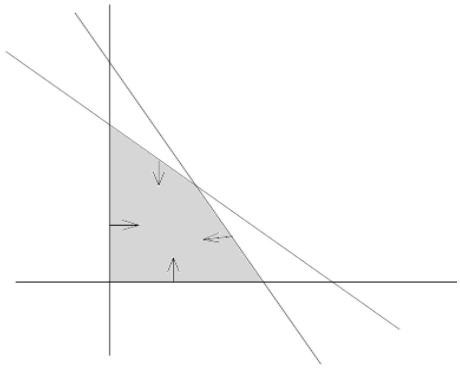
$25000x + 40000y \leq 700000$

$\Rightarrow 5x + 8y \leq 1400$

$x \geq 0, \; y \geq 0.$

Profit is maximum = 1150000

When 250 desktop computers and 50 portable computers are stocked.



Ans 09. Let the diet contain x unit of food F_1 and y units of food F_2.

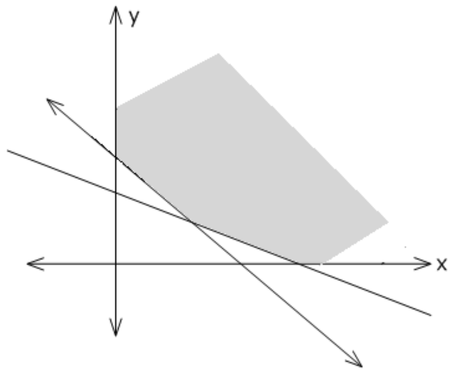
$Z = 4x + 6y$

$3x + 6y \geq 80$

$4x + 3y \geq 100$

$X \geq 0, y \geq 0$

Z is minimum when 24 units of food F_1 and $\frac{4}{3}$ unit of food F_2 are mixed minimum cost = 104 .



Ans 10. Let the farmer use x Kg of F_1 and y Kg of F_2.

$Z = 6x + 5y$

$\frac{10x}{100} + \frac{5y}{100} \geq 14$

$\frac{6x}{100} + \frac{10y}{100} \geq 14$

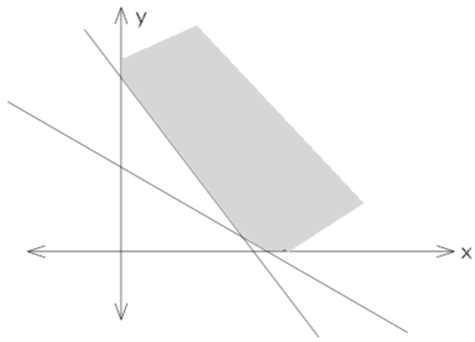
$x \geq 0, y \geq 0.$

Minimum cost

Z = 100

at x = 100

y = 80



CBSE TEST PAPER-02

CLASS - XII MATHEMATICS (Linear Programming)

Topic: - Linear Programming

1. Two tailors A and B earn Rs 150 and Rs 200 per day respectively. A can stitch 6 shirts and 4 pants per day while B can stich 10 shirts and 4 Pants per day. How many days each work if it is desired to produce at least 60 shirts and 32 pants at a minimum labour cast? [6]

2. A farmer mixes two brands p and Q of cattle feed. Brand P, costing Rs 250 per bag, contains 3 units of nutritional elements A, 2.5 units of element B and 2 units of element C. Brand Q costing Rs 200 per bag contains 1.5 units of nutritional element A, 11.25 units of element B and 3 units of element C. The minimum requirements of nutrients A, B and C are 18 units, 45 units and 24 units respectively. Determine the number of bags of each brand which should be mixed in order to produce a mixture having a minimum cost per bag? What is the minimum cost of the mixture per bag? [6]

3. A dietician wishes to mix together two kinds of food X and Y in such a way that the mixture contains at least 10 units of vitamin A, 12 units of vitamin B and 8 units of vitamin C. The vitamin contents of one kg food are given below: [6]

Food	**Vitamin A**	**Vitamin B**	**Vitamin C**
X	1	2	3
Y	2	2	1

4. A manufacture makes tow types of toys A and B. three machines are needs for this purpose and the time (in minutes) required for each toy on the machines is given below: [6]

Types of Toys	**Machines**		
	I	II	III
A	12	18	6
B	6	0	9

Each machine is available for a maximum of 6 hours per day. If the profit on each toy on of type A is Rs 7.50 and that on each toy of type B is Rs 5, show that 15 toys f type A and 30 of type A and 30 of type B should be manufacture in a day to get maximum.

5. An aeroplane can carry a maximum of 200 passengers. A profit of Rs 1000 is made on each executive class ticket and a profit of Rs 600 is made on each economy class ticket. The airline reserves at least 20 seats for executive class. However, at least 4 times as many passengers prefer to travel by economy class than by the executive class. Determine how many tickets of each type must be sold in order to maximise the profit for the airline. What is the maximum profit? [6]

6. Two godowns A and B have grain capacity of 100 quintals and 50 quintals respectively. They supply to 3 ration shops, D, E and F whose requirements are 60, 50 and 40 quintals respectively. The cost of transportation per quintal from the godowns to the shops are given in the following table: [6]

Transportation cost per quintal (in Rs)		
From/To	**A**	**B**
D	6	4
E	3	2
F	2.50	3

How should be the supplies be transported in order that the transportation cost is minimum? What is the minimum cost?

7. An oil company has two depots A and B with capacities of 7000 L and 4000 L respectively. The company is to supply oil to three petrol pumps, D, E and F whose requirements are 4500L, 3000L and 3500L respectively. The distances (in km) between the depots and the petrol pumps is given in the following table: [6]

Transportation cost per quintal (in Rs)		
From/To	**A**	**B**
D	6	4
E	3	2
F	2.50	3

Assuming that the transportation cost of 10 litres of oil Rs 1 per km, how should the delivery be scheduled in order that the transportation cost is minimum? What is the minimum cost?

8. A fruit grower can use two types of fertilizer in his garden, brand P and Q. The amounts (in kg) of nitrogen, phosphoric acid, potash, and chlorine in a bag of each brad are given in the table. Testes indicate that the garden needs at least 240 kg of phosphoric acid, at least 270 kg of potash and at most 310 kg of chlorine. [6]

If the grower wants to minimize the amount of nitrogen added to the garden, how many bags of each brand should be used? What is the minimum amount of nitrogen added in the garden?

Kg per bag		
	Brand P	**Brand Q**
Nitrogen	3	3.5
Phosphoric acid	1	2
Potash	3	1.5
Chlorine	1.5	2

9. Anil wants to invest at most Rs 12,000 in bonds A and B. According to the rules he has to invest at least Rs 2000 in bond A and at least Rs 4000 in bond B. If the rate of interest on bond A is 8% per annum and on bond B, it is 10% per annum, how should be invest the money for maximum interest. [6]

10. A toy company manufactures two types of dolls. A and B market tests and available resources have indicated that the combined production level should not exceed 1200 dolls per week and the demand for dolls of type B is at most half of that for dolls of type A. Further, the production level of dolls of type A can exceed three times the production of dolls of other type by at most 600 units. If the company makes profit of Rs 12 and Rs 16 per doll respectively on dolls A and B, how many of each should be produced weekly in order to maximise the profit? [6]

Ans 01. Let the two tailors work for x days and y days respectively

Z = 150x + 200y

6x + 10y ≥ 60

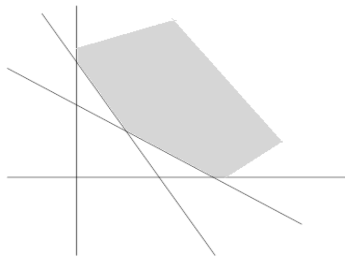
$\Rightarrow 3x+5y \geq 30$

$4x+4y \geq 32$

$\Rightarrow x+y \geq 8$ and x ≥ 0, y ≥ 0

Z is minimum = 1350

When A work for 5 days B work for 3 days



Ans 02. Let P = x

Q = y

Z = 250x + 200y

3x + 1.5y ≥ 18

2.5x + 11.25y ≥ 45

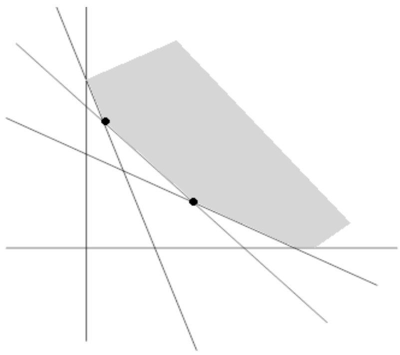
2x + 3y ≥ 24

x ≥0, y ≥ 0

Z = Rs 1950

When x = 3 y = 6

Number of bags of brand P = 3 bags of brand Q = 6



Ans 03. Let the dietician mix x Kg of food X and y Kg of food Y.

Z = 16x + 20y

x + 2y ≥ 10

2x + 2y ≥12

$\Rightarrow x+y \geq 6$

$3x+y \geq 8$

Cost is minimum = 112 When 2 Kg of food X and 4 Kg of food Y are mixed.

Ans 04. Let x toys of type A and y toys of type B

$Z = \frac{15}{2}x+5y$

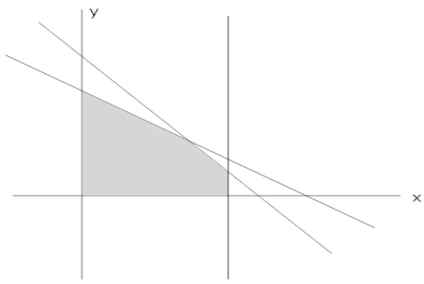
$12x+6y \leq 360$

$\Rightarrow 2x+y \leq 60$

$18x \leq 360$

$\Rightarrow x \leq 20$

$6x+9y \leq 360$



$\Rightarrow 2x+3y \le 120$

$x \ge 0,\ y \ge 0$

profit is maximum = 262.5 at A = 15 B = 30

Ans 05. X passengers travel by executive class and y passengers travel by economy class. L

Z = 1000x + 600y

$x + y \le 200$

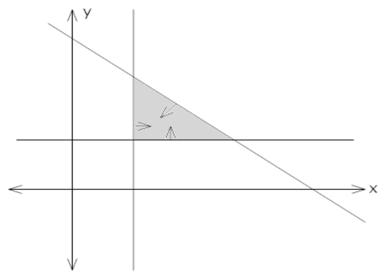
$x \ge 20$

$y \ge 80$

$x \ge 0,\ y \ge 0$

profit is maximum = 168000

When x = 120, y = 80



Ans 06. $Z = \frac{5}{2}x + \frac{3}{2}y + 410$

$60 - x \ge 0$

$50 - y \ge 0$

$100 - (x+y) \ge 0$

$x + y - 60 \ge 0$

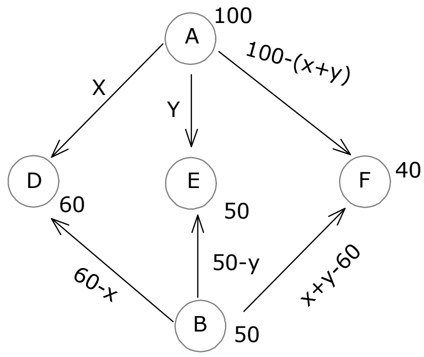
$x, y \ge 0$

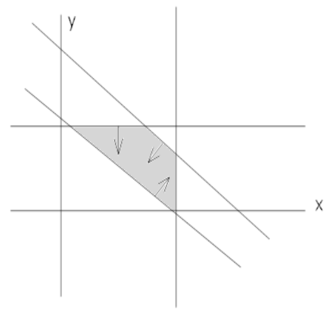
minimum = 510

when D → 10

E → 50

F → 40





Ans 07. $Z = \frac{3x}{10} + \frac{y}{10} + 3950$

$4500 - x \ge 0$

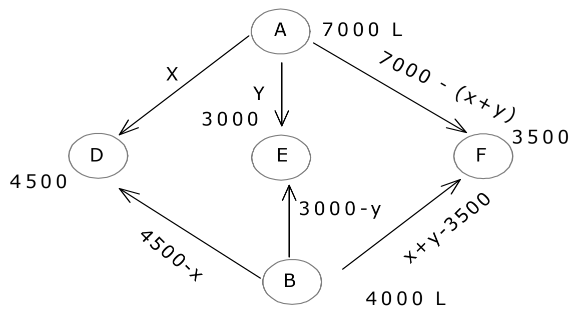
$3000 - y \ge 0$

$x + y - 3500 \ge 0$

$7000 - (x+y) \ge 0$

$x \ge 0, y \ge 0$

Minimum = 4400



Ans 08. Let the fruit grower mix x bags of brand P and Y bags of brand Q

$Z = 3x + \frac{7}{2}y$

$x + 2y \geq 240$

$3x + \frac{3}{2}y \geq 270$

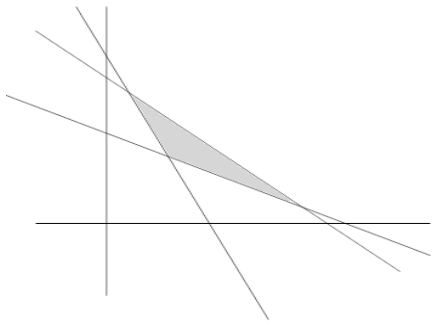
$\frac{3}{2}x + 2y \leq 310$

$x \geq 0, y \geq 0$

minimum = 470 Kg

P = 40

Q = 100



Ans 09. Let Anil invest x in bond A and Y in bond B,

$P = \frac{8x}{100} + \frac{10y}{100}$

x + y $\leq$12000

x $\geq$ 2000

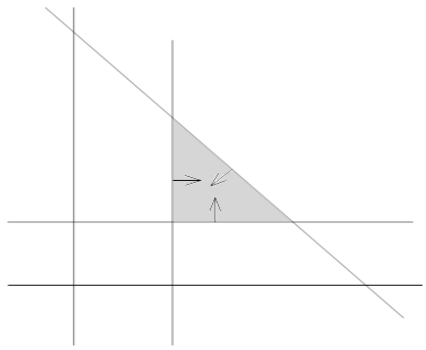
y $\geq$ 4000

x $\geq$ 0, y $\geq$ 0

P is maximum = 1160

x = 2000

y = 10,000



Ans 10. X dolls of type A and y dolls of type B.

Z = 12x + 16y

X + y $\leq$ 1200

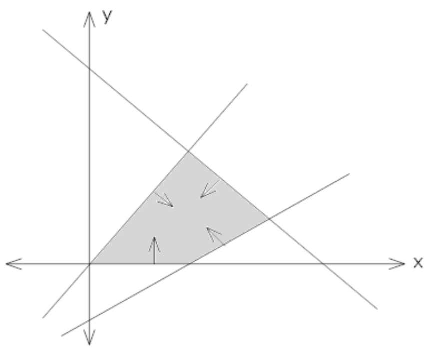
$y \leq \frac{x}{2}$

$x \leq 3y + 600$

Profit is maximum = 16000

A = 800

B = 400



CBSE TEST PAPER-01
CLASS - XII MATHEMATICS (Probability)

Topic: Probability

1. In a school there are 1000 students, out of which 430 are girls. It is known that out of 430, 10% of the girls study in class XII. What is the probability that a student chosen randomly studies in class XII given that the chosen student is a girl? [4]

2. A die thrown three times. Events A and B are defined as below. [4]

   A : 4 on the third throw

   B : 6 on the first and 5 on the second throw.

   Find the probability of A given that B has already occurred.

3. Mother, father and son line up at random for a family picture [4]

   E : Son on one end

   F : Father in middle

   Find (E |F)

4. An instructor has a question bank consisting of 300 easy True / False questions, 200 difficult True / False questions, 500 easy multiple choice questions and 400 difficult multiple choice questions. If a question is selected at random from the question bank, what is the probability that it will be an easy question given that it is a multiple choice question? [4]

5. If A and B are two independent events, then the probability of occurrence of at least one of A and B is given by 1 – P (A') P (B'). [4]

6. A – Box of oranges is inspected by examining three randomly selected oranges drown without replacement. It all the three oranges are good, the box is approved for sale, other wise, it is rejected. Find the probability that a box containing 15 oranges out of which 12 are good and 3 are bad ones will be approve for sale. [4]

7. A fair coin and an unbiased die are tossed. Let A be the event head appear on the coin and B be the event 3 on the die. [4]

   Check weather A and B are independent events or not.

8. Probability of solving specific problem independently by A and B are $\frac{1}{2}$ and $\frac{1}{3}$ respectively of both try to solve the problem independently, find the probability that [4]

(i) the problem is solved

(ii) Exactly one of them solves the problem.

9. In a hostel 60% of the students read Hindi news paper, 40% read English news paper and 20% read both Hindi and English news papers. A student is selected et random. [4]

(a) Find the probability that she read neither Hindi nor English news papers.

(b) If the reads Hindi news paper, find the probability that she reads English news paper.

(c) If she reads English news papers, find the probability that she reads Hindi news paper.

10. Three cards are drawn successively, without replacement from a pack of 52 well shuffled cards. What is the probability that first two cards are kings and the third card drawn is ace. [4]

Topic: Probability {answers}

Ans 01. Let E, student chosen randomly studies in class XII, F randomly chosen student is girl.

P (E|F) = ?

$$P(F) = \frac{430}{1000} = 0.43$$

$$P(E \cap F) = \frac{43}{1000} = 0.043$$

$$P(E/F) = \frac{P(E \cap F)}{P(F)}$$

$$= \frac{0.043}{0.43} \qquad = 0.1$$

Ans 02. Total sample space = 216

$$A = \begin{bmatrix} (1,1,4) & (1,2,4)....(1,6,4) & (2,1,4) & (2,2,4)....(2,6,4) \\ (3,1,4) & (3,2,4)....(3.6,4) & (4,1,4) & (4,2,4)....(4,6,4) \\ (5,1,4) & (5,2,4)....(5,6,4) & (6,1,4) & (6,2,4)....(6,6,4) \end{bmatrix}$$

$$B = \{(6,5,1)\ (6,5,2)\ (6,5,3)\ (6,5,4)\ (6,5,5)\ (6,5,6)\}$$

$$A \cap B = \{6,5,4\}$$

$$P\ (B)\ = \frac{6}{216},\ \text{P}\ (\text{A} \cap \text{B}) = \frac{1}{216}$$

$$P\ (\text{A/B}) = \frac{P\ (\text{A} \cap \text{B})}{P\ (\text{B})} = \frac{\frac{1}{216}}{\frac{6}{216}} = \frac{1}{6}$$

Ans 03.

$$S = \{mfs, msf, fms, fsm, smf, sfm\}$$

$$E = \{\text{mfs, fms, smf, sfm}\}$$

$$F = \{\text{mfs, sfm}\}$$

$$E \cap F = \{mfs, sfm\}$$

$$P\ (E/F) = \frac{\text{P}\ (\text{E} \cap \text{F})}{\text{P}\ (\text{F})} \quad = \frac{\frac{2}{6}}{\frac{2}{6}} = 1$$

Ans 04. let E : easy question

F : multiple choice question

total questions = 300 + 200 + 500 + 400 = 1400

$$P\ (\text{F})\ \frac{500 + 400}{1400} = \frac{9}{14}$$

$$P\ (\text{E} \cap \text{F}) = \frac{500}{1400} = \frac{5}{14}$$

$$P\ (\text{E/F}) = \frac{P\ (\text{E} \cap \text{F})}{P\ (\text{F})}$$

$$= \frac{\frac{5}{14}}{\frac{9}{14}}$$

$$= \frac{5}{9}$$

Ans 05. P (at least one of A and B) = P (A∪B)

= P (A) + P (B) – P (A∩B)

= P (A) + P (B) – P (A). P (B)

= P (A) + P (B) [1 – P (A)]

= P (A) + P (B) P (A')

= 1 – P (A') + P (B) P (A')

= 1 – P (A') [1 – P (B)]

= 1 – P (A') P ((B'))

Ans 06. required

$$\textit{probability} = \frac{12}{15} \times \frac{11}{14} \times \frac{10}{13}$$

$$= \frac{44}{91}$$

Ans 07. $$S = \begin{Bmatrix} (H,1) & (H,2) & (H,3) & (H,4) & (H,5) & (H,6) \\ (T,1) & (T,2) & (T,3) & (T,4) & (T,5) & (T,6) \end{Bmatrix}$$

A : Head appear on the coin

B : 3 appear on the lice

$A = \{(H1),\ (H2),\ (H3),\ (H4),\ (H5),\ (H6)\}$

$B = \{(H,3),\ (T,3)\}$

$A \cap B = \{(H,3)\}$

$P\ (\text{A}) = \frac{6}{12} = \frac{1}{2}, P\ (B) = \frac{2}{12} = \frac{1}{6}$

$P\ (\text{A} \cap \text{B}) = \frac{1}{12}$

$P\ (\text{A}) \times \text{P (B)} = \frac{1}{2} \times \frac{1}{6}$

$= \frac{1}{16}$

$= \text{P}\ (A \cap B)$

Hence A and B are independent

Ans 08. E_1 : A solves the problem

E_2 : B solves the problem

$P(E_1) = \frac{1}{2}$ and $P(E_2) = \frac{1}{3}$

(i) P (the problem is solved)

= 1 – P (the problem is not solved)

$= 1 - P\left[\left(1-\frac{1}{2}\right)\ \left(1-\frac{1}{3}\right)\right]$

$= 1 - \frac{1}{3} = \frac{2}{3}$

(ii) P Exactly one of their solves the problem

$= \text{P}(E_1)\ (1-P(E_2)) + \text{P}\ (E_2)\ (1-P(E_1))$

$= \frac{1}{2}\left(1-\frac{1}{3}\right) + \frac{1}{3}\left(1-\frac{1}{2}\right) = \frac{1}{2}$

Ans 09. E : Student read Hindi newspaper

F : Student read English newspaper

$P(E) = \frac{60}{100} = \frac{3}{5}$, $P(F) = \frac{40}{100} = \frac{2}{5}$

$P(E \cap F) = \frac{20}{100} = \frac{1}{5}$

(i) $P(E \cup F)' = 1 - P(E \cup F)$

$= 1 - \left(P(E) + P(F) - P(E \cap F)\right)$

$= \frac{1}{5}$

(ii) $P(F/E) = \frac{P(E \cap F)}{P(E)}$

$= \frac{\frac{1}{5}}{\frac{3}{5}} = \frac{1}{3}$

(iii) $P(E/F) = \frac{P(E \cap F)}{P(F)}$

$= \frac{\frac{1}{5}}{\frac{2}{5}} = \frac{1}{2}$

Ans 10. By multiplication theorem

$\text{prob} = \frac{4}{52} \times \frac{3}{51} \times \frac{4}{50}$

$= \frac{2}{5525}$

CBSE TEST PAPER-02
CLASS - XII MATHEMATICS (Probability)

Topic: Probability

1. Given three identical boxes I, II and III each containing two coins. In box-I both coins are gold coins, in box-II, both are silver coins and in the box-III, there is one gold and one silver coin. A person chooses a box at random and takes out a coin. If the coin is of gold, what is the probability that the other coin in the box is also of gold. [6]

2. Suppose that the reliability of a HIV test is specified as follows of people having HIV, 90% of the test detect the disease but 10% go undetected of people free of HIV, 99% of the test are Judged HIV – tive but 1% are diagnosed as showing HIV +tive. From a large population of which only 0.1% have HIV, one person is selected at random, given the HIV test, and the pathologist reports him/her is HIV +tive what is the probability that the person actually has HIV. [6]

3. In a factory which manufactures bolts, machines. A, B and C manufacture respectively 25%, 35% and 40% of the bolts. Of their output 5,4 and 2 percent are respectively defective bolts. A bolt is drown at random from the product and is found to be defective. What is the probability that it is manufactured by the machine B. [6]

4. A doctor is to visit a patient. From the past experience, it is known that the probabilities that he will come by train, bus, scooter or by other mean of transport are respectively $\frac{3}{10}$, $\frac{1}{5}$, $\frac{1}{10}$ and $\frac{2}{5}$. The probabilities that he will be late are $\frac{1}{4}$, $\frac{1}{3}$, and $\frac{1}{12}$ if he comes by train, bus and scooter respectively, but he comes by other means of transport, that he will not the late. When he arrives he is late. What is the probability that he comes by train. [6]

5. A man is known to speak truth 3 out of 4 times. He throws a die and reports that it is a six. Find the probability that it is actually a six. [6]

6. In answering a question on a multiple choice test a student either knows the answer or guesses Let $\frac{3}{4}$ be the probability that he knows the answer and ¼ be the probability he guesses. Assuming that a student who guesses at the answer will be correct with probability $\frac{1}{4}$. What is the probability that the student knows the answer given that he answered it correctly. [6]

7. A laboratory blood test is 99% effective in detecting a certain disease when it is in fact, present. However, the test also yields a false positive result for 0.5% of the healthy person tested (i. e if a healthy person is test then with probability 0.005 the test will imply he has the disease) If 0.1 percent of the population actually has the disease, what is the probability that a person has the disease given that his test result is positive. [6]

8. An insurance company insured 2000 scooter drivers, 4000, car drivers and 6000 truck drivers. The probability of accidents is 0.01, 0.03 and 0.15 respectively. One of the insured persons meet with an accident what is the probability that he is scooter driver. [6]

9. A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn and are found to be both diamonds. Find the probability of the lost card being a diamond. [6]

10. Suppose a girl throws a die. If she gets a 5 or 6, she tosses a coin three times and notes the number of heads. If she gets 1, 2, 3, 4, she tosses a coin once and notes whether a head or tail is obtained. If she obtained exactly one had, what is the probability that she threw 1, 2, 3 or 4 with the die? [6]

Topic: Probability {answers}

Ans 01. let E_1, E_2 and E_3 be the events that boxes I, II and III are chosen.

$P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$

let A be the event the coin drawn is of gold.

$$p(A|E_1) = \frac{2}{2} = 1$$
$$P(A|E_2) = 0$$
$$P(A|E_3) = \frac{1}{2}$$
$$P(E_1|A) = \frac{P(E_1)\,P(A|E_1)}{P(E_1)\,P(A|E_1) + P(E_2)\,P(A|E_2) + P(E_3)\,P(A|E_3)}$$
$$= \frac{2}{3}$$

Ans 02. let E denote the event that the person selected is actually having HIV and A the event that the person's HIV test is diagnosed as + tive.

let E' not having HIV.

$$P(E) = 0.1\% = \frac{0.1}{150} = 0.001$$
$$P(E') = 1 - P(E) = 0.999$$
$$P(A|E) = 90\% = \frac{90}{100} = 0.9$$
$$P(A|E') = 1\% = \frac{1}{100} = 0.01$$
$$P(E|A) = \frac{P(E)\,P(A|E)}{P(E)\,P(A|E) + P(E')\,P(A|E')}$$
$$= 0.083$$

Ans 03. let B_1 = bolt is manufactures by A

B_2 = bolt is manufactured by B

B_3 = bolt is manufactured by C

let E bolt is defective

$P(B_1) = 25\% = 0.25$

$P(B_2) = 0.35$

$P(B_3) = 0.40$

$P(E|B_1) = 5\% = 0.05$

$P(E|B_2) = 0.04$

$P(E|B_3) = 0.02$

$$P(B_2|E) = \frac{P(B_2)\,P(E|B_2)}{P(B_1)\,P(E|B_1) + P(B_2)\,(P\,E|B_2) + P(B_3)\,P(E|B_3)}$$

$$= \frac{28}{69}$$

Ans 04. let E be the event that the doctor visits the patient late and let T_1, T_2, T_3, T_4, be the event that the doctor comes by train, bus, scooter and other means of Transport respectively.

$$P(T_1) = \frac{3}{10},\ P(T_2) = \frac{1}{5},\ P(T_3) = \frac{1}{10},\ P(T_4) = \frac{2}{5}$$

$$P(E|T_1) = \frac{1}{4},\ P(E|T_2) = \frac{1}{3},\ P(E|T_3) = \frac{1}{12},\ P(E|T_4) = 0$$

$$P(T_1|E) = \frac{P(T_1)\,P(E|T_1)}{P(T_1)\,P(E|T_1) + P(T_2)P((E|T_2) + P(T_3)\,P(E|T_3) + P(T_4)\,P(E|T_4)}$$

$$= \frac{1}{2}$$

Ans 05. let E be the event that the man reports that six occurs in the throwing of the dice and let S_1 be the event that six occurs and S_2 be the event six does not occur.

$$P(S_1) = 1/6\quad P(S_2) = 5/6$$

$$P(E|S_1) = \frac{3}{4},\ \ P(E|S_2) = 1 - \frac{3}{4} = \frac{1}{4}$$

$$P(S_1|E) = \frac{P(S_1)\,P(E|S_2)}{P(S_1)\,P(E|S_1) + P(S_2)\,P(E|S_2)}$$

$$= \frac{3}{8}$$

Ans 06. E_1 : the student knows the answer

E_2 : the student guesses the answer

A the answer is correct

$P(A/E_1) = 1$, $P(A/E_2) = \frac{1}{4}$

$$P\ (E_1 / A) = \frac{P\ (E_1)\ P\ (A / E_1)}{P\ (E_1)\ P\ (A / E_1)\ +\ P\ (E_2)\ P\ (A / E_2)}$$

$$= \frac{1 \times \frac{3}{4}}{1 \times \frac{3}{4} + \frac{1}{4} \times \frac{1}{4}} = \frac{12}{13}$$

Ans 07. E_1 : the person has the disease

E_2 : the person is healthy

A : test is positive

P (E_1) = 0.1 $\frac{1}{10}$, P (E_2) = 1-$\frac{1}{10}$ = $\frac{9}{10}$

P (A/E_1) = $\frac{99}{100}$, P (A/E_2) = 0.005

$$= \frac{5}{1000}$$

$$P\ (\mathrm{E_1/A}) = \frac{\mathrm{P\ (E_1)\ P\ (A/E_1)}}{\mathrm{P\ (E_1)\ P\ (A/E_1) + P\ (E_2)\ P\ (A/E_2)}}$$

$$= \frac{\frac{99}{100} \times \frac{1}{10}}{\frac{99}{100} \times \frac{1}{10} + \frac{5}{1000} \times \frac{9}{10}}$$

$$= \frac{22}{23}$$

Ans 08. E_1 : Insured person is a scooter driver

E_2 : Insured person is a car driver

E_3 : Insured person is a truck driver

$$P\ (\mathrm{E_1}) = \frac{2000}{2000 + 4000 + 6000} = \frac{2000}{12000} = \frac{1}{6}$$

$$P\ (\mathrm{E_2}) = \frac{1}{3}$$

$$P\ (\mathrm{E_3}) = \frac{1}{2}$$

let A Insured person meets with an accident

$P\ (A/E_1) = 0.01 = \dfrac{1}{100}$

$P\ (A/E_2) = 0.03 = \dfrac{3}{100}$

$P\ (A/E_3) = 0.15 = \dfrac{15}{100}$

$$P\ (E_1/A) = \frac{P\ (E_1)\ P\ (A/E_2)}{P\ (E_1)\ P\ (A/E_1) + P\ (E_2)\ P\ (A/E_2) + P\ (E_3)\ P\ (A/E_3)}$$

$$= \frac{1}{52}$$

Ans 09. E1 : lost card is diamond

E2 : lost card is not diamond

let A : two cards drawn from the remaining pack are diamonds.

$P\ (E_1) = \dfrac{13}{52} = \dfrac{1}{4},\ P(E_2) = \dfrac{39}{52} = \dfrac{3}{4}$

$P\ (A/E_1) = \dfrac{12C_2}{51C_2} = \dfrac{12\times11}{51\times50}$

$P\ (A/E_2) = \dfrac{13C_2}{51C_2} = \dfrac{13\times12}{51\times50}$

$$P\ (E_1/A) = \frac{P\ (E_1)\ P\ (A/E_1)}{P\ (E_1)\ P\ (A/E_1) + P\ (E_2)\ P\ (A/E_2)}$$

$$= \frac{11}{50}$$

Ans 10. E_1 : 1, 2, 3, 4 is shown on dice

E_2 : 5 or 6 is shown on dice

$P\ (E_1) = \dfrac{4}{6} = \dfrac{2}{3},\ P\ (E_2) = \dfrac{2}{6} = \dfrac{1}{3}$

let A exactly one head shown up

$P\ (A/E_1) = \dfrac{1}{2},\ P\ (A/E_2) = \dfrac{3}{8}$

$$P\ (E_1/A) = \frac{P\ (E_1)\ P\ (A/E_1)}{P\ (E_1)\ P\ (A/E_1) + P\ (E_2)\ P\ (A/E_2)} = \frac{8}{11}$$

CBSE TEST PAPER-03
CLASS - XII MATHEMATICS (Probability)

Topic: - Probability

1. Find the probability distribution of number of doublets in three throws of a pair of dice. [4]

2. Let X denote the no of hours you study during a randomly selectee school day. The probability that X can take the values x, has the following form where K is some unknown constant [4]

$$p(\chi = x) = \begin{cases} 0.1 \text{ if } x = 0 \\ kx \text{ if } x = 1, \text{ or } 2 \\ K(5 - x) \text{ if } x = 3 \text{ or } 4 \\ 0, \text{ otherwise} \end{cases}$$

(a) Find the value of K
(b) What is the probability that you study at least two hours.
Exactly two hours? At most 2 hr.

3. Find the variance of the number obtained on a throw of an unbiased die. [4]

4. Two cards are drawn simultaneously (or successively without replacement) for a well shuffled of 52 cards. Find the mean, variance and standard deviation of the number of kings. [6]

5. From a lot of 30 bulbs which include 6 defectives, a sample of 4 bulbs is drawn at random with replacement. Find the probability distribution of the number of defective bulbs. [6]

6. In a meeting 70% of the members favour and 30% oppose a certain proposal. A member is selected at random and we take x = 0 if he opposed and x = 1 if he is in favour. Find E (x) and var (x). [6]

7. A and B throw a die alternatively till one of them gets a '6' and win the game. Find their respective probabilities of winning if A starts first. [6]

8. Find the mean of the Binomial distribution B $\left(4, \frac{1}{3}\right)$ [6]

9. If a leap year is selected at random, what is the chance that it will contain 53 Tuesdays? [4]

10. Bag I contain 3 red and 4 black balls and bag II contain 4 red and 5 black balls. One ball is transferred from Bag I to Bag II and then a ball is drawn from Bag II. The ball so drawn is fund to be red in colour. Find the probability that the transferee ball is black. [6]

CBSE TEST PAPER-03

CLASS - XII MATHEMATICS (Probability)
Topic: - Probability [ANSWERS]

Ans 01. Let x denote the number of doublets x = 0, 1, 2, 3

Probability of getting doublet = $\frac{6}{36} = \frac{1}{6}$

Probability of not getting doublet = $1 - \frac{1}{6} = \frac{5}{6}$

X	0	1	2	3
P (x)	$\frac{125}{216}$	$\frac{75}{216}$	$\frac{15}{216}$	$\frac{1}{216}$

Ans 02. The probability distribution of x is

x	0	1	2	3	4
P (x)	0.1	k	2k	2k	k

(a) $\sum_{i=1}^{n}$ pi = 1

0.1 + K + 2K + 2K + K = 1
K = 0.15

(b) p (study atleast two hr) = p (x ≥ 2)

= 2K + 2K + K
= 5K
= 5×0.15
= 0.75

p (Study exactly two hr) = p (x = 2)

= 2K
= 2×0.15
= 0.3

p (Study et most two hr) = p (x $\leq$ 2)

= 0.1 + K + 2K
= 0.55

Ans 03. S = {1, 2, 3, 4, 5, 6}

$p(1) = p(2) = p(3) = p(4) = p(5) = p(6) = \frac{1}{6}$

x	1	2	3	4	5	6
P (x)	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

$$\sum (x) = 1\times\frac{1}{6}+2\times\frac{1}{6+}+3\times\frac{1}{6}+4\times\frac{1}{6}+5\times\frac{1}{6}+6\times\frac{1}{6}=\frac{21}{6}$$

$$E(x^2) = 1^2\times\frac{1}{6}+2^2\times\frac{1}{6}+3^2\times\frac{1}{6}+4^2\times\frac{1}{6}+5^2\frac{1}{6}+6^2\frac{1}{6}=\frac{91}{6}$$

$$\text{Var}(x) = E(x^2) - (\Sigma (x))^2$$

$$= \frac{91}{6}-\left(\frac{21}{6}\right)^2$$

$$= \frac{91}{6}-\frac{441}{36}$$

$$= \frac{35}{12}$$

Ans 04. Let x denote the number of kings in a draw of two cards.

$$p(x=0) = \frac{48C_2}{52C_2}=\frac{188}{221}$$

$$p(x=1) = \frac{4C_1 \times 48C_1}{52C_2}=\frac{32}{221}$$

$$p(x=2) = \frac{4C_2}{52C_2}=\frac{1}{221}$$

x	0	1	2
p (x)	$\frac{188}{221}$	$\frac{32}{221}$	$\frac{1}{221}$

$$x = E(x) = 0\times\frac{188}{221}+1\times\frac{32}{221}+2\times\frac{1}{221}$$

$$=\frac{34}{221}$$

$$E = (x^2) = 0^2 \times \frac{188}{221} + 1^2 \times \frac{32}{221} + 2^2 \times \frac{1}{221} = \frac{36}{221}$$

$$Var\ (x) = \sum\ (x^2) - \left(\sum(x)\right)^2$$

$$= \frac{36}{221} - \left(\frac{34}{221}\right)^2$$

$$= \frac{6800}{(221)^2}$$

$$\sigma\ x = \sqrt{\text{var}\ (x)}$$

$$= \frac{\sqrt{6800}}{221} = 0.37$$

Ans 05. S = 30

A = 6 defective bulbs

$$P = \frac{6}{30} = \frac{1}{6} \text{ defective bulbs}$$

$$q = \frac{24}{30} = \frac{4}{5} \text{ non defective bulb}$$

$$p\ \left(x = 0\right) = \frac{4}{5} \times \frac{4}{5} \times \frac{4}{5} \times \frac{4}{5} = \frac{256}{625}$$

$$p\left(x = 1\right) = 4c_1\ p^1\ q^3 = 4\ \left(\frac{1}{5}\right)\ \left(\frac{4}{5}\right)^3 = \frac{256}{625}$$

$$p\left(x = 2\right) = 4c_2\ p^2\ q^2 = 6\ \left(\frac{1}{5}\right)^2\ \left(\frac{4}{5}\right)^2 = \frac{96}{625}$$

$$p\left(x = 3\right) = 4c_3\ p^3\ q^1 = 4\ \left(\frac{1}{5}\right)^3\ \left(\frac{4}{5}\right) = \frac{16}{625}$$

$$p\left(x = 4\right) = 4c_0\ p^4\ q^0 = 1\ \left(\frac{1}{5}\right)^4 = \frac{1}{625}$$

x	0	1	2	3	4
p (x)	$\frac{256}{625}$	$\frac{256}{625}$	$\frac{96}{625}$	$\frac{16}{625}$	$\frac{1}{625}$

Ans 06.

x	0	1
p (x)	$\frac{30}{100}$	$\frac{70}{100}$

$$E(x) = o \times \frac{30}{100} + 1 \times \frac{70}{100} = \frac{7}{10}$$

.

$$E(x^2) = o^2 \times \frac{30}{100} + 1^2 \times \frac{70}{100} = \frac{70}{100} = \frac{7}{10}$$

$$\text{Var} = \text{E}(x^2) - (E(x))^2 = \frac{7}{10} - \frac{49}{100} = \frac{70-49}{100} = \frac{21}{100}$$

Ans 07. S denote the success

F denote the failure

$$P(S) = \frac{1}{6},\ \text{P}(F) = \frac{5}{6}$$

$$P \text{ (A win in the first throw) = (S) = } \frac{1}{6}$$

$$P \text{ (A win in the thirs throw) } = \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6}$$

$$P \text{ (A win in the 5th throw) = } \left(\frac{5}{6}\right)^4 \left(\frac{1}{6}\right)$$

$$P \text{ (A wins) = } \frac{1}{6} + \left(\frac{5}{6}\right)^2 \left(\frac{1}{6}\right) + \left(\frac{5}{4}\right)^2 \left(\frac{1}{6}\right) + \ldots\ldots$$

$$= \frac{\frac{1}{6}}{1 - \frac{25}{36}} \quad \left[\because \qquad S = \frac{a}{1-r}\right.$$

$$= \frac{6}{11}$$

Ans 08. Let x be the random variable whose probability distribution is B $\left(4 \frac{1}{3}\right)$

$$n = 4,\ \text{p} = \frac{1}{3},\ \text{q} = 1 - \frac{1}{3} = \frac{2}{3}$$

$$P(X = x) = 4c_x \left(\frac{2}{3}\right)^{4-x} \left(\frac{1}{3}\right)^x,\ \text{x = 0, 1, 2, 3, 4,}$$

$$\text{P}(X = 0) = 4c_0 \left(\frac{2}{3}\right)^4$$

$$\text{P}(X = 1) = 4c_1 \left(\frac{2}{3}\right)^3 \left(\frac{1}{3}\right)$$

$$P(X=2)=4c_2\left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)^2$$

$$P(X=3)=4c_3\left(\frac{2}{3}\right)\left(\frac{1}{3}\right)^3$$

$$P(X=4)=4c_4\left(\frac{1}{3}\right)^4$$

meen = $\sum_{i=1}^{4}$ xi p (xi)

$$= 4\times\frac{2^3}{3^4}+2\times6\times\frac{2^2}{3^4}+3\times4\times\frac{2}{3^4}+4\times1\times\frac{1}{3^4}$$

$$= \frac{108}{81}=4/3$$

Ans 09. A leap contain 52 weeks and two additional days which can be Sunday and Monday, Monday and Tuesday, Tuesday and Wednesday, Wednesday and Thursday, Thursday and Friday, Friday and Saturday, Saturday and Sunday.

req. prob. = $\frac{2}{7}$

Ans 10. Let E_1: red ball is transferred from bag I to bag II.

E_2 : black is transferred from bag I to bag II.

$$P(E_1)=\frac{3}{3+4}=\frac{3}{7}$$

$$P(E_2)=\frac{4}{3+4}=\frac{4}{7}$$

Let A Red ball is drawn from bag II

$$P\ (A/E_1)=\frac{4+1}{(4+1)+5}=\frac{5}{10}=\frac{1}{2}$$

$$P\ (A/E_2)=\frac{4}{4+(5+1)}=\frac{4}{10}=\frac{2}{5}$$

req. probability $P\ (E_2/A)=\frac{P(E_2)P(A/E_2)}{P(E_1)P(A/E_1)+P(E_2)P(A/E_2)}$

$$= \frac{\frac{4}{10}\times\frac{4}{7}}{\frac{1}{2}\times\frac{3}{7}+\frac{4}{10}\times\frac{4}{7}}=\frac{16}{31}$$

CBSE MIXED TEST PAPER-01

(First Terminal Examination)

CLASS - XII MATHEMATICS

[Time : 1.50 hrs.] **[M. M.:** 50]

General Instructions:-

(i) All questions are compulsory.

(ii) There is no overall choice. However internal choice has been provided.

(iii) Marks allotted to each question are indicated against it.

Q1. If $A=\begin{bmatrix}2 & -1\\ 3 & 4\end{bmatrix}$ Find $A^2 + 2A$ 2 marks

Q2. Construct a 3 x 3 matrix A = [aij] whose elements are given by aij = $\frac{(i+2j)^2}{2}$ 2 marks

Q3. Find the value λ which makes the following points collinear (2,3) , (5, λ) & (1,1) 2 marks

Q4. Find the value f x for which $\begin{vmatrix}3 & x\\ x & 1\end{vmatrix}=\begin{vmatrix}3 & 2\\ 4 & 1\end{vmatrix}$ 2 marks

Q5. If A = $\begin{bmatrix}1 & -2 & 3\\ -4 & 2 & 5\end{bmatrix} and B=\begin{bmatrix}2 & 3\\ 4 & 5\\ 2 & 1\end{bmatrix}$. Find AB and BA. Is AB = BA? 2 marks

Q6. Express the following matrix as the sum of a symmetric & a skew symmetric matrix.

4 marks

$$A=\begin{bmatrix}2 & 1 & 3\\ 1 & 1 & 4\\ -1 & 6 & 2\end{bmatrix}$$

Q7. Find the inverse of following matrix using elementary row transformation. 4 marks

$$\begin{bmatrix} 2 & -1 & 3 \\ 1 & 3 & -1 \\ 3 & 2 & 1 \end{bmatrix}$$

Q8. If A = $\begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix}$ verify that $A^2 - 4A - I_2 = 0$ and hence find A^{-1}. 4 marks

Q9. Prove that $\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ac & bc & c^2+1 \end{vmatrix} = 1+a^2+b^2+c^2$ 4 marks

Q10. If a, b, c are in A.P, show that : 4 marks

$$\begin{bmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{bmatrix} = 0$$

Q11. Prove that: $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+1\right)$ 4 marks

Q12. Find the equation of line joining A (1,3) and B(0,0) using determinants and find k if D(k,a) is a point such that area of triangle ABD is 3 sq: units. 4 marks

Q13. Solve the following system of eq ms using matrix method 6 marks

4 x – 5y – 2z = 2

5x – 4y + 2z = 3

2x + 2y + 8z = 1

Q14. Solve the following system of equation using matrix method. 6 marks

X + 2y = 3z = -4

2x + 3y + 2z = 2

3x – 3y – 4z = 11

CBSE MIXED TEST PAPER-02

(First Terminal Examination)

CLASS - XII MATHEMATICS

[Time : 3.00 hrs.] **[M. M.:** 100]

General Instructions:-

(i) All questions are compulsory.

(ii) There is no overall choice. However internal choice has been provided.

(iii) The questions paper consist of 29 questions divided into three sections, A, B and C. Section A comprises of 10 question of 1 mark each, section B comprises of 12 question of 4 marks each. Section C comprises of 7 questions of 6 marks each.

Section A

1. Find the value of $\sin\left[\frac{\pi}{3}-\sin^{-1}\left(-\frac{1}{2}\right)\right]$

2. For what value of x, the following matrix is singular. $\begin{bmatrix} 5-x & x+1 \\ 2 & 4 \end{bmatrix}$

3. If $A=\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}, 0<\theta<\frac{\pi}{2}$ Find θ when A + A1 =1.

4. The total revenue (in Rs.) from sale of x units is given by R(x) = $3x^2$ + 36 x + 5. Find the marginal revenue when x = 15.

5. A balloon which always remains spherical has a variable diameter $\frac{3}{2}(2x+1)$, find the rate of change of volume w.r.t.x.

6. $If\ \frac{d}{dx}(f(x)) = 4x^3 - \frac{3}{x^4}$ such that f(2) = 0. Find f(x).

7. Find the values of a and b if the slope of tangent to the curve xy + ax + by = 2 at (1,1) is 2.

8. Evaluate: $\int \sin^2 x dx$

9. If $y = \sqrt{x + \sqrt{x + \sqrt{x + ...\infty}}}$ prove that $\frac{dy}{dx} = \frac{1}{2y-1}$

10. Using determinants, find the area of triangle whose vertices are (-2,4);(2,-6) and (5, 4)

Section B

11. Solve for x: $\tan^{-1}\left(\frac{x-1}{x-2}\right) + \tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}$

12. Find $\frac{dy}{dx}$ if y = $\tan^{-1}\left[\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right], 0 < |x| < 1$

13. A practical moves along the curve 6y = x3 + 2. Find the points on the curve at which the y co-ordinate is changing 8 times as fast as x co-ordinate.

Or

Find the interval in which f(x) = $x^3 + 3x^2 - 105\,x + 25$ is increasing or decreasing .

14. Evaluate: $\int \log(1+x^2)dx$ Or Evaluate $\int \frac{x^2+5x+3}{x^2+3x+2}dx$

15. Verify Rolle's Theorem for the function f(x) = $\cos 2\left(x - \frac{\pi}{4}\right)$ on the interval $\left[0, \frac{\pi}{4}\right]$

16. Prove that:

$$\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (b-c)(c-a)(a-b)$$

17. If y = $\frac{\log x}{x}$, prove that $\frac{d^2y}{dx^2} = \frac{2\log x - 3}{x^3}$

18. Evaluate : $\int \frac{dx}{\sin(x-a)\cos(x-b)}$ Or Evaluate: $\int_0^{\pi/4} \log(1+\tan x)dx$

20. Find a point on the curve y = (x-3)2 where the tangent is parallel to the line joining (4, 1) and (3,0).

21. Evaluate $\int_0^4 \frac{dx}{\sqrt{x^2+2x+3}}$

22. Find $\frac{dy}{dx}$ if y = $x^{\cos x}$ + $(\cos x)^x$

Section – C

23. $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ and use it to solve the following system of equations.

x– y + z = 4, x – 2y – 2z = 9, 2x + y + 3z = 1

24. The sum of three numbers is 6. If we multiply third number by 3 and add second number to it. We get 11. By adding first and third numbers, we get double of second number, representing it algebraically and the number using matrix method.

25. A helicopter of enemy is flying along the curve y = x^2 + 7. A soldier, paced at (3, 7) wants to scoot down the helicopter when it is nearest to him. Find the nearest distance .

26. A rectangular sheet of tin 45 cm by 24 cm is to be made into a box outshout top, by cutting off squares from the corners and folding up the flaps. What should be the side of the squarer to be cut off so that the volume of ht boxes maximum? **Or**

Show that the right circular cylinder of given surface and maximum volume is such that its height is equal to the diameter of the base.

27. Find the area of the region enclosed by the crave y^2 = 4x and the line y = x.

28. Find the local maximum and local minimum values of the function given by : f(x) $3x^4 + 4x^3 - 12x^2 + 12$

29. Show that the matrix A = $\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ satisfies the equation $A^2 - 4A - 5l = 0$ hence find A^{-1}

CBSE MIXED TEST PAPER-03

(First Unit Test)

CLASS - XII MATHEMATICS

[Time : 1.50 hrs.] **[M. M.:** 40]

General Instructions:-

(i) All questions are compulsory.

(ii) There is no overall choice. However internal choice has been provided.

(iii) Marks allotted to each question are indicated against it.

1. Check the relation R in the set of R of real number, defined as R = {(a,b): a$\leq$ b^2} is reflexive — 1 mark

2. If a*b = a^2 + 2b-3, find 3*2. — 1 mark

3. Find the principal value of $Cot^{-1}\left(-\frac{1}{\sqrt{3}}\right)$ — 1 mark

4. Find the value of $Sin^{-1}\left(\sin\frac{3\pi}{5}\right)$ — 1 mark

5. Consider $f : R_+ \to [5, \infty)$ given by f(x) = 9x^2 + 6x – 5. Show that f is invertible. Find its inverse. — 4 marks

6. Define a binary operation * on the set A = {1, 2, 3, 4} as $a * b = \begin{cases} a+b & if \quad a+b<5 \\ a+b \quad -4 & if \geq 5 \end{cases}$ Find the identify for this operation and inverse of all elements of A (If exists) — 4 marks

7. Prove that $Sin^{-1}\frac{12}{13} + Cos^{-1}\frac{4}{5} + \tan^{-1}\frac{63}{16} = \pi$, find x. — 4 marks

8. If $Sin^{-1}(1-x)-2Sin^{-1}x=\frac{\pi}{2,}$ 4 marks

9. Simplify $Cot^{-1}\left[\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right], where x\in\left(\frac{\pi}{2},\pi\right)$ 4 marks

10. If a, b & c are real numbers, and $\begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix}=0$

Using properties of determinants show that either a+ b + c = 0 or a = b = c 4marks

11. (i) Express $\begin{bmatrix} 3 & -4 \\ 6 & 5 \end{bmatrix}$ as a sum of symmetric & skew symmetric matrix. 2 marks

(ii) Using properties of determinants prove that $\begin{vmatrix} 1 & 3x & 9x^2 \\ 9x^2 & 1 & 3x \\ 3x & 9x^2 & 1 \end{vmatrix}=(1-27x^3)^2$

12. Find the values of a, b & c if the matrix

$A=\begin{bmatrix} a-3b & a & -b \\ -a & 2b+c & c \\ b & -c & 2a-6 \end{bmatrix}$ is a skew – symmetric matrix.

Also find matrix B = A + 21 and inverse of matrix B, where 1 is a 3 x 3 identity matrix.

6 marks

CBSE MIXED TEST PAPER-04

(Unit Test)

CLASS - XII MATHEMATICS

[Time : 1.50 hrs.] **[M. M.:** 50]

General Instructions:-

(i) All questions are compulsory.

(ii) There is no overall choice. However internal choice has been provided.

(iii) Marks allotted to each question are indicated against it.

1. (a) Find $\frac{dy}{dx}, if x = a\cos^4\theta, y = a\sin^4\theta$. 1mark

(b) Differentiate: $\tan^{-1}\sqrt{\frac{1-\cos x}{1+\cos x}}$ w.r.t.x 1mark

(c) Differentiate y =log(sec x + tan x) w.r.t.x 1mark

(d) Find the principle value of $\cos^{-1}\frac{\sqrt{3}}{2}$ 1mark

(e) If $\sin(\sin^{-1}\frac{1}{5} + \cos^{-1}x) = 1$, find the value of x. 1mark

(f) find the value of $\cot(\tan^{-1}\alpha + \cot^{-1}\alpha)$ 1mark

2. Show that $\sin^{-1}\frac{3}{5} - \sin^{-1}\frac{8}{17} = \cos^{-1}\frac{84}{85}$ 6 marks

3. Differentiate $\sin^{-1}\left(\frac{2x}{1+x^2}\right) w.r.t \cos^{-1}\left(\frac{1-x^2}{x+x^2}\right)$ 3 marks

4. Express $\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right), \frac{-\pi}{2} < x < \frac{\pi}{2}$ in simplest form. 3 marks

Or

Express $\tan^{-1}\left(\frac{\cos x - \sin x}{\cos x + \sin x}\right), x < \pi$ in simplest form

5. Solve $\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$ 3 marks

6. If $y = \sqrt{\cos x + \sqrt{\cos x + \sqrt{\cos x + ...\infty}}}$ prove that $\frac{dy}{dx} = \frac{\sin x}{1-2y}$. 3 marks

7. If $x^a y^b = (x+y)^{a+b}$, *find* $\frac{dy}{dx}$. 3 marks

8. Find the equations of tangent to the curve $y = x^2 - 2x + 7$ which is parallel to $2x - 4y + 9 = 0$

   Or Find the equations of normal to the curve $y^2 = 4x$ at the point (1,2) 3 marks

9. Find the intervals in which the function given by.

   $F(x) = 4x^3 - 6x^2 - 72x + 50$ is (i) strictly increasing (ii) strictly decreasing. 3 marks

10. The radius of a circular plate increases at eh rate of 0.1 cm/sec. At what rate does the area increases when the radius of the plate is 25 cm. 3 marks

11. Find the approximate value of $\sqrt[4]{81.5}$ using differentials find the intervals in which f(x) = sinx – cos x, $x \in (0, 2\pi)$ is increasing or decreasing. 3 marks

12. Find two positive numbers whose sum is 60 and the product of one with cube of other is maximum. 4 marks

13. An open topped box is to be constricted by removing equal squares from each corner of a 3 m by 8 m rectangular sheet of aluminum and folding sides, find the maximum volume of box.

    Or Prove that the area of right angle triangle of given hypotenuse is maximum when the triangle is isosceles. 4 marks

14. Prove that the rt. Circular cone of maximum volume which can be inscribed in a sphere of radius R has an altitude equal to $\frac{4}{3}$R. Find the maximum volume and show that maximum volume of cone is $\frac{8}{27}$ of the volume of the sphere. 6 marks

CBSE MIXED TEST PAPER-05

(Unit Test)

CLASS - XII MATHEMATICS

[Time : 3.00 hrs.] **[M. M.:** 100]

General Instructions:-

(i) All questions are compulsory.

(ii) There is no overall choice. However internal choice has been provided.

(iii) Questions in Section A are of 1 mark each.

(iv) Questions in Section B are of 4 marks each.

(v) Questions in Section C are 6 marks each.

Section – A

1. If f: R --→ r is given by $f(x) = x^3 + 3$, find f^{-1}.
2. If A is a square matrix of order 3 such that $|adjA| = 441,\ find\ |A|$
3. Evaluate: $\int \frac{\tan^{-1}}{1+x^2} dx$
4. If f(x) = $|x|$ and g(x) = sin, then find fog.
5. Find the value of x and y if : $2\begin{bmatrix} 1 & 3 \\ 0 & 2 \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$
6. Find the principal value of $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$
7. Find the point on the curve $y = x^2 - 2x + 3$ where the tangent is parallel to x-axis.
8. Evaluate: $\int_{-\pi/2}^{\pi/2} \sin^7 x \ dx$

9. Find $\frac{dy}{dx}$ if $x = a(\theta - \sin\theta) and y = a(1 - \cos\theta)$. **Or**

Write the value of $\tan^{-1} x + \tan^{-1}\frac{1}{x}$ for x>0.

10. Evaluate: $\int x^2 \frac{\tan^{-1} x^3}{1+x^6}$

Section B

11. Solve for x: $\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$

12. If y = $(\log x)^x + x^{\log x}$, find $\frac{dy}{dx}$.

13. Solve the differential equation: $\left(x + 2y^2\right)\frac{dy}{dx} = y; y > 0$

14. Solve the difference equation: $(x^2 - y^2)\, dx + 2\, xy\, dy = 0$. Given that y = 1 when x = 1.

15. Given a binary operation defined on set N given by: $a * b = \frac{a+b}{2}\ for a, b \in N$

(i) Is it commutative? (ii) Is it associative?

16. For what value of k is the following functions continuous at x = 2.

$$f(x)\begin{cases} 2x+1 & if \quad x < 2 \\ k & if \quad x = 2 \\ 3x-1 & if \quad x > 2 \end{cases}$$

17. Evaluate: $\int e^x \left(\frac{2+\sin 2x}{1+\cos 2x}\right) dx$

18. Evaluate: $\int_0^{\pi/4} \log(1+\tan x)\, dx$

19. Using differential find approximate value of (82)1/4. **Or**

Find the interval in which the function: F(x) $2x^3 - 15\, x^2 + 36\, x + 1$ Is increasing or decreasing?

20. Find $\frac{dy}{dx} if \sqrt[x]{1+y} + \sqrt[y]{1+x} = 0$.

21. For what matrix $A = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$, find numbers ‘a’ and ‘b’ such that A^2 + ab + bl = 0.

22. The surface area of a spherical bubble is increasing at the rate of 2cm^2/sec. Find the rate at which the volume of bubble is increasing at the instant if its radius is 6 cm.

Section C

23. Solve the system of linear equation using matrix method.

x – y + z = 4

2x + y – 3z = 0

x + y + z = 2

24. Evaluate: $\int_0^3 f(x)dx$ where $f(x) = |x| + |x-1| + |x-2|$

25. Find the area of the region bounded by the curve y^2 = 4x and the line x = 4y – 2.

OR

Find the area enclosed by curves:

y^2 = 4ax and x^2 = 4ay

26. Using elementary transformation, find inverse of the matrix: $\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$

27. Using properties of determinants, prove that:

$$\begin{vmatrix} b+c & a-b & a \\ c+a & b-c & B \\ a+b & c-a & c \end{vmatrix} = 3abc - a^3 - b^3 - c^3$$

28. Prove that a conical tent of a given capacity will require the least curved surface area when height is $\sqrt{2}$ times the radius of the base.

29. Show that the right circular cylinder of given surface and maximum volume is such that its height is equal to the diameter of the base.

CBSE MIXED TEST PAPER-06

(Unit Test)

CLASS - XII MATHEMATICS

[Time : 1.50 hrs.] **[M. M.:** 40]

General Instructions:-

(i) All questions are compulsory.

(ii) There is no overall choice. However internal choice has been provided.

1 Check the relation R in the set of R of real number, defined as 1-mark

R = {(a,b): a≤ b2} is reflexive

2 If a*b = a2 + 2b-3, find 3*2. 1-mark

3 Find the principal value of 2-mark

$$Cot^{-1}\left(-\frac{1}{\sqrt{3}}\right)$$

4 Find the value of 2-mark

$$Sin^{-1}\left(\sin\frac{3\pi}{5}\right)$$

5 Consider given by 4-marks

$$f: R_+ \rightarrow [5,\infty)$$

f(x) = 9x2 + 6x – 5. Show that f is invertible. Find its inverse.

6 Define a binary operation * on the set 4-marks

A = {1, 2, 3, 4} as

$$a*b = \begin{cases} a+b & if \quad a+b<5 \\ a+b-4 & if \geq 5 \end{cases}$$

Find the identify for this operation and inverse of all elements of A (If exists)

7 Prove that 4-marks

$$Sin^{-1}\frac{12}{13}+Cos^{-1}\frac{4}{5}+\tan^{-1}\frac{63}{16}=\pi$$

8 Find x, if 4-marks

$$Sin^{-1}(1-x)-2Sin^{-1}x=\frac{\pi}{2},$$

9 Simplify 4-marks

$$Cot^{-1}\left[\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right], where x\in\left(\frac{\pi}{2},\pi\right)$$

10 If a, b & c are real numbers, and 4-marks

$$\begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix}=0$$

Using properties of determinants show that either a+ b + c = 0 or a = b = c

11 (i)Express the matrix as a sum of symmetric & skew symmetric matrix. 4-marks

$$\begin{bmatrix} 3 & -4 \\ 6 & 5 \end{bmatrix}$$

(ii) Using properties of determinants prove that

$$\begin{vmatrix} 1 & 3x & 9x^2 \\ 9x^2 & 1 & 3x \\ 3x & 9x^2 & 1 \end{vmatrix}=(1-27x^3)^2$$

12 Find the values of a, b & c if the matrix is a skew – symmetric matrix. Also find matrix B = A + 21 and inverse of matrix B, where 1 is a 3 x 3 identity matrix. 6-marks

$$A=\begin{bmatrix} a-3b & a & -b \\ -a & 2b+c & c \\ b & -c & 2a-6 \end{bmatrix}$$

CBSE MIXED TEST PAPER-07

(Unit Test)

CLASS - XII MATHEMATICS

[Time : 1.50 hrs.] **[M. M.:** 40]

General Instructions:-

(i) All questions are compulsory.

(ii) There is no overall choice. However internal choice has been provided.

1. If a matrix has 18 elements, what are possible order it can have? What if it has 5 elements? 3-marks

2. Use matrix method of show that the system of equations 3-marks

   X + 2y + 9

   2x + 4y = 7 is inconsistent

3. Construct a 3 x 4 matrix, whose elements are given by $a_{ij} = \frac{1}{2} - |3i + j|$ 3-marks

4. Find the value of a, b, c and d from the equations 3-marks

   $$\begin{pmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{pmatrix} = \begin{pmatrix} -1 & 5 \\ 0 & 13 \end{pmatrix}$$

5. Find the matrices X and Y if 3-marks

   $$2Y + Y = \begin{bmatrix} 4 & 4 & 7 \\ 7 & 3 & 4 \end{bmatrix} and X - 2Y \begin{bmatrix} -3 & 2 & 1 \\ 1 & -1 & 2 \end{bmatrix}$$

6. Let A and B be two matrices such that they commute. By mathematical induction, prove $AB^n = B^n A$. 3-marks

7. If A = $\begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$ and B = (1 5 7), verify that (AB)' and B'A'. 4-marks

8. Show that the matrix B' AB is symmetric or skew-symmetric according as A is symmetric or skew-symmetric 4-marks

9. Find the inverse of 4-marks

$$\begin{pmatrix} 2 & -6 \\ 1 & -2 \end{pmatrix}$$

if it exist, by using row transformation.

10. Prove that 5-marks

$$\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & a & 1+c \end{vmatrix} = abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{b}\right)$$

Or

If x, y, z are different and triangle =

$$\begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix}$$

= 0 show that 1 + xyz = 0

11. Compute (AB)-1 where 5-marks

$$A = \begin{pmatrix} 1 & 1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{pmatrix}, B^{-1}\begin{pmatrix} 1 & 2 & 0 \\ 0 & 3 & -1 \\ 1 & 0 & 2 \end{pmatrix}$$

Or

If A = $\begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix} and B = \begin{pmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{pmatrix}$

12. Find the product and use this result to solve the following system of Linear equations:- 3-marks

2x – y + z = -1, -x + 2 y – z = 4, x – y + 2z = -3

FIRST UNIT TEST PAPER 08

CLASS - XII MATHS

Time: $1^1/_2$ hours Maximum Marks: 40

Note:

i. Q1 to Q4 carry 1 mark each

ii. Q5 to Q10 carry 4 marks each.

iii. Q11 and Q12 carry 6 marks each.

1. Let A = R – {3} and B = R – {1]. Consider the function f: A → B defined by $f(x) = \dfrac{x-2}{x-3}$ find the inverse of f.

2. Find the value of $\sin^{-1}\left(\dfrac{-\sqrt{3}}{2}\right) + \cos^{-1}\dfrac{\sqrt{3}}{2}$.

3. If |A| = -6. Then find |2A|.
   A is a matrix of order 2×2.

4. Find the value of p in order, that the points (10, 7), (p, 1) and (5, 5) are collinear.

5. Let *: $N\times N$ → N be a binary operation on A defined by
   (a, b) * (c, d) = (ac, bd) for all a, b, c, d$\in$ N show that* is commutative and associative operation on A. Find the identify element if exist.

6. Show that $\sin^{-1}\dfrac{8}{17} + \sin^{-1}\dfrac{3}{5} = \tan^{-1}\dfrac{77}{36}$

7. Find the inverse of $\begin{bmatrix} 4 & 7 \\ 1 & 2 \end{bmatrix}$ by elementary transformation.

8. If $\cos^{-1}\dfrac{x}{a} + \cos^{-1}\dfrac{y}{b} = \alpha$, then, prove that $\dfrac{x^2}{a^2} - \dfrac{2xy}{ab}\cos\alpha + \dfrac{y^2}{b^2} = \sin 2\alpha.$

   OR

   Solve $\tan^{-1}(x+1) + \tan^{-1}(x-1) = \tan^{-1}\dfrac{8}{31}(x>0)$

9. If A = $\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ then prove that

   $A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$ Where n is a positive integer

10. If f and g be two function from R to R given by f(x) = 2x2 – x + 7 and g(x) = x – 4. for all $x \in$ R. find

(i) Fog (ii) got (iii) fof (iv) gog

11. Using properties of determinants, prove that

$$\begin{bmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{bmatrix} = \left(1+a^2+b^2\right)^3$$

12. Find A^{-1}, where $\begin{bmatrix} -1 & 2 & -1 \\ 2 & -3 & 1 \\ 5 & 1 & 1 \end{bmatrix}$

Hence solve the equations

-x + 2y + 5z = 2

-2x – 3y + z = 15

-x + y + z = 3

OR

Show that the following system of equations is consistent.

x – y + z = 3

2x + y – z = 2

-x – 2y + 2z = 1

Also find the solution.

CBSE UNIT TEST PAPER-09

CLASS - XII (MATHEMATICS)

FIRST TERM UNIT TEST

Time :1.5 Hours **M.M.40**

Instructions:

(i) All questions are compulsory.

(ii) The question paper is divided into three sections. A, B and C.

Section 'A' consists of 4 question of 1 mark each.

Section 'B' consists of 6 question of 4 marks each.

Section 'C' consists of 2 question of 6 marks each.

SECTION 'A' **1X4 = 4**

1. If f (x) = x + 7 and g (x) = x – 7, find fog (7)

2. Evaluate,(without expanding)

$$\begin{vmatrix} 0 & 1 & 2 \\ -1 & 0 & -3 \\ -2 & 3 & 0 \end{vmatrix}$$

3. If A in a square matrix of order 3 such that ladjAl = 64 find lAl.

4. Find the principal value of $\cos^{-1}\left(\frac{1}{2}\right)$

SECTION 'B' **4 X 6 =24**

5. Using properties of determinants prove that.

$$\begin{vmatrix} a+b+2c & c & c \\ a & b+c+2a & a \\ b & b & \text{c+a+2b} \end{vmatrix} = 2(a+b+c)^3$$

OR

$$\begin{vmatrix} \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \\ \beta+\gamma & \gamma+\alpha & \alpha+\beta \end{vmatrix} = (\alpha-\beta)\ (\beta-\gamma)\ (\gamma-\alpha)\ (\alpha+\beta+\gamma)$$

6. If A $\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ show that A^2 -5A +7l = 0. Hence find A^{-1}.

7. Let A = $\begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$. Express A as the sum of symmetric and a skew symmetric matrix.

8. Show that the relation R defined by (a, b) R (c, d) $\Rightarrow$ a + d = b + c on the set N x N in an equivalence relation.

9. Prove that.

$$\tan^{-1}\left(\frac{1}{3}\right)+\tan^{-1}\left(\frac{1}{5}\right)+\tan^{-1}\left(\frac{1}{7}\right)+\tan^{-1}\left(\frac{1}{8}\right)=\frac{\pi}{4}.$$

OR

Prove that.

$$\tan^{-1}\left(\frac{\sqrt{1+X-}\quad\sqrt{1-X}}{\sqrt{1+X+}\quad\sqrt{1-X}}\right)=\frac{\pi}{4}-\frac{1}{2}\cos^{-1}X,\frac{1}{2}\le X\le 1$$

10. Show that the function f : R $\rightarrow$ R defined by f (x) = $\frac{2X-1}{3}$, $X \in$ R in one – one and onto function. Also find the inverse of the function f.

SECTION 'C'

11. Let A =N x N and *be the binary operation on a defined by (a, b) *(c, d) = (a + c, b + d). Show that *in commutative and associative. Find the identity element for * on A_1 if any.

12. If A $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ *and* B = $\begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & -1 \\ -1 & 1 & 3 \end{bmatrix}$ find AB use the result to solve the following system of linear equations.
2x – y + z = -1
-x + 2y –z = 4
X – y + 2z = -3

OR

Solve the system of equations.
2x -3y +5z = 11
3x + 2y – 4z =-5
X + y – 2z = -3 s s

CBSE UNIT TEST PAPER-10

CLASS - XII (MATHS)

Time : 1.5 Hours **M.M.40**

All questions are compulsory.

Section A

1. Construct a 2x2 matrix A = [aij] whose elements are given by aij = $\frac{(i+2j)^2}{2}$ [1]

2. For what value of 'x' te matrix $\begin{bmatrix} 5-x & x+1 \\ 2 & 4 \end{bmatrix}$ is singular? [1]

3. If f: $R \to R$ is an invertible function defined by f (x) $\frac{3x-2}{5}$ find f^{-1} (x). [1]

4. If A = $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$, find the value of θ satisfying the equation A^T+A = l_2. [1]

5. Let a $\odot$ b a binary operation defined by 2a+ 3b, find b $\odot$ a if a = 3 and b=2. [1]

6. Find the principal value of $\sin^{-1}\left[\sin \frac{3\pi}{5}\right]$. [1]

7. If A is a square matrix of order 3 such that |adjA|=64, find |A| **OR** [1]

   A matrix 'P' of order 3x3 has determinant 5, what is the value of |3p|

8. Using properties, show that $\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}$ =0 [1]

Section B

9. If A = $\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, show that A^2 – 5A +71 = 0 Hence find A^{-1}. [4]

10. Let = R –{3}and B= R –{1}and the function f: $A \to B$ defined by f (x)=$\left(\frac{x-2}{x-3}\right)$. Is [4]

    'f' one – one and ONTO, Justify your answer.

11. Let f: {2,3,4,5} $\to${3,4,5,9} and g : {3,4,5,9}$\to${7,11,15}, find gof. [4]

12. Find the value of x if $\sin\left(\sin^{-1}\frac{1}{5}+\cos^{-1}x\right)=1$ [4]

13. Express $\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)\left(\frac{\cos x}{1+\sin x}\right)$ in the simplest of form. [4]

OR

Prove that $\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)=\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x$

Section C

14. If $A=\begin{bmatrix}2 & -1 & 1\\ -1 & 2 & -1\\ 1 & -1 & 2\end{bmatrix}$ [6]

and $B=\begin{bmatrix}3 & 1 & -1\\ 1 & 3 & 1\\ -1 & 1 & 3\end{bmatrix}$, find AB

using this result solve

2x-y+z=-1, -x+2y-z=4,x-y+2z= -3

OR

If x,y,z are different and

$\begin{vmatrix}x & x^2 & 1+x^3\\ y & y^2 & 1+y^3\\ z & z^2 & 1+z^3\end{vmatrix}=0$, show that xyz = -1

15. Show that a binary operation $Z^{\odot}$ defined as a $\odot$ b=a+b+1 $\forall a,b\in z$ satisfies [5]

i. Closure properties

ii. Commutative Property

iii. Associative property

Also find identify element of $Z^{\odot}$, if any.

CBSE UNIT TEST PAPER-11

CLASS - XII (MATHS)

Time : 2.5 Hours **M.M.50**

Note : Attempt all Questions.
Q. from 1 to 5 are of 1 Mark.
Q. 6 to 9 are 2 Marks each.
Q. 10 to 16 are of 3 Marks and 17 to 20 are of 4 marks each.

1. Simplify : $\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$. [1]

2. If A is a square matrix of order 3 and |A| = - 5, Find the value of |-4A|. [1]

3. Evaluate $\int e^x \sin(e^x)dx$ [1]

4. Find point on the curve y =(x – 3)^2. Where tangent is parallel to x- axis. [1]

5. Write the order and degree of differential equation. $\left(\frac{ds}{dt}\right)^4 + 3s\frac{d^2s}{dt^2} = 0$ [1]

6. Prove that $\tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{8} = \frac{1}{4}$ [1]

7. Using properties of determinant evaluate. [2]

$$\begin{vmatrix} o & a & -1 \\ -a & o & -c \\ b & c & 0 \end{vmatrix}$$

8. Find $\frac{dy}{dx}, if \frac{x = a(\theta - \sin\theta)}{y = a(1+\cos\theta)}$ [2]

9. The radius of an air the bubble increasing at the rate of $\frac{1}{2}cm/s$. At what rate is the volume of the bubble increasing when the radius is 1 cm. [2]

10. Show that the relation R = {(a-b) :|a-b| is given, a, b $\in$ z) is an equivalence relation. [3]

OR

Consider f : R $\rightarrow$R given by f (x) = 4x+3. Show that f is invertible. Find the inverse of f.

11. Find the inverse of the matrix. [3]

$\begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$ Using elementary Transformation.

12. Find a, b such that the function. [3]

F (x) $\begin{matrix} 5 \\ ax+b \\ 21 \end{matrix}\begin{cases} if \ x \le 2 \\ if \ 2 < x \\ if \ x > 10 \end{cases}$ is a continuous function.

13. Differentiate w.r.t,x (x cos)^x + (x sin x)$^{1/x}$ [3]

14. Find the intervals in which the function f (x) = 2x^3 – 3x^2 – 36x +7 is increasing or decreasing. [3]

15. $\int \frac{(3\sin\ \theta - 2)\cos\theta}{5-\cos^2\theta - 4\sin\theta}$ dθ [3]

16. Solve differential $x\frac{dy}{dx} + 2y = x^2 \log x$ or ydx+x log $\left(\frac{y}{x}\right) dy - 2xdy = 0$ [3]

17. Solve by matrix method. [4]

2x - 3y + 5z = 11

3x + 2y – 4z = -5

X + y - 2z = -3.

18. Using properties of determinant prove. [4]

$$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3$$

19. Prove that the volume of the largest cone be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere. [4]

20. Find the area of the circle 4x^2 + 4y^2 =9 Which is interior to the parabola x^2 = 4y. [4]

CBSE UNIT TEST PAPER-12

CLASS - XII (MATHS)

Time : 3 Hours

M.M.100

General Instructions

1. All questions are compulsory.
2. The question paper consists of 29 questions divided into three sections A, B and C. Section A comprises of 10 questions of1 mark each, Section B comprises of 12 questions of 4 marks each and Section C comprises of 07 questions of 6 marks each.
3. All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.

Section A

1. Let f : A $\rightarrow$B be a function such that range of f is a proper subset of B. If f onto? [1]

2. If $\text{Cos}^{-1} x + \text{Cos}^{-1} y = 0$, find x and y. [1]

3. If A is a 2 x 3 matrix, can we find A^2? What if A is a diagonal matrix? [1]

4. If $x\begin{bmatrix} 2 \\ 3 \end{bmatrix} + y\begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$, *find* x and y.

5. If det A = - 5 and A is of order 3 x 3, find det (adj A) [1]

6. Find $\frac{dy}{dx}$, when $y = \log_{10}(x^2 + 1)$ [1]

7. Write a value of $\int e^{3\log x} x^4\, dx$ [1]

8. Write a value of $\int \frac{1-\tan x}{x + \log(\cos x)} dx$ [1]

9. Evaluate $\int (10^x + x^{10} + 10^{10}) dx$ [1]

10. If f be the greatest integer function and g be the absolute value function. Find the value of (fog) $\left(-\frac{3}{2}\right)$ [1]

Section B

11. Let * be a binary operation of N x N defined by (a, b) * (c, d) = (a + c, b + d). Show that * is commutative and associative. Find the identity element for * on N x N if any. [4]

12. Prove that Tan-1 $\sqrt{x = \frac{1}{2}Cos^{-1}\left(\frac{1-x}{1+x}\right)}, x \geq 0$ [4]

13. Solve the equation [4]

$$\text{Tan}^{-1}\left(\frac{x-1}{x-2}\right) + \text{Tan}^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}$$

$|x| < 1$

14. Solve the matrix equation for p where [4]

$$\begin{bmatrix} 5 & 4 \\ 1 & 1 \end{bmatrix} p = \begin{bmatrix} 1 & -2 \\ 1 & 3 \end{bmatrix}$$

15. By using property of determinant, show that [4]

$$\begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix} = (1-x^3)^2$$

OR

$$\begin{vmatrix} y+k & y & y \\ y & y+k & y \\ y & y & y+k \end{vmatrix} = k^2(3y+k)$$

16. Differentiate $\text{Sin}^2 x$ w.r.t. $e^{\cos x}$. [4]

17. If x = a (Cos t + t Sin t) [4]

Y= a (Sin t – t Cos t)

Find $\frac{d^2y}{dx^2}$

18. For what value of K is the following functions continous at x = 0? [4]

F(x) = $\begin{cases} \dfrac{1-Cos4x}{8x^2} & x \neq 0 \\ K & x = 0 \end{cases}$

19. Find the intervals in which the function f given by [4]

F (x) = Sin x – Cos x, $0 \leq x \leq 2\pi$

1 is increasing

2 is decreasing

20. Prove that the curve x = y^2 and xy = K Cut at right angles if $8K^2 = 1$. [4]

21. A water tank as the shape of an inverted right circular cone with its axis vertical and vertex lower most. Its semi vertical angle if Tan^{-1} (1/2) water is poured into it at a constant rate of 5 cubic meter minute. Find the rate at which the level of the water is rising at the instant when the dept of water in the tank is 10 m. [4]

22. Evaluate $\int \frac{\sqrt{Tan \text{ x}}}{Sin \text{ x Cos x}} dx$ [4]

OR

$\int \frac{Sin \text{ x}}{Sin \text{ (x + a)}} dx$

23. Evaluate $\int \frac{1}{Co \text{ sec x + Cos x}} dx$ [6]

24. Evaluate $\int \frac{x^4}{(x-1)(x^2+1)} dx$ [6]

25. Let f: N $\rightarrow$R be a function defined as f (x) = $4x^2 + 12x + 15$. Show that f: N $\rightarrow$S where, S is the range of f, is invertible find the inverse of f. [6]

26. If A = $\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ *and* B = $\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$ are two square matrices, [6]

Find AB and solve the following system of equations.

X – y = 3, 2x + 3y + 4z = 17, y + 2z = 7

27. Show that of all the rectangles inscribed in a given fixed circle the square as the maximum area. [6]

28. If $(x - a)^2 + (y - b)^2 = C^2$ for some C > O. Prove tat [6]

$$\left[1+\frac{dy}{dx}\right]^{\frac{3}{2}}$$

$$\frac{d^2y}{dx^2}$$

Is a constant independent of a and b.

29. Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere.

OR

Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of Radius R is $\frac{2R}{\sqrt{3}}$. Also find the maximum volume.

CBSE UNIT TEST PAPER-13

CLASS - XII (MATHS)

Time : 3 Hours **M.M.100**

General Instructions

1. All questions are compulsory.
2. The question paper consists of 29 questions divided into three sections A, B and C. Section A comprises of 10 questions of1 mark each, Section B comprises of 12 questions of 4 marks each and Section C comprises of 07 questions of 6 marks each.
3. All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
4. There is no overall choice. However, internal choice has been provided in 4 questions of four marks each and 2 questions of six marks each. You have to attempt only one of the alternatives in all such questions.
5. Use of calculators in not permitted.

SECTION – A

1. If A = $\begin{bmatrix} 2 & -1 \\ 3 & 0 \end{bmatrix}$, what is the value of $\left|A^{-1}\right|$? [1]

2. If A = $\begin{bmatrix} 4 & x+2 \\ 2x-3 & x+1 \end{bmatrix}$ is a symmetric matrix, then [1]

   What is the value of x?

3. If $f(x) = x^2 + 2$, g(x) = 3x, xεR, find fog (x). [1]

4. Evaluate Cos $\left[Cos^{-1}(\frac{-\sqrt{3}}{2}) + \frac{\pi}{6} \right]$. [1]

5. If = A= $\begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix}, B = \begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix}$, Write matrix x such that 2x + A = 2B. [1]

6. Evaluate $\pi / 2^{\int \pi/2} x^7 Cos$ x dx . [1]

7. Write projection of $\vec{a} = 8i + \hat{j}$ on $\vec{b} = \hat{i} - \widehat{2j} + \hat{k}$ [1]

8. Evaluate : $\int \frac{1}{x + x\log x} dx$ [1]

9. It $\left|\vec{a} + \vec{b}\right| = \left|\vec{a} - \vec{b},\right|$, what is the angle between $\vec{a}$ and $\vec{b}$? [1]

10. Write the equation of a plane when foot of perpendicular from origin to the plane is (2, 1, 1). [1]

SECTION - B

11. Using properties of determinants, prove that [4]

$$\begin{vmatrix} a^2+b^2 & c^2 & c^2 \\ a^2 & b^2+c^2 & a^2 \\ b^2 & b^2 & c^2+a^2 \end{vmatrix} = 4a^2b^2c^2$$

OR

Using elementary transformations, find inverse of the matrix $\begin{bmatrix} 3 & 7 \\ 2 & 4 \end{bmatrix}$

12. Check the continuity of the function at x = 1 [4]

$$f(x) = \begin{cases} (x-1)Sin(\frac{1}{x-1}) & if\ x \neq 1 \\ 0 & if\ x = 1 \end{cases}$$

13. Consider $f : R_+ \to [-5, \infty)$ given by f(x) $=9x^2+6x-5$. Show that f is invertible function and also find the inverse of the function f. [4]

14. Show that $f(x) = \tan^{-1}(Sinx + Cosx), x > o$ is strictly increasing function on $(o\frac{\pi}{4})$ [4]

OR

For the curve y = $4x^3 - 2x^5$, Find all the points on curve at which the tangents drawn, also passes through origin.

15. Solve the following differential equation: [4]

$X \log x \frac{dy}{dx} + y - \log x = 0 \qquad x > o$

16. Solve the following differential equation: [4]

$(x^2 - y^2)\,dx + 2xy\,dy = 0$, given that y = 1 when x = 1

17. In a game, a man wins a rupee for a six and looses a rupee for any other number [4]

when a fair die is thrown. The man decided to throw a die thrice, but to quit as and when he gets a six. Find the expected value of the amount he wins/ looses.

OR

A and B in turns a die, till one of them throw a six and wins the game. Find their respective probabilities of winning if A starts the game.

18. Show that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$ intersect. [4]

Also find the point of intersection.

19. Let $\vec{a} = \hat{i} - \hat{j}, \vec{b} = 3\hat{j} - k$ and $\vec{C} = 7\hat{i} - \hat{k}$. Find a vector $\vec{d}$ which is perpendicular to $\vec{a}$ and $\vec{b}$ and $\vec{C}.\vec{d}. = 1$ [4]

OR

If $\vec{\infty} = 3\hat{i} - \hat{j}$ and $\vec{\beta} = 2\hat{i} + \hat{j} - 3\hat{k}$, then express $\vec{\beta}$ in the from $\vec{\beta} = \vec{\beta_1} + \vec{\beta_2}$,

Where $\vec{\beta_1}$, is parallel to $\vec{\infty}$ and $\vec{\beta_2}$ is perpendicular to $\vec{\infty}$.

20. Prove that $\tan^{-1}\left[\frac{\sqrt{1+x^2} + \sqrt{1-x^2}}{\sqrt{1+x^2} - \sqrt{1-x^2}}\right] = \frac{\pi}{4} + \frac{1}{2}Cos^{-1}(x^2)$. [4]

21. If $(x+y)^{m+n} = x^m . y^n$ then prove that $\frac{dy}{dx} = \frac{y}{x}$. [4]

22. Evaluate $\int \frac{Sinx + Cosx}{\sqrt{9+16Sin2x}}dx$ [4]

SECTION – C

23. Find the matrix P, for which [6]

$$\begin{bmatrix} 3 & 2 \\ 7 & 5 \end{bmatrix} P \begin{bmatrix} -1 & 1 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 0 & 4 \end{bmatrix}$$

OR

If A= $\begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}$, find A^{-1}. Hence solve the system of linear equations:

x + 2y-3z =-4,2x+3y+2z=2, 3x-3y-4z =11.

24. Using integration, find the area of the region bounded by parabola $y^2 = 4x$ and the circle $4x^2 + 4y^2 = 9$. [6]

25. Using properties of definite integrates, evaluate $\int_0^{\pi} \frac{x \tan x}{Secx + \tan x} dx$ [6]

26. A wire of length 28m is to be cut into two pieces. One of the piece is to be made into a square and other into a circle. What should be length of the two pieces, so that the combined area of the square and the circle is minimum? [6]

27. Bag A contains 5 white and 6 black balls and a bag B contains 5 black and 6 white balls. One ball is transferred from bag A to bag B without Seeing its colour and then a ball is drawn from bag B. If the ball drawn from bag B is white, find the probability that ball transferred from bag A to bag B was a shits ball. [6]

28. If a line makes angles $\infty, \beta, Y and\ \delta$ with the four diagonals of cube. Prove that [6]

$Cos^2 \infty + Cos^2\beta + Cos^2 Y + Cos^2\delta = \frac{4}{3}$

OR

Find the equation of the plane passing through the intersection of planes $2x+3y-z=-1$ and $x+y-2z+3 = 0$, and perpendicular to plane $3x-y-2z = 4$. Also find the inclination of this plane with xy plane.

29. An Airoplane can carry a maximum of 200 passengers. A profit of Rs. 400 is made on each first class ticket and a profit of Rs. 300 is made on each second class ticket. The airline reserves at least 20 first class ticket. However atleast, four times as many passengers prefer to travel by second class ticket than by first class ticket Determine how many tickets of each type must be sold to maximize the profit for the airline form of this L. P. P. and solve it graphically. [6]

CBSE UNIT TEST PAPER-14

CLASS - XII (MATHS)

FIRST TERM UNIT TEST

Time :3 Hours **M.M.100**

General Instructions :

1. All the questions are compulsory.
2. The question paper consists of 29 questions divided into three sections A, B and C.
   Section A Q. 1 to 10 are of 1 mark each.
   Section B Q 11 to Q 22 are of 4 marks each.
   Section C Q 23 to Q 29 are of 6 marks each.
3. There is no overall choice. However internal choice has been provided. You have to attempt only one of the alternatives in all such questions.
4. Use of calculators is not permitted.

Section A

1. Find a unit vector parallel to the sum of the vectors $\vec{a} = \widehat{2i} + \widehat{4j} - \widehat{5k} and \vec{b} = \hat{i} + \widehat{2j} + \widehat{3k}$ [1]

2. If A is a square matrix of order 3 such that |adj A| = 100, find |A|. [1]

3. Give an example of two non zero matrices s. t. AB =0. [1]

4. For any two non zero vectors $\vec{a}$ and $\vec{b}$ write when $\left|\vec{a}+\vec{b}\right| = \left|\vec{a}-\vec{b}\right|$ holds. [1]

5. Evaluate $\int_{-1}^{1} \frac{|x|}{x} dx$ [1]

6. Find the value of cot $\left[\sin^{-1} x + \cos^{-1} x\right], \left|x \le 1\right|$ [1]

7. If A= {1, 2, 3} and B = {3, 5}, then find the number of functions from A to B. [1]

8. Cartesian equations of a line AB are $\frac{2x-1}{2} = \frac{4-y}{7}, \frac{z+1}{2}$. Write the direction ratios of a line parallel to AB. [1]

9. Find a, for which f (x) =a (x+sinx)+ a is increasing. [1]

10. Two balls are drawn from a bag containing 4 red, 4 blue and 5 green balls. What is the probability that both are green. [1]

Section B

11. Show that f : R – {-1} $\rightarrow$ R – {1} given by f (x) = $\dfrac{x}{x+1}$ is invertible. Also find f[1]. [4]

12. Solve the equation $\tan^{-1}\dfrac{x-1}{x-2}+\tan^{-1}\dfrac{x+1}{x+2}=\dfrac{\pi}{4}$ [3]

13. Evaluate $\int\dfrac{\sin x\cos x}{a^2\cos^2 x+b^2\sin^2 x}dx, a\neq\pm b$ [4]

OR

Evaluate $\int\dfrac{dx}{\left(e^x-1\right)^2}$

14. If y = sin $\left[2\tan^{-1}\sqrt{\dfrac{1-x}{1+x}}\right]$, *prove* that $\dfrac{dy}{dx}=\dfrac{-x}{\sqrt{1-x^2}}$ [4]

15. Using properties of determinants, prove that [4]

$$\begin{vmatrix} -bc & b^2+bc & c^2+bc \\ a^2+ac & -ac & c^2+ac \\ a^2+ab & b^2+ab & -ab \end{vmatrix}=(ab+bc+ca)^3$$

16. The CBSE has a list of examiners of 150 persons. Out of these 50 are women, 125 of the examiners know Hindi. & the remaining do not know Hindi. 90 of the examiners are working teachers & the remaining are tiered teachers. What is the probability of selecting a Hindi knowing working woman teacher as examiner. [4]

OR

A can hit a target 4 times in 5 shots. B 3 times in 4 shots and C they fire a volley. What is the probability that at least 2 shots hit the target.

17. Find the relation between α and β s.t. $\overrightarrow{\alpha a}+\overrightarrow{\beta b}$ is perpendicular to $\vec{c}$ where $\vec{a}=\widehat{3i}-\widehat{2j}+\hat{k},\vec{b}=\hat{i}+\widehat{2j}-\widehat{3k}$ and $\vec{c}=-\hat{i}+\hat{j}+\widehat{2k}$. [4]

18. If f (x) = $\begin{cases} \dfrac{\cos^2 x - \sin^2 x - 1}{\sqrt{x^2+1} - 1} & x \neq 0 \\ K, & x = 0 \end{cases}$ [4]

Is its at (x) =0. Find the value of K.

19. Find the vector equation of the line parallel to the $\frac{x-1}{5} = \frac{3-y}{2} = \frac{\cancel{z}+1}{4}$ and passing through (3, 0, 4). Also find the distance these two lines. [4]

20. Find the interval in which the function f (x) =x^3-12x^2+36x+17 is strictly (a) increasing (b) decreasing. Also find the points at which the tangents are parallel to x-axis. [4]

21. Solve the differential equation [4]

$$\frac{dy}{dx} = \sin(x+y) + \cos(x+y)$$

Or

Solve the differential equation

$$\left[1 + e^{x/y}\right] dx + e^{x/y}\left[1 - \frac{x}{y}\right] = 0$$

22. Form the differential equation representing the family of ellipses having foci on x-axis and centre at origin. [4]

Section C

23. Find the equation of the plane passing through the point (2,-1,5) and perpendicular to each of the planes $\vec{r}.(\hat{i} + \widehat{2j} - \hat{k}) = 1$ and $\vec{r}.(\widehat{3i} - \widehat{4j} + \hat{k}) = 5$ [6]

24. If a = $\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix} and B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$ Find AB and hence solve the system of equations x-y=3; $2x + 3y + 4\cancel{z} = 17$ and y+2$\cancel{z}$=7. [6]

OR

Using elementary transformations, find the inverse of the matrix $\begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{bmatrix}$

25. Find the area of the region bounded by $x=y^3$, the y-axis and the lines $y=-1$ and $y=2$. [6]

OR

Find the smaller of the two areas in which the circle $x^2+y^2=4$ is divided by the parabola $y^2=3[2x-1]$

26. A brick manufacturer has two depots, A and B which stocks of 30,000 and 20,000 bricks respectively. He receives orders from three builders P, Q and R for 15,000, 20,000 and 15,000 bricks respectively. The cost in transporting 1000 briks to the builders from the depots are given below. [6]

To From	P	Q	R
A	40	20	30
B	20	60	40

How should the manufacturer fulfil the orders so as to keep the cost of transportation minimum. Formulate a LPP and solve it graphically

27. A fair die is rolled. If 1 turns up, a ball is picked up at random form bag A. If 2 or 3 turns up, a ball is picked up from bag B. If 4, 5 or 6 turns up a ball is picked from bag C. Bag A contains 3 red and 2 white balls; bag B contains 3 red and 4 white balls, bag C contains 4 red and 5 white balls. The die is rolled and a bag is packed up and a ball is drawn. [6]

(i) What are the chances of drawing a red ball?

(ii) If the ball drawn is red, what are the chances that bag B was picked up?

28. A given rectangular area is to be fenced off in a field whose length lies along a straightriver. If no fencing is needed along the river show that the least amount of fencing will be required when the length of the field is twice its breadth. [6]

29. Evaluate $\int_0^{\pi/2} (2long \sin x - \log \sin 2x)dx$ [6]

ANNUAL EXAMINATION Test Paper 15

Class : 12

Subject : Mathematics

Time : 3 Hrs. **M.M. : 100**

General Instructions :

(i) All questions are compulsory.

(ii) Q.No. 1 to Q.No. 10 of section 'A' are of 1 mark each.

(iii) Q.No. 11 to Q.No.22 of section 'B' are of 4 marks each.

(iv) Q. No. 11 to Q. No. 29 of section 'C' are of 6 marks of 6 marks each.

(v) In some questions, internal choice is given.

Section 'A'

1. Write power set of the set A= {a, b, c}
2. Express – 47^0 30 in radian measure.
3. What is the value of cot $\left(\frac{-15\pi}{4}\right)$?
4. What is the probability that a letter chosen at random from word 'EQUATIONS' is a consonant?
5. A cord is drawn from the pack of 52 cards. What is the probability that it is a king or queen?
6. Find the derivative of x sin x with respect to x.
7. Find the derivative of $\frac{1}{ax^2+b}$ with respect to x.
8. Write the component statements of the compound statement. "All prime numbers are either even or odd"
9. Write contra positive of the statement. "If you are born in India, you are a citizen of India".
10. What id the eccentricity of hyperbola whose vertices and foci are $(\pm 2, 0)$ and $(\pm 3, 0)$ respectively?

Section 'B'

11. Evaluate: $\lim_{x \to \frac{\pi}{2}} \frac{1+\cos 2x}{(\pi - 2x)^2}$

OR

For the function $f(x) = \begin{cases} a+bx, x<1 \\ 4, x \quad =1 \\ b-ax, x>1 \end{cases}$

$\lim_{x \to 1} f(x) = f(1)$ .Find the possible values of a and b.

12 Find the equation of circle of circle which passes through (2,-2) and (3,4) and whose centre lies on the line x+y =2.

13. If $\sin x = \frac{3}{5}, \frac{\pi}{2} < x < \pi$ then find the value of cos x, tan x, sec x and cot x.

14. Find the value of sin 15^0.

15. Find the ratio in which yz-plane divides the line segment joining points (-2,4,7) and (3,-5,8).also find the co –ordinates of the point of intersection.

16. Using principle of mathematical induction, prove that:

$1.3+2.3^2+3.3^3+$………… $+n.3^n=$ $\frac{(2n-1)3^{n+1}+3}{4} \forall n \in N.$

OR

Using principle of mathematical induction, prove that:

$(1+x)^N \geq (1+nx)$ $\forall n \in N; x > -1$

17. Write the complex number $\frac{1+2i}{1-3i}$ in its polar from.

18. Find image of the point p(-8,12) with respect to the line mirror 4x+7y+13 =0.

OR

Find the equation of the lines through the point (3,2) which makes an angle of 45^0 with the line x-2y=3.

19. Two students Anil and Ashima appeared in an examination. The probability that Anil wilol qualify the examination is 0.05 and that Ashima will qualify the examination is 0.10.The probability that atleast one of them qualify the examination is 0.13. Find the probability that only one of them will qualify the examination.

20. How many words can be made by using all letters of the word 'MATHEMATICS' in which all vowels are never together?

21. A mathematic question paper consists of 10 questions divided two parts I and II, each containing 5 questions. A student is required to attempt 6 questions in all, taking at least 2 questions from each part. In how many ways can the student select the questions?

22. Find the sum to n terms of terms of the series: 0.5+0.55+0.555+……………n terms.

OR

Find the value of n so that $\dfrac{a^{n+1}+b^{n+1}}{a^n+b^n}$ may be G.M between a and b.

Section 'C'

23. Prove that: $\sin^2 A+\sin^2\left(A+\frac{\pi}{3}\right)+\sin 2\left(A-\frac{.\pi}{3}\right)=\frac{3}{2}.$

24. In the expansion of $\left(x^2-\frac{1}{x}\right)^{12}$, find (i) 4th term (ii) Middle term and (iii) term independent of x.

OR

Find $(a+b)^4-(a-b)^4$. hence, evaluate $\left(\sqrt{3}+\sqrt{2}\right)^4-\left(\sqrt{3}-\sqrt{2}\right)^4$

25. the ratio of the A.M. and G.M. of two positive numbers a and b is m : n. Show that: $a:b=\left(m+\sqrt{m^2-n^2}\right):\left(m-\sqrt{m^2-n^2}\right).$

26 In a survey of 5000 people in a town, 2250 were listed as reading as reading English Newspaper, 1750 as reading Hindi Newspaper and 875 were listed both Hindi as

well as English 1 Find how many people do not read Hindi or English Newspaper. Find how many people read only English Newspaper?

27. Draw the graph of $f(x)=[x-2], x \in R.$ What are the domain and range of $f(x)=[x-2]$?

28. Solve the following system of inequalities graphically:
$x+2y \leq 10, x+y \geq 1, x-y \leq 0, x \geq 0, y \geq 0.$

29. Calculate mean and standard deviation for the following data:

	30-40	40-50	50-60	60-70	70-80	80-90	90-100
Frequency	3	7	12	15	8	3	2

OR

The mean and standard deviation of 20 observations are found to be 10 and 2 respectively. On rechecking, it was found that an observation 8 was incorrect. Calculate the correct mean and standard deviation if observation 8 is replaced by 12.

CBSE UNIT TEST PAPER-16

CLASS - XII (MATHS)

Time :2.5Hours **M.M.60**

General Instructions:

(i) All questions are compulsory.

(ii) Question paper consists of 20 questions.

Q. No. 1 to Q. No. 6 carry 1 mark each.

Q. No. 7 to Q. No. 11 carry 2 marks each.

Q. No. 12 to Q. No. 16 carry 4 marks each.

Q. No. 17 to Q. No. 20 carry 6 marks each.

1. If a $\left(\hat{i}+\hat{j}+\hat{k}\right)$ is a unit vector then find the value of a. [1]

2. Find P $\left(A\cap\right)$ *if* P $\left(B\right)=\frac{5}{13}$ *and* P $\left(A/B\right)=\frac{2}{5}$. [1]

3. Write the value of : $\hat{i}\left(\hat{j}x\hat{k}\right)+\hat{j}.\left(\widehat{K}x\hat{i}\right)+\hat{k}.\left(\hat{j}x\hat{i}\right)$. [1]

4. The probability that an event happens in one trial of an experiments 0.4. Three independent trials of experiment are performed. Find the probability that the event happens least once. [1]

5. Reduce equation of the plane 2x + 3y + 4z = 12 to intercept from and find intercepts on co-ordinate axes. [1]

6. The Cartesian equation of line is $\frac{2x-1}{\sqrt{3}}=\frac{y+2}{2}=\frac{z-3}{3}$. Find the direction ratios of the line. [1]

7. If the probability of defective bolts is 0.1. Find the mean and the standard deviation for distribution of defective bolts in a total of 500 bolts. [2]

8. Find a unit vector perpendicular to both vectors $\hat{i}-2\hat{j}+3\hat{k}$ and $\hat{i}+2\hat{j}-\hat{k}$. [2]

9. Find the vector and Cartesian equation of line passing through (1, 2, 3) and parallel to the line: [2]
$$\frac{-x-2}{1}=\frac{y+3}{7}=\frac{2z-6}{3}$$

10. Show that the point A $\left(-2\hat{i}+3\hat{j}+5\hat{k}\right)$ and B $\left(\hat{i}+2\hat{j}3\hat{k}\right)$ and C $\left(7\hat{i}-\hat{k}\right)$ are collinear. [2]

11. Find the projection of [2]

$\vec{b}+\vec{c}$ on $\vec{a}$ Where $\vec{a}=2\hat{i}-2\hat{j}+\hat{k}$, $\vec{b}=\hat{i}+2\hat{j}-2\hat{k}$ and $\vec{c}=2\hat{i}-\hat{j}+4\hat{k}$.

12. Find the value of p so that the lines [4]

$\frac{1-x}{3}=\frac{7y-14}{2p}=\frac{z-3}{2}$ and $\frac{7-7x}{3p}=\frac{y-5}{1}=\frac{6-z}{5}$ are perpendicular to each other.

13. A bag contains 50 tickets numbered 1, 2, 3, 4, ………, 50 of which five are drawn at random and arranged in ascending order appearing on the tickets ($x_1 < x_2 < x_3 < x_4 < x_5$). Find the probability that $x_3 = 30$. [4]

Or

Two godowns A and B have grain storage capacity of 100 quintals and 50 quintals respectively. They supply to three ration shops D, E and F whose requirements are 60, 50, 40 quintals respectively. The cost of transportation pre quintal from godown to the shops are given in the following table:

To \ From	Transporation (A)	Cost per quintal (in Rs.) (B)
D	6	4
E	3	2
F	2.5	3

Formulate the problem as a linear programming problem in order to minimize the transportation cost.

14. Find the area of triangle whose vertices are A (3, - 1, 2) B (1, -1, -3) and C (+4, -3, 1) by using vectors. [4]

15. If $\vec{a}+\vec{b}+\vec{c}=0,\ |a|=3, |b|=5 and |c|=7$. Then find the angle between $\vec{a} and \vec{b}$. [4]

16. Find the foot of perpendicular from the point (0, 2, 3) on the line $\frac{x-3}{5}=\frac{y-1}{2}=\frac{z+4}{3}$ [4]

Or

Find the shortest distance between two line whose vector equations are:

$\vec{r}=\left(\hat{i}+2\hat{j}+3\hat{k}\right)+\lambda\left(\hat{i}-3\hat{j}+2\hat{k}\right)$

$\vec{r}=\left(4\hat{i}+5\hat{j}+6\hat{K}\right)+\mu\left(2\hat{i}+3\hat{j}+\hat{k}\right)$

17. Suppose that 5% of man and 0.25% of women have grey hair. A grey haired person is selected at random. What is the probability of this person being male? Assume that these are equal number of males and females. [6]

18. Find the equation of plane containing the line of intersection of two planes x + y + z – 6 = 0 and 2x + 3y + 4z + 5 = 0 and passing through the point (1, 1, 1). [6]

19. Solve the following LPP by graphical method: [6]

Minimize $z = 20x + 10y$

Subject to $x + 2y \leq 40$

$3x + y \geq 30$

$4x + 3y \geq 60$

And $x \geq 0, y \geq 0$

20. Two dice are thrown simultaneously. If X denotes number of sixes then find the expectation and variance of X. [6]

CBSE UNIT TEST PAPER-17

CLASS - XII (MATHS)

Time: 3Hours **M.M.100**

General Instructions:

(i) All questions are compulsory.

(ii) The question paper consist of 29 questions divided into three sections A, B, and C. Section A comprises of 10 questions of one mark each, section B comprises of 07 questions of six marks each.

(iii) All questions in Section A are to be answered in one word, one sentence or as par the exact requirement of the question.

(iv) There is no overall choice. However, internal choice has been provided in 04 questions of four marks each and 02 questions of six marks each. Your have to attempt only one of the alternatives in all such questions.

(v) Use of calculators is not permitted. You may for logarithmic tables, if required.

SECTION – A

1. Cartesian equations of a line AB are:- [1]
$$\frac{2x-1}{2}=\frac{4-y}{7}=\frac{z+1}{2}$$
Write the equation of a line passing through (1,2,3) parallel to AB.

2. Evaluate : $\int e^{3\log x}\left(x^4\right)dx$ [1]

3. Find the point on the curve y = x^2 -7x + 12, where the tangent is parallel to x-axis. [1]

4. Write the position vector of a point dividing the line segment joining points A and B with position vectors $\vec{a}$ and $\vec{b}$ externally in the ratio 1:3, where $\vec{a}=\hat{i}-3\hat{j}+\hat{k}$ and $\vec{b}=-2\hat{i}+3\hat{j}+\hat{k}$. [1]

5. Let * be a binary operation be a * b = m2a.b-7. Find 3 * 4 [1]

6. A matrix A of order 3 x 3 has determinant 3. What is the value of | adj A |? [1]

7. If B is a skew symmetric matrix, write whether the matrix (ABA) is symmetric or skew symmetric. [1]

8. If A = $\begin{bmatrix}\cos x & \text{sinx}\\ -\sin x & \cos x\end{bmatrix}$, *find* x, 0<x<$\frac{\pi}{2}$ when A + A = I. [1]

9. If $\vec{a}=\hat{i}+\hat{j}$; $\vec{b}=\hat{j}+\hat{k}$; $\vec{c}=\hat{k}+\hat{i}$, find a unit vector in the direction of $2\vec{a}+\vec{b}+\vec{c}$. [1]

10. Write the range of one branch of $\sin^{-1} x$, other than the Principal Branch. [1]

SECTION – B

11. A water tank has the shape of an inverted right circular cone with its axis vertical and Lower most. Its semi vertical angle is $\tan^{-1}\left(\frac{1}{2}\right)$. Water is poured into it at a constant rate of 5 cubic mater per minute. Find the rate at which the level of the water is rising at the instant when the depth of water in the tank is 10m. [4]

Or

Discuss applicability of Rolle's Theorem for the function f (x) = cos x + sin x in [0,2π] and find point at which tangent is parallel to X axis.

12. Solve the following differential equation: [4]

$\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$, give that y(0) = 0

13. Evaluate $\int_0^{\pi} 2\frac{x + xin\ \mathrm{x}}{1+\cos\ \mathrm{x}} dx$ [4]

Or

Evaluate $\int_0^{\pi} \frac{x}{a^2\cos^2 x + b^2 \sin^2 x} dx$

14. If the function f:R →defined by f (x) = $2x^3 + 7$, Prove that f is one-one and onto function. Also find the inverse of the function f and f^1 (23). [4]

15. Prove that $\cot^{-1}\left[\frac{\sqrt{1+\sin x} \quad + \quad \sqrt{1-\sin x}}{\sqrt{1+\sin x} \quad - \quad \sqrt{1-\sin x}}\right] = \frac{x}{2}$ [4]

Or

Prove that: $4\tan^{-1}\frac{1}{5} - \tan^{-1}\frac{1}{70} + \tan^{-1}\frac{1}{99} = \frac{\pi}{4}$

16. Find the value of which function f defined by f (x) = $\begin{cases} \frac{1-\cos ax}{x\sin x}, & x \neq 0 \\ 2, & x = 0 \end{cases}$ *is* continuous at x = 0. [4]

17. 14 If y = b $\tan^{-1}\left(\frac{x}{a} + \tan^{-1}\frac{y}{x}\right)$, find $\frac{dy}{dx}$. [4]

Or

If y = $(\sin x)^{\cos x} + (\cos x)^{\sin x}$, find $\frac{dy}{dx}$

18. Solve the following differential equation [4]

$$\cos^2 x\frac{dy}{dx}+y=\tan x$$

19. Find the values of λ and μ if $\left(3\hat{i}-6\hat{j}+\hat{k}\right)x\left(\hat{i}+\lambda\hat{j}+\mu\hat{k}\right)=\vec{0}$ [4]

20. Find the shortest distance between the lines, whose equations are [4]

$$\frac{x-8}{3}=\frac{y+9}{-16}=\frac{10-z}{-7} \text{ and } \frac{x-15}{3}=\frac{58-2y}{-16}=\frac{z-5}{-5}$$

21. Using properties of determinants prove that [4]

$$\begin{vmatrix} 1 & a^2+bc & a^3 \\ 1 & b^2+ca & b^3 \\ 1 & c^2+ab & c^3 \end{vmatrix} = -(a-b)(b-c)(c-a)\left(a^2+b^2+c^2\right)$$

22. A football match may be either won, drawn or lost by the host country's team. So there are three ways of forecasting the result of any one match, one correct and two incorrect. Find the probability of forecasting at least three correct results for four matches. [4]

SECTION - C

23. Find the area of the region $\left\{(x,y):x^2\le y\le |x|\right\}$ [6]

Or

Evaluate the following integral as a limit of a sum

$$\int_1^5\left(6x^2-2x-7\right)dx$$

24. Find the equation of the plane containing the lines, [6]

$\vec{r}=\hat{i}+\hat{j}+\lambda\left(\hat{i}+2\hat{j}-\hat{k}\right)$ and $\vec{r}=2\hat{i}+\hat{j}+\hat{k}-\mu\left(-\hat{i}+\hat{j}-2\hat{k}\right)$.

Find the distance of this plane from origin and also from the point (2,-2,3).

25. Evaluate $\int\frac{(3\sin\varnothing-2)\cos\varnothing}{5-\cos^2\varnothing-4\sin\varnothing}$ [6]

26. A company uses three machines to manufacture and sell two types of shirts-half sleeves and full sleeves. Machines M_1, M_2 and M_3 take 1 hour, 2 hours and $1\frac{3}{5}$ hours to make a half sleeve shirt and 2 hours, 1 hours and $1\frac{3}{5}$ hours to make a full sleeve shirt. The profit on each half sleeve shirt is Rs. 1.00 and on a full sleeve shirt is Rs. 1.50. No machine can work for more than 40 hours per week. How many shirts of each type should be made to maximize the company's profit? Solve the problem graphically. [6]

27. For matrix $\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$, find A^{-1} and hence solve the system of equations [6]

X + 2y + z = 4, -x+ y + z = 0, x- 3y + z = 2

Or

Using elementary transformations, find the inverse of the following matrix:

$$\begin{bmatrix} 3 & 2 & 3 \\ 5 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix}$$

28. An insurance Company insured 3000 scooter drivers, 4000 car drivers and 5000 truck drivers. The probability of an accident involving a scooter, a car and a truck is 0.02, 0.03 and 0.1 respectively. If a driver meets an accident, What is the chance that the person is scooter driver? [6]

29. A square piece of tin of side 48 cm is to be made into a box without top, by cutting a square from each corner and folding up the flaps to from the box. What should be the side of the square to be cut off, so that the volume of the box is the maximum possible? Also find the maximum volume. [6]